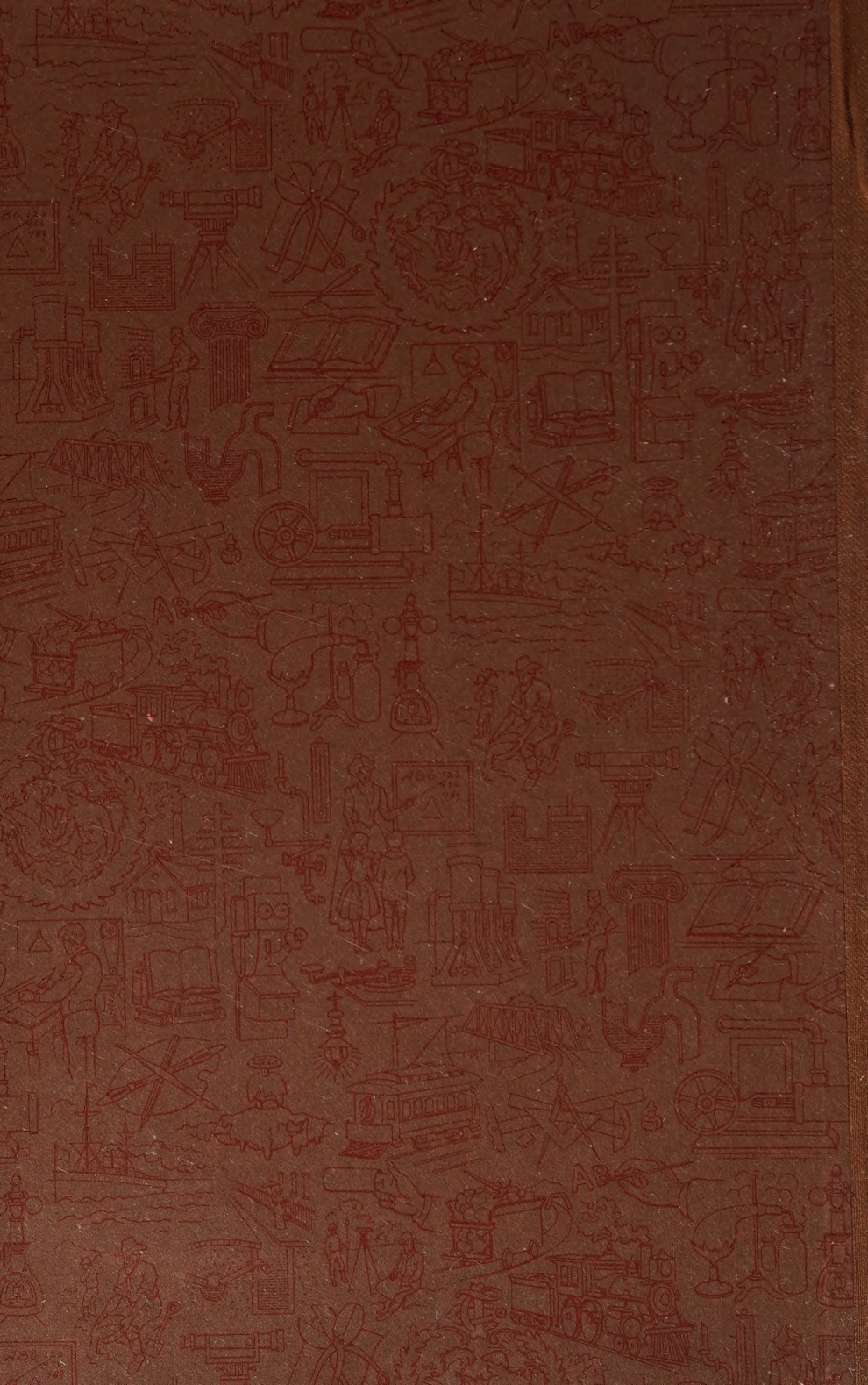
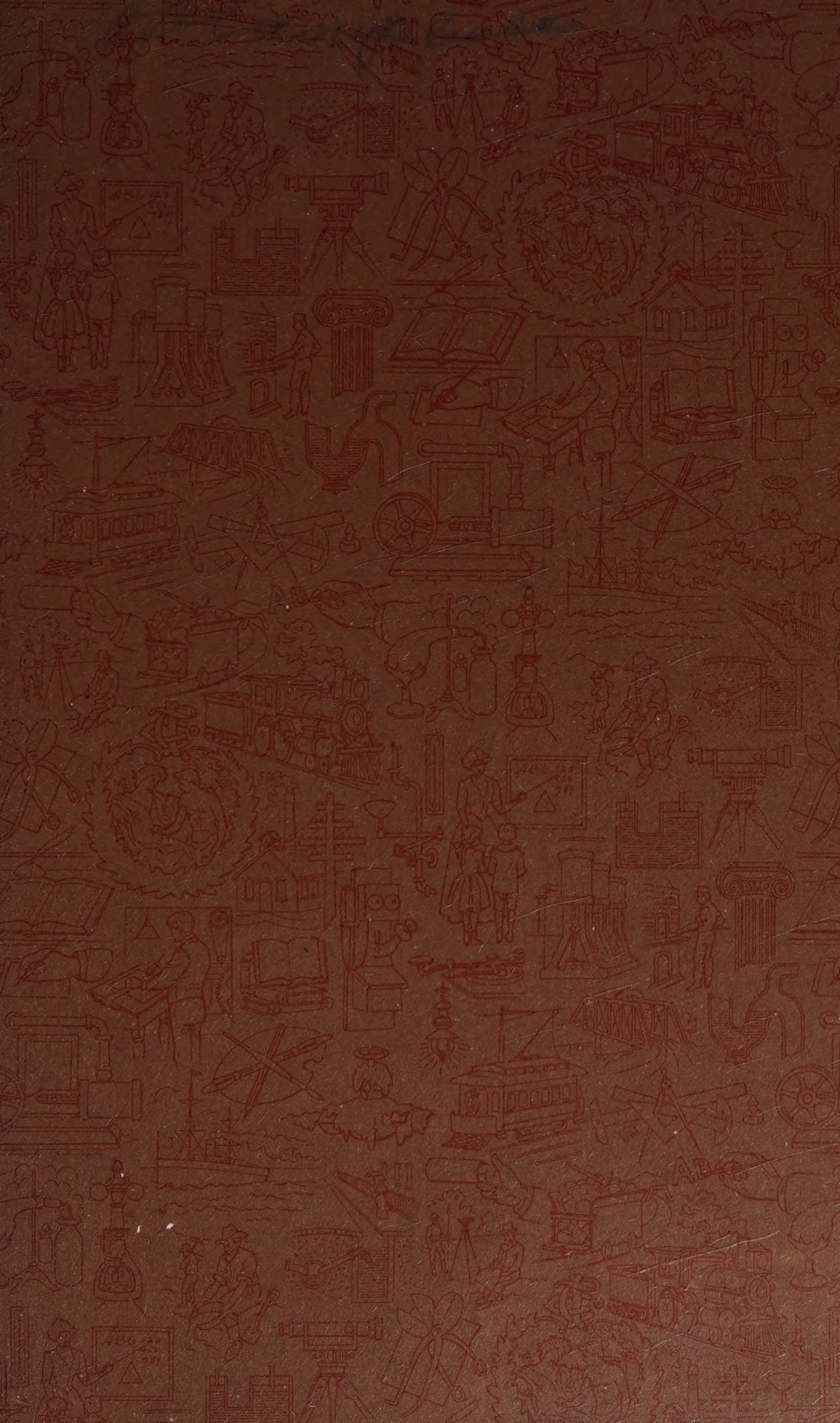




Digitized by the Internet Archive
in 2025

https://archive.org/details/textbookonarchit0000vari

A TEXTBOOK

ON

ARCHITECTURE AND BUILDING CONSTRUCTION

INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

ARITHMETIC
FORMULAS
GEOMETRY AND MENSURATION
ARCHITECTURAL ENGINEERING

WITH PRACTICAL QUESTIONS AND EXAMPLES

SCRANTON
INTERNATIONAL TEXTBOOK CO.
A-3

7108

GEORGE E. MAYCOCK

Copyright, 1897, 1898, 1899, by THE COLLIERY ENGINEER COMPANY, under the title of A TREATISE ON ARCHITECTURE AND BUILDING CONSTRUCTION.

Arithmetic, Sections 1, 2, 3, 4, and 6: Copyright, 1892, 1893, 1894, 1895, 1896, 1898, 1899, by THE COLLIERY ENGINEER COMPANY.

Arithmetic, Section 5: Copyright, 1902, by INTERNATIONAL TEXTBOOK COMPANY. Entered at Stationers' Hall, London.

Formulas: Copyright, 1898, 1899, by THE COLLIERY ENGINEER COMPANY.

Geometry and Mensuration: Copyright, 1899, by THE COLLIERY ENGINEER COMPANY.

Architectural Engineering: Copyright, 1898, 1899, 1900, by THE COLLIERY ENGINEER COMPANY.

All rights reserved.

bi

Press of EATON & MAINS

NEW YORK

PREFACE.

In the first six volumes of the seven volumes of this set are comprised all the Instruction Papers and Examination Questions used in our Complete Architectural Course; they form a thorough, progressive, and comprehensive treatise on the subject of Architecture.

While the individual Instruction Papers are not in themselves exhaustive in their treatment of the particular subjects named in their titles, yet they are so closely interrelated, one with another, that when they are joined together in one harmonious whole (as in these volumes), they constitute a treatise that is complete in all those details of design and construction that are likely to be met with in general architectural practice. The student can therefore use them as works of reference in connection with any of the numerous problems that so frequently arise in all branches of architectural work.

The method of numbering the pages, cuts, articles, etc. is such that each paper and part is complete in itself; hence, in order to make the indexes intelligible, it was necessary to give each paper and part a number. This number is placed at the top of each page, on the headline, opposite the page number; and to distinguish it from the page number, it is preceded by the printer's section mark §. Consequently, a reference such as § 8, page 29, would be readily found by looking along the headlines until § 8 is found, and then through § 8 until page 29 is found.

The pages of the Examination Questions are given the same section numbers as the Instruction Papers to which

they belong, and are grouped together at the end of the volumes containing the Instruction Papers to which they refer.

The volumes of the present Course, the Complete Architectural, are seven in number:

The first volume in the order of study contains the Instruction and Question Papers on Arithmetic, Formulas, Geometry and Mensuration, and Architectural Engineering.

The second volume in the order of study contains the Instruction and Question Papers on Masonry, Carpentry, and Joinery.

The third volume in the order of study contains the Instruction and Question Papers on Stair Building, Ornamental Ironwork, Roofing, Sheet-Metal Work, and Electric-Light Wiring and Bellwork.

The fourth volume in the order of study contains the Instruction and Question Papers on Plumbing and Gas-Fitting, Heating and Ventilation, Painting and Decorating, and Estimating and Calculating Quantities.

The fifth volume in the order of study contains the Instruction and Question Papers on History of Architecture, Architectural Design, Specifications, Building Superintendence, and Contracts and Permits.

Of the two remaining volumes, one contains the Drawing Plates and the instructions for drawing them. Nothing equal to this volume has ever before been published. It forms a complete course in Architectural Drawing.

The other volume contains the tables and formulas given in the various Instruction Papers, conveniently arranged for reference, so that the student can save the labor and time of hunting them out in the Instruction Papers.

This volume also contains the answers to the questions and solutions to the examples in the Question Papers. Whenever it has been deemed inadvisable to answer a question, a reference to the proper article in the Instruction Paper has been given, the reading of which will enable the student to answer the question himself.

INTERNATIONAL CORRESPONDENCE SCHOOLS.

CONTENTS.

	Section.	Page.
ARITHMETIC.		
Definitions	1	1
Notation and Numeration	1	1
Addition	1	4
Subtraction	1	9
Multiplication	1	11
Division	1	16
Cancelation	1	19
Fractions	1	22
Decimals	1	35
Symbols of Aggregation	1	49
Percentage	2	1
Denominate Numbers	2	7
Involution	2	23
Evolution	2	25
Ratio	2	54
Proportion	2	58
FORMULAS.		
Use and Application of Formulas	3	1
GEOMETRY AND MENSURATION.		
Lines and Angles	4	1
Plane Figures	4	6
The Triangle	4	7
The Circle	4	16
Inscribed and Circumscribed Polygons	4	21

GEOMETRY AND MENSURATION—*Continued.*	*Section.*	*Page.*
Mensuration	4	23
Conversion Tables	4	25
Mensuration of Plane Surfaces	4	26
Mensuration of Solids	4	44
Symmetrical and Similar Figures	4	54
ARCHITECTURAL ENGINEERING.		
Introduction	5	1
The Elements of Mechanics	5	3
Definitions	5	3
Effects of a Force	5	3
Composition of Forces	5	6
Resolution of Forces	5	14
Equilibrium	5	16
Moments of Forces	5	17
The Lever	5	23
Center of Gravity	5	26
Loads Carried by Structures	5	27
Stresses and Strains	5	39
Strength of Building Materials	5	42
Foundations	5	48
Columns	5	53
Beams	5	62
Reactions	5	62
Stresses in Beams	5	69
Strength of Beams	5	84
Trussed Beams	5	101
Deflection of Floorbeams	5	110
Graphical Statics	5	111
Design for a Large Building	5	142
Properties of Sections	6	1
The Neutral Axis	6	1
The Moment of Inertia	6	8
Resisting Moment	6	14
Radius of Gyration	6	15
Steel Columns	6	16
Strength of Rivets and Pins	6	43

ARCHITECTURAL ENGINEERING—*Continued.*	*Section.*	*Page.*
Plate Girders	6	57
Deflection of Beams	6	102
Flitch Plate Girders	6	106
Roof Trusses	6	110
Determination of Stresses in the Fink Truss	6	110
Design of a Composite Pin-Connected Roof Truss	6	118
Design of a Structural Steel Roof Truss	6	124
General Notes Regarding the Design of a Roof Truss	6	131
Elements of Usual Sections: Table	6	135
Value of Rivets: Table	6	136
Areas of Angles: Table	6	137
Properties of Angles, Equal Legs: Table	6	138
Properties of Angles, Unequal Legs: Table	6	140
Properties of Z Bars: Table	6	141
Properties of I Beams: Table	6	143
Properties of Channels: Table	6	144
Radii of Gyration for Two Angles: Table	6	145
Moduli of Elasticity: Table	6	148
Resisting Moments of Pins: Table	6	149
Deflection of Beams: Table	6	152

EXAMINATION QUESTIONS.

ARITHMETIC.

(SECTION 1.)

DEFINITIONS.

1. **Arithmetic** is the art of reckoning, or the study of numbers.

2. A **unit** is *one*, or a single thing, as *one*, *one* boy, *one* horse, *one* dozen.

3. A **number** is a unit or a collection of units, as *one*, *three* apples, *five* boys.

4. The **unit of a number** is one of the collection of units which constitutes the number. Thus, the unit of *twelve* is *one*, of *twenty* dollars is *one* dollar.

5. A **concrete number** is a number applied to some particular kind of object or quantity, as three *horses*, five *dollars*, ten *pounds*.

6. An **abstract number** is a number that is not applied to any object or quantity, as *three*, *five*, *ten*.

7. **Like numbers** are numbers which express units of the *same kind*, as six *days* and ten *days*, two *feet* and five *feet*.

8. **Unlike numbers** are numbers which express units of *different kinds*, as ten *months* and eight *miles*, seven *dollars* and five *feet*.

NOTATION AND NUMERATION.

9. Numbers are expressed in three ways: (1) by words; (2) by figures; (3) by letters.

10. **Notation** is the art of expressing numbers by figures or letters.

11. **Numeration** is the art of reading the numbers which have been expressed by figures or letters.

For notice of copyright, see page immediately following the title page.

12. The **Arabic notation** is the method of expressing numbers by figures. This method employs ten different **figures** to represent numbers, viz.:

Figures	0	1	2	3	4	5	6	7	8	9
Names	*naught, cipher, or zero*	*one*	*two*	*three*	*four*	*five*	*six*	*seven*	*eight*	*nine*

The first character (0) is called **naught, cipher,** or **zero,** and when standing alone has no value.

The other nine figures are called **digits,** and each has a value of its own.

Any whole number is called an **integer.**

13. As there are only ten *figures* used in expressing numbers, each figure must have a different *value* at different times.

14. The value of a figure depends upon its *position* in relation to others.

15. Figures have **simple** values and **local,** or **relative,** values.

16. The **simple** value of a figure is the value it expresses when standing alone.

17. The **local,** or **relative,** value of a figure is the *increased* value it expresses by having other figures placed on its right.

For instance, if we see the figure 6 standing alone, thus 6

we consider it as *six units*, or simply **six.**

Place another 6 to the *left* of it; thus 66

The original figure is still *six units*, but the second figure is *ten times* 6, or 6 **tens.**

If a third 6 be now placed still one place further to the *left*, it is increased in value *ten times* more, thus making it 6 **hundreds** 666

A fourth 6 would be 6 **thousands** 6666

A fifth 6 would be 6 **tens of thousands,** or **sixty thousands** 66666

A sixth 6 would be 6 **hundreds of thousands** . 666666

A seventh 6 would be 6 **millions** 6666666

The entire line of seven figures is read *six millions six hundred sixty-six thousands six hundred sixty-six.*

18. The **increased value** of each of these figures is its *local*, or *relative*, value. Each figure is *ten times* greater in value than the one immediately on its *right.*

19. The **cipher** (0) has no value in itself, but it is useful in determining the place of other figures. To represent the number *four hundred five*, two digits only are necessary, one to represent *four hundred*, and the other to represent *five units;* but if these two digits are placed together, as 45, the 4 (being in the second place) will mean 4 *tens.* To mean 4 *hundreds*, the 4 should have two figures on its right, and a *cipher* is therefore inserted in the place usually given to *tens*, to show that the number is composed of *hundreds* and *units* only, and that there are no *tens.* *Four hundred five* is therefore expressed as 405. If the number were *four thousand and five*, two ciphers would be inserted; thus, 4005. If it were *four hundred fifty*, it would have the *cipher* at the right-hand side to show that there were no *units*, and only *hundreds and tens;* thus, 450. *Four thousand and fifty* would be expressed 4050, the first cipher indicating that there are no units and the second that there are no hundreds.

20. In *reading* numbers that have been represented by figures, it is usual to *point off* the number into groups of three figures each, beginning with the right-hand, or **units,** column, a comma (,) being used to point off these groups.

Billions.			*Millions.*			*Thousands.*			*Units.*		
Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds of Units.	Tens of Units.	Units.
4	3	2 ,	1	9	8 ,	7	6	5 ,	4	3	2

In *pointing off* these figures, begin at the right-hand figure and count—*units*, *tens*, *hundreds;* the next group of three figures is *thousands;* therefore, we insert a comma (,) before beginning with them. Beginning at the figure 5, we say *thousands*, *tens of thousands*, *hundreds of thousands*, and insert another comma. We next read *millions*, *tens of millions*, *hundreds of millions* (insert another comma), *billions*, *tens of billions*, *hundreds of billions.*

The entire line of figures would be read: *four hundred thirty-two billions one hundred ninety-eight millions seven hundred sixty-five thousands four hundred thirty-two.* When we thus *read* a line of figures it is called **numeration,** and if the **numeration** be changed back to *figures*, it is called **notation.**

For instance, the writing of the following figures,

72,584,623,

would be the **notation,** and the **numeration** would be *seventy-two millions five hundred eighty-four thousands six hundred twenty-three.*

21. Note.—It is customary to leave the *s* off the words millions, thousands, etc., in cases like the above, both in speaking and writing; hence, the above would usually be expressed seventy-two million five hundred eighty-four thousand six hundred twenty-three.

22. The four fundamental processes of arithmetic are **addition, subtraction, multiplication,** and **division.** They are called fundamental processes because all operations in arithmetic are based upon them.

ADDITION.

23. **Addition** is the *process of finding* the *sum of two or more numbers.* The sign of addition is +. It is read *plus*, and means *more.* Thus, 5 + 6 is read *5 plus 6*, and means that 5 and 6 are to be added.

24. The sign of equality is =. It is read *equals* or *is equal to.* Thus, 5 + 6 = 11 may be read *5 plus 6 equals 11.*

25. *Like* numbers can be added, but *unlike* numbers cannot be added. Thus, 6 dollars can be added to 7 dollars, and the sum will be 13 dollars; but 6 dollars cannot be added to 7 feet.

26. The following table gives the sum of any two numbers from 1 to 12:

1 and 1 is 2	2 and 1 is 3	3 and 1 is 4	4 and 1 is 5
1 and 2 is 3	2 and 2 is 4	3 and 2 is 5	4 and 2 is 6
1 and 3 is 4	2 and 3 is 5	3 and 3 is 6	4 and 3 is 7
1 and 4 is 5	2 and 4 is 6	3 and 4 is 7	4 and 4 is 8
1 and 5 is 6	2 and 5 is 7	3 and 5 is 8	4 and 5 is 9
1 and 6 is 7	2 and 6 is 8	3 and 6 is 9	4 and 6 is 10
1 and 7 is 8	2 and 7 is 9	3 and 7 is 10	4 and 7 is 11
1 and 8 is 9	2 and 8 is 10	3 and 8 is 11	4 and 8 is 12
1 and 9 is 10	2 and 9 is 11	3 and 9 is 12	4 and 9 is 13
1 and 10 is 11	2 and 10 is 12	3 and 10 is 13	4 and 10 is 14
1 and 11 is 12	2 and 11 is 13	3 and 11 is 14	4 and 11 is 15
1 and 12 is 13	2 and 12 is 14	3 and 12 is 15	4 and 12 is 16
5 and 1 is 6	6 and 1 is 7	7 and 1 is 8	8 and 1 is 9
5 and 2 is 7	6 and 2 is 8	7 and 2 is 9	8 and 2 is 10
5 and 3 is 8	6 and 3 is 9	7 and 3 is 10	8 and 3 is 11
5 and 4 is 9	6 and 4 is 10	7 and 4 is 11	8 and 4 is 12
5 and 5 is 10	6 and 5 is 11	7 and 5 is 12	8 and 5 is 13
5 and 6 is 11	6 and 6 is 12	7 and 6 is 13	8 and 6 is 14
5 and 7 is 12	6 and 7 is 13	7 and 7 is 14	8 and 7 is 15
5 and 8 is 13	6 and 8 is 14	7 and 8 is 15	8 and 8 is 16
5 and 9 is 14	6 and 9 is 15	7 and 9 is 16	8 and 9 is 17
5 and 10 is 15	6 and 10 is 16	7 and 10 is 17	8 and 10 is 18
5 and 11 is 16	6 and 11 is 17	7 and 11 is 18	8 and 11 is 19
5 and 12 is 17	6 and 12 is 18	7 and 12 is 19	8 and 12 is 20
9 and 1 is 10	10 and 1 is 11	11 and 1 is 12	12 and 1 is 13
9 and 2 is 11	10 and 2 is 12	11 and 2 is 13	12 and 2 is 14
9 and 3 is 12	10 and 3 is 13	11 and 3 is 14	12 and 3 is 15
9 and 4 is 13	10 and 4 is 14	11 and 4 is 15	12 and 4 is 16
9 and 5 is 14	10 and 5 is 15	11 and 5 is 16	12 and 5 is 17
9 and 6 is 15	10 and 6 is 16	11 and 6 is 17	12 and 6 is 18
9 and 7 is 16	10 and 7 is 17	11 and 7 is 18	12 and 7 is 19
9 and 8 is 17	10 and 8 is 18	11 and 8 is 19	12 and 8 is 20
9 and 9 is 18	10 and 9 is 19	11 and 9 is 20	12 and 9 is 21
9 and 10 is 19	10 and 10 is 20	11 and 10 is 21	12 and 10 is 22
9 and 11 is 20	10 and 11 is 21	11 and 11 is 22	12 and 11 is 23
9 and 12 is 21	10 and 12 is 22	11 and 12 is 23	12 and 12 is 24

This table should be carefully committed to memory. Since 0 has no value, the sum of any number and 0 is the number itself; thus 17 and 0 is 17.

27. For addition, place the numbers to be added directly under each other, taking care to place *units* under *units*, *tens* under *tens*, *hundreds* under *hundreds*, and so on.

When the numbers are thus written, the *right-hand figure of one number is placed directly under the right-hand figure of the one above it*, thus bringing units under units, tens under tens, etc. Proceed as in the following examples:

28. EXAMPLE.—What is the sum of 131, 222, 21, 2, and 413?

SOLUTION.—

$$\begin{array}{r} 131 \\ 222 \\ 21 \\ 2 \\ 413 \\ \hline \textit{sum}\ 789 \end{array}$$ Ans.

EXPLANATION. — After placing the numbers in proper order, begin at the bottom of the right-hand, or units, column, and add, mentally repeating the different sums. Thus, three and two are five and one are six and two are eight and one are nine, the sum of the numbers in units column. Place the 9 directly beneath as the first, or units, figure in the sum.

The sum of the numbers in the next, or tens, column equals 8 tens, which is the second, or tens, figure in the sum.

The sum of the numbers in the next, or hundreds, column equals 7 hundreds, which is the third, or hundreds, figure in the sum.

The sum, or answer, is 789.

29. EXAMPLE.—What is the sum of 425, 36, 9,215, 4, and 907?

SOLUTION.—

$$\begin{array}{r} 425 \\ 36 \\ 9215 \\ 4 \\ 907 \\ \hline 27 \\ 60 \\ 1500 \\ 9000 \\ \hline \textit{sum}\ 10587 \end{array}$$ Ans.

EXPLANATION.—The sum of the numbers in the first, or

units, column is seven and four are eleven and five are sixteen and six are twenty-two and five are twenty-seven, or 27 units; i. e., two tens and seven units. Write 27 as shown. The sum of the numbers in the second, or tens, column is six tens, or 60. Write 60 underneath 27, as shown. The sum of the numbers in the third, or hundreds, column is 15 hundreds, or 1,500. Write 1,500 under the two preceding results as shown. There is only one number in the fourth, or thousands, column, 9, which represents 9,000. Write 9,000 under the three preceding results. Adding these four results, the sum is 10,587, which is the sum of 425, 36, 9,215, 4, and 907.

NOTE.—It frequently happens when adding a long column of figures, that the sum of two numbers, one of which does not occur in the addition table, is required. Thus, in the first column above, the sum of 16 and 6 was required. We know from the table that $6+6=12$; hence, the first figure of the sum is 2. Now, the sum of any number less than 20 and of any number less than 10 must be less than 30, since $20+10=30$; therefore, the sum is 22. Consequently, in cases of this kind, add the first figure of the larger number to the smaller number, and if the result is greater than 9, increase the second figure of the larger number by 1. Thus, $44+7=?$ $4+7=11$; hence, $44+7=51$.

30. The addition may also be performed as follows:

	425
	36
	9215
	4
	907
sum	10587 Ans.

EXPLANATION.—The sum of the numbers in units column is 27 units, or 2 tens and 7 units. Write the 7 units as the first, or right-hand, figure in the sum. Reserve the two tens and add them to the figures in tens column. The sum of the figures in the tens column, plus the 2 tens reserved and carried from the units column, is 8, which is written down as the second figure in the sum. There is nothing to carry to the next column, because 8 is less than 10. The sum of the numbers in the next column is 15 hundreds, or 1 thousand and 5 hundreds. Write down the 5 as the third, or hundreds, figure in the sum and carry the 1 to the next

column. 1 + 9 = 10, which is written down at the left of the other figures.

The second method saves space and figures, but the first is to be preferred when adding a long column.

31. EXAMPLE.—Add the numbers in the column below:

SOLUTION.—

	890
	82
	90
	393
	281
	80
	770
	83
	492
	80
	383
	84
	191
sum	3899 Ans.

EXPLANATION.—The sum of the digits in the first column equals 19 units, or 1 ten and 9 units. Write down the 9 and carry 1 to the next column. The sum of the digits in the second column + 1 is 109 tens, or 10 hundreds and 9 tens. Write down the 9 and carry the 10 to the next column. The sum of the digits in this column plus the 10 reserve is 38.

The entire sum is 3,899.

32. **Rule.—I.** *Begin at the right, add each column separately, and write the sum, if it be only one figure, under the column added.*

II. *If the sum of any column consists of two or more figures, put the right-hand figure of the sum under that column and add the remaining figure or figures to the next column.*

33. **Proof.**—*To prove addition, add each column from top to bottom. If you obtain the same result as by adding from bottom to top, the work is probably correct.*

EXAMPLES FOR PRACTICE.

34. Find the sum of:

(*a*)	104 + 203 + 613 + 214.	Ans.	(*a*) 1,134.
(*b*)	1,875 + 3,143 + 5,826 + 10,832.		(*b*) 21,676.
(*c*)	4,865 + 2,115 + 8,173 + 40,084.		(*c*) 55,267.
(*d*)	14,204 + 8,173 + 1,065 + 10,042.		(*d*) 33,484.
(*e*)	10,832 + 4,145 + 3,133 + 5,872.		(*e*) 23,982.
(*f*)	214 + 1,231 + 141 + 5,000.		(*f*) 6,586.
(*g*)	123 + 104 + 425 + 126 + 327.		(*g*) 1,105.
(*h*)	6,354 + 2,145 + 2,042 + 1,111 + 3,333.		(*h*) 14,985.

SUBTRACTION.

35. In arithmetic, **subtraction** is the process of finding how much greater one number is than another.

The greater of the two numbers is called the **minuend.**

The smaller of the two numbers is called the **subtrahend.**

The number left after subtracting the subtrahend from the minuend is called the **difference,** or **remainder.**

36. The sign of subtraction is −. It is read *minus*, and means *less*. Thus, 12 − 7 is read *12 minus 7*, and means that 7 is to be taken from 12.

37. EXAMPLE.—From 7,568 take 3,425.

SOLUTION.—

minuend	7 5 6 8	
subtrahend	3 4 2 5	
remainder	4 1 4 3	Ans.

EXPLANATION.—Begin at the right-hand, or units, column and subtract in succession each figure in the subtrahend from the one directly above it in the minuend, and write the remainders below the line. The result is the entire remainder.

38. When there are more figures in the minuend than in the subtrahend, and when some figures in the minuend are *less* than the figures directly under them in the subtrahend, proceed as in the following example:

EXAMPLE.—From 8,453 take 844.

SOLUTION.—

minuend	8 4 5 3	
subtrahend	8 4 4	
remainder	7 6 0 9	Ans.

EXPLANATION.—Begin at the right-hand, or units, column to subtract. We cannot take 4 from 3, and must, therefore, borrow 1 from 5 in tens column and annex it to the 3 in units column. The 1 ten = 10 units, which added to the 3 in units column = 13 units. 4 from 13 = 9, the first, or units, figure in the remainder.

Since we borrowed 1 from the 5, only 4 remains; 4 from 4 = 0, the second, or tens, figure. We cannot take 8 from 4, and must, therefore, borrow 1 from 8 in thousands column. Since 1 thousand = 10 hundreds, 10 hundreds + 4 hundreds = 14 hundreds, and 8 from 14 = 6, the third, or hundreds, figure in the remainder.

Since we borrowed 1 from 8, only 7 remains, from which there is nothing to subtract; therefore, 7 is the next figure in the remainder, or answer.

The operation of borrowing is performed by mentally placing 1 before the figure following the one from which it is borrowed. In the above example the 1 borrowed from 5 is placed before 3, making it 13, from which we subtract 4. The 1 borrowed from 8 is placed before 4, making 14, from which 8 is taken.

39. EXAMPLE.—Find the difference between 10,000 and 8,763.

SOLUTION.—

minuend	10000
subtrahend	8763
remainder	1237 Ans.

EXPLANATION.—In the above example we borrow 1 from the second column and place it before 0, making 10; 3 from 10 = 7. In the same way we borrow 1 and place it before the next cipher, making 10; but as we have borrowed 1 from this column and have taken it to the units column, only 9 remains from which to subtract 6; 6 from 9 = 3. For the same reason we subtract 7 from 9 and 8 from 9 for the next two figures, and obtain a total remainder of 1,237.

40. **Rule.**—*Place the subtrahend (or smaller number) under the minuend (or larger number), in the same manner as for addition, and proceed as in Arts.* **37, 38,** *and* **39.**

41. Proof.—*To prove an example in subtraction, add the subtrahend and the remainder. The sum should equal the minuend. If it does not, a mistake has been made, and the work should be done over.*

Proof of the above example:

subtrahend	8763
remainder	1237
minuend	10000

EXAMPLES FOR PRACTICE.

42. From:

(*a*)	94,278 take 62,574.	(*a*)	31,704.
(*b*)	53,714 take 25,824.	(*b*)	27,890.
(*c*)	71,832 take 58,109.	(*c*)	13,723.
(*d*)	20,804 take 10,408.	(*d*)	10,396.
(*e*)	310,465 take 102,141.	(*e*)	208,324.
(*f*)	(81,043 + 1,041) take 14,831.	(*f*)	67,253.
(*g*)	(20,482 + 18,216) take 21,214.	(*g*)	17,484.
(*h*)	(2,040 + 1,213 + 542) take 3,791.	(*h*)	4.

Ans.

MULTIPLICATION.

43. To **multiply** a number is to *add* it to itself a certain number of times.

44. **Multiplication** is the process of multiplying one number by another.

The number thus added to itself, or the number to be multiplied, is called the **multiplicand.**

The number which shows how many times the *multiplicand* is to be taken, or the number by which we *multiply*, is called the **multiplier.**

The result obtained by multiplying is called the **product.**

45. The sign of multiplication is $\times$. It is read *times* or *multiplied by*. Thus, 9×6 is read *9 times 6*, or *9 multiplied by 6*.

46. It matters not in what order the numbers to be multiplied together are placed. Thus, 6×9 is the same as 9×6.

47. In the following table, the product of any two numbers (neither of which exceeds 12) may be found:

1 times 1 is 1	2 times 1 is 2	3 times 1 is 3
1 times 2 is 2	2 times 2 is 4	3 times 2 is 6
1 times 3 is 3	2 times 3 is 6	3 times 3 is 9
1 times 4 is 4	2 times 4 is 8	3 times 4 is 12
1 times 5 is 5	2 times 5 is 10	3 times 5 is 15
1 times 6 is 6	2 times 6 is 12	3 times 6 is 18
1 times 7 is 7	2 times 7 is 14	3 times 7 is 21
1 times 8 is 8	2 times 8 is 16	3 times 8 is 24
1 times 9 is 9	2 times 9 is 18	3 times 9 is 27
1 times 10 is 10	2 times 10 is 20	3 times 10 is 30
1 times 11 is 11	2 times 11 is 22	3 times 11 is 33
1 times 12 is 12	2 times 12 is 24	3 times 12 is 36
4 times 1 is 4	5 times 1 is 5	6 times 1 is 6
4 times 2 is 8	5 times 2 is 10	6 times 2 is 12
4 times 3 is 12	5 times 3 is 15	6 times 3 is 18
4 times 4 is 16	5 times 4 is 20	6 times 4 is 24
4 times 5 is 20	5 times 5 is 25	6 times 5 is 30
4 times 6 is 24	5 times 6 is 30	6 times 6 is 36
4 times 7 is 28	5 times 7 is 35	6 times 7 is 42
4 times 8 is 32	5 times 8 is 40	6 times 8 is 48
4 times 9 is 36	5 times 9 is 45	6 times 9 is 54
4 times 10 is 40	5 times 10 is 50	6 times 10 is 60
4 times 11 is 44	5 times 11 is 55	6 times 11 is 66
4 times 12 is 48	5 times 12 is 60	6 times 12 is 72
7 times 1 is 7	8 times 1 is 8	9 times 1 is 9
7 times 2 is 14	8 times 2 is 16	9 times 2 is 18
7 times 3 is 21	8 times 3 is 24	9 times 3 is 27
7 times 4 is 28	8 times 4 is 32	9 times 4 is 36
7 times 5 is 35	8 times 5 is 40	9 times 5 is 45
7 times 6 is 42	8 times 6 is 48	9 times 6 is 54
7 times 7 is 49	8 times 7 is 56	9 times 7 is 63
7 times 8 is 56	8 times 8 is 64	9 times 8 is 72
7 times 9 is 63	8 times 9 is 72	9 times 9 is 81
7 times 10 is 70	8 times 10 is 80	9 times 10 is 90
7 times 11 is 77	8 times 11 is 88	9 times 11 is 99
7 times 12 is 84	8 times 12 is 96	9 times 12 is 108
10 times 1 is 10	11 times 1 is 11	12 times 1 is 12
10 times 2 is 20	11 times 2 is 22	12 times 2 is 24
10 times 3 is 30	11 times 3 is 33	12 times 3 is 36
10 times 4 is 40	11 times 4 is 44	12 times 4 is 48
10 times 5 is 50	11 times 5 is 55	12 times 5 is 60
10 times 6 is 60	11 times 6 is 66	12 times 6 is 72
10 times 7 is 70	11 times 7 is 77	12 times 7 is 84
10 times 8 is 80	11 times 8 is 88	12 times 8 is 96
10 times 9 is 90	11 times 9 is 99	12 times 9 is 108
10 times 10 is 100	11 times 10 is 110	12 times 10 is 120
10 times 11 is 110	11 times 11 is 121	12 times 11 is 132
10 times 12 is 120	11 times 12 is 132	12 times 12 is 144

This table should be carefully committed to memory.

Since 0 has no value, the product of 0 and any number is 0.

48. To multiply a number by one figure only:

EXAMPLE.—Multiply 425 by 5.

SOLUTION.—

multiplicand	425	
multiplier	5	
product	2125	Ans.

EXPLANATION.—For convenience, the multiplier is generally written under the right-hand figure of the multiplicand. On looking in the multiplication table, we see that 5×5 are 25. Multiplying the first figure at the right of the multiplicand, or 5, by the multiplier, 5, it is seen that 5 times 5 units are 25 units, or 2 tens and 5 units. Write the 5 units in units place in the product, and reserve the 2 tens to add to the product of tens. Looking in the multiplication table again, we see that 5×2 are 10. Multiplying the second figure of the multiplicand by the multiplier, 5, we see that 5 times 2 tens are 10 tens, and 10 tens plus the 2 tens reserved are 12 tens, or 1 hundred plus 2 tens. Write the 2 tens in tens place, and reserve the 1 hundred to add to the product of hundreds. Again, we see by the multiplication table that 5×4 are 20. Multiplying the third, or last, figure of the multiplicand by the multiplier, 5, we see that 5 times 4 hundreds are 20 hundreds, and 20 hundreds plus the 1 hundred reserved are 21 hundreds, or 2 thousands and 1 hundred, which we write in thousands and hundreds places, respectively.

Hence, the product is 2,125.

This result is the same as adding 425 five times. Thus,

	425
	425
	425
	425
	425
sum	2125 Ans.

EXAMPLES FOR PRACTICE.

49. Find the product of:

(*a*) $61,483 \times 6$.
(*b*) $12,375 \times 5$.
(*c*) $10,426 \times 7$.
(*d*) $10,835 \times 3$.

Ans. (*a*) 368,898. (*b*) 61,875. (*c*) 72,982. (*d*) 32,505.

(*e*)	98,376 × 4.	Ans.	(*e*)	393,504.
(*f*)	10,873 × 8.		(*f*)	86,984.
(*g*)	71,543 × 9.		(*g*)	643,887.
(*h*)	218,734 × 2.		(*h*)	437,468.

50. To multiply a number by two or more figures:

EXAMPLE.—Multiply 475 by 234.

SOLUTION.—

multiplicand	475
multiplier	234
	1900
	1425
	950
product	111150 Ans.

EXPLANATION.—For convenience, the multiplier is generally written under the multiplicand, placing units under units, tens under tens, etc.

We cannot multiply by 234 at one operation; we must, therefore, multiply by the *parts* and then *add* the **partial products.**

The parts by which we are to multiply are 4 units, 3 tens, and 2 hundreds. 4 times 475 = 1,900, the *first partial product;* 3 times 475 = 1,425, the *second partial product, the right-hand figure of which is written directly under the figure multiplied by*, or 3; 2 times 475 = 950, the *third partial product*, the right-hand figure of which is written directly under the figure multiplied by, or 2.

The sum of these three partial products is 111,150, which is the *entire product.*

51. Rule.—I. *Write the multiplier under the multiplicand, so that units are under units, tens under tens, etc.*

II. *Begin at the right and multiply each figure of the multiplicand by each successive figure of the multiplier, placing the right-hand figure of each partial product directly under the figure used as a multiplier.*

III. *The sum of the partial products will equal the required product.*

52. Proof.—*Review the work carefully, or multiply the multiplier by the multiplicand; if the results agree, the work is correct.*

53. When there is a *cipher* in the *multiplier*, multiply by it the same as with the other figures. Thus,

(*a*)	(*b*)	(*c*)	(*d*)
0	2	15	708
× 0	× 0	× 0	× 0
0 Ans.	0 Ans.	00 Ans.	000 Ans.

(*e*)	(*f*)	(*g*)
3114	4008	31264
203	305	1002
9342	20040	62528
0000	0000	00000
6228	12024	00000
632142 Ans.	1222440 Ans.	31264
		31326528 Ans.

When multiplying by a number containing a cipher, the work may be shortened by writing the first cipher of the partial product, then multiplying by the next figure of the multiplier and writing the partial product alongside of the cipher. Thus, examples (*e*) and (*g*) above might have been solved in the following manner:

3114	31264
203	1002
9342	62528
62280	3126400
632142 Ans.	31326528 Ans.

EXAMPLES FOR PRACTICE.

54. Find the product of:

(*a*) 3,842 × 26.
(*b*) 3,716 × 45.
(*c*) 1,817 × 124.
(*d*) 675 × 38.
(*e*) 1,875 × 33.
(*f*) 4,836 × 47.
(*g*) 5,682 × 543.
(*h*) 3,257 × 246.

Ans. (*a*) 99,892. (*b*) 167,220. (*c*) 225,308. (*d*) 25,650. (*e*) 61,875. (*f*) 227,292. (*g*) 3,085,326. (*h*) 801,222.

(*i*) $2,875 \times 302$.
(*j*) $17,819 \times 1,004$.
(*k*) $38,674 \times 205$.
(*l*) $18,304 \times 100$.
(*m*) $7,834 \times 10$.
(*n*) $87,543 \times 1,000$.
(*o*) $48,763 \times 100$.

Ans. (*i*) 868,250. (*j*) 17,890,276. (*k*) 7,928,170. (*l*) 1,830,400. (*m*) 78,340 (*n*) 87,543,000. (*o*) 4,876,300.

DIVISION.

55. **Division** is the process of finding how many times one number is contained in another of the same kind.

The number to be *divided* is called the **dividend.**

The number by which we *divide* is called the **divisor.**

The number which *shows* how many times the *divisor is contained in the dividend* is called the **quotient.**

56. The sign of division is $\div$. It is read *divided by.* $54 \div 9$ is read *54 divided by 9.* Another way to write 54 divided by 9 is $\frac{54}{9}$. Thus, $54 \div 9 = 6$, or $\frac{54}{9} = 6$.

In both of these cases, 54 is the *dividend* and 9 is the *divisor.*

Division is the reverse of multiplication.

57. **To divide when the divisor consists of but one figure,** proceed as in the following example:

EXAMPLE.—What is the quotient of $875 \div 7$?

SOLUTION.—

divisor dividend quotient

$$7)875(125 \text{ Ans.}$$
$$7$$
$$17$$
$$14$$
$$35$$
$$35$$
remainder 0

EXPLANATION.— 7 is contained in 8 hundreds, 1 hundred times. Place the 1 as the first, or left-hand, figure of the quotient. Multiply the divisor, 7, by the 1 hundred of the

quotient, and place the product, 7 hundreds, under the 8 hundreds in the dividend, and subtract. Beside the remainder, 1, bring down the next, or tens, figure of the dividend, in this case 7, making 17 tens; 7 is contained in 17, 2 times. Write the 2 as the second figure of the quotient. Multiply the divisor, 7, by the 2 in the quotient, and subtract the product from 17. Beside the remainder, 3, bring down the units figure of the dividend, making 35 units. 7 is contained in 35, 5 times, which is placed in the quotient. Multiplying the divisor by the last figure of the quotient, 5 times 7 = 35, which subtracted from 35, under which it is placed, leaves 0. Therefore, the quotient is 125. This method is called **long division.**

58. In **short division,** only the divisor, dividend, and quotient are written, the operations being performed mentally.

dividend
divisor 7) 8 1 7 3 5
quotient 1 2 5 Ans.

The mental operation is as follows: 7 is contained in 8, once and 1 remainder; imagine 1 to be placed before 7, making 17; 7 is contained in 17, 2 times and 3 over; imagine 3 to be placed before 5, making 35; 7 is contained in 35, 5 times. These partial quotients, placed in order as they are found, make the entire quotient 125.

59. If the divisor consists of *two or more* figures, proceed as in the following example:

EXAMPLE.—Divide 2,702,826 by 63.

divisor *dividend* *quotient*
SOLUTION.— 63) 2702826 (42902 Ans.
252
182
126
568
567
126
126

EXPLANATION.—As 63 is not contained in the first two figures, 27, we must use the first three figures, 270. Now, by trial we must find how many times 63 is contained in 270. 6 is contained in the first two figures of 270, 4 times. Place the 4 as the first, or left-hand, figure in the quotient. Multiply the divisor, 63, by 4, and subtract the product, 252, from 270. The remainder is 18, beside which we write the next figure of the dividend, 2, making 182. Now, 6 is contained in the first two figures of 182, 3 times, but on multiplying 63 by 3, we see that the product, 189, is too great, so we try 2 as the second figure of the quotient. Multiplying the divisor, 63, by 2 and subtracting the product, 126, from 182, the remainder is 56, beside which we bring down the next figure of the dividend, making 568. 6 is contained in 56 about 9 times. Multiply the divisor, 63, by 9 and subtract the product, 567, from 568. The remainder is 1, and bringing down the next figure of the dividend, 2, gives 12. As 12 is smaller than 63, we write 0 in the quotient and bring down the next figure, 6, making 126. 63 is contained in 126, 2 times, without a remainder. Therefore, 42,902 is the quotient.

60. Rule.—I. *Write the divisor at the left of the dividend, with a line between them.*

II. *Find how many times the divisor is contained in the lowest number of the left-hand figures of the dividend that will contain it, and write the result at the right of the dividend, with a line between, for the first figure of the quotient.*

III. *Multiply the divisor by this quotient; write the product under the partial dividend used, and subtract, annexing to the remainder the next figure of the dividend. Divide as before, and thus continue until all the figures of the dividend have been used.*

IV. *If any partial dividend will not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend, and proceed as before.*

V. *If there be at last a remainder, write it after the quotient, with the divisor underneath.*

61. Proof.—*Multiply the quotient by the divisor and add the remainder, if there be any, to the product. The result will be the dividend.* Thus,

divisor *dividend* *quotient*

63) 4235 (67$\frac{14}{63}$ Ans.

378

455

441

remainder 14

PROOF.— *quotient* 67

divisor 63

201

402

4221

remainder 14

dividend 4235

EXAMPLES FOR PRACTICE.

62. Divide the following:

(*a*) 126,498 by 58.
(*b*) 3,207,594 by 767.
(*c*) 11,408,202 by 234.
(*d*) 2,100,315 by 581.
(*e*) 969,936 by 4,008.
(*f*) 7,481,888 by 1,021.
(*g*) 1,525,915 by 5,003.
(*h*) 1,646,301 by 381.

Ans. (*a*) 2,181. (*b*) 4,182. (*c*) 48,753. (*d*) 3,615. (*e*) 242. (*f*) 7,328. (*g*) 305. (*h*) 4,321.

CANCELATION.

63. Cancelation is the process of shortening operations in division by casting out equal factors from both dividend and divisor.

64. The **factors** of a number are those numbers, which, when multiplied together, *produce the given number.* Thus, 5 and 3 are the factors of 15, since $5 \times 3 = 15$. Likewise, 8 and 7 are the factors of 56, since $8 \times 7 = 56$.

65. A **prime number** is one which cannot be divided by any number except itself and 1. Thus, 2, 3, 11, 29, etc. are prime numbers.

66. A **prime factor** is any factor that is a prime number.

Any number that is not a prime is called a **composite** number, and may be produced by multiplying together its prime factors. Thus, 60 is a composite number, and is equal to the product of its prime factors, $2 \times 2 \times 3 \times 5$.

67. *Canceling equal factors from both dividend and divisor does not change the quotient.*

The canceling of a factor in *both* dividend and divisor is the same as *dividing them both by the same number*, and this, evidently, *does not change the quotient.*

Write the numbers forming the dividend above a horizontal line, and those forming the divisor below it; then cancel the equal factors.

68. EXAMPLE.—Divide $4 \times 45 \times 60$ by 9×24.

SOLUTION.—Placing the dividend over the divisor, and canceling,

$$\frac{\overset{5}{\cancel{4}} \times \cancel{45} \times \overset{10}{\cancel{60}}}{\underset{6}{\cancel{9} \times \cancel{24}}} = \frac{50}{1} = 50. \quad \text{Ans.}$$

EXPLANATION.—The 4 in the dividend and the 24 in the divisor are both divisible by 4, since 4 divided by 4 equals 1, and 24 divided by 4 equals 6. Cross off the 4 and write the 1 over it; also, cross off the 24 and write the 6 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times 45 \times 60}{9 \times \underset{6}{\cancel{24}}} =$$

60 in the dividend and 6 in the divisor are divisible by 6, since 60 divided by 6 equals 10, and 6 divided by 6 equals 1. Cross off the 60 and write 10 over it; also, cross off the 6 and write 1 under it. Thus,

$$\frac{1 \qquad \overset{10}{\cancel{60}}}{\cancel{4} \times 45 \times \cancel{60}} =$$
$$\frac{}{9 \times \underset{6}{\cancel{24}}}$$

Again, 45 in the dividend and 9 in the divisor are divisible by 9, since 45 divided by 9 equals 5, and 9 divided by 9 equals 1. Cross off the 45 and write the 5 over it; also, cross off the 9 and write the 1 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times 60}{\underset{1}{\cancel{9}} \times \underset{1}{\cancel{24}}} =$$

Since there are no two remaining numbers (one in the dividend and one in the divisor) divisible by any number except 1, without a remainder, it is impossible to cancel further.

Multiply all the uncanceled numbers in the dividend together and divide their product by the product of all the uncanceled numbers in the divisor. The result will be the quotient. The product of all the uncanceled numbers in the dividend equals $5 \times 1 \times 10 = 50$; the product of all the uncanceled numbers in the divisor equals $1 \times 1 = 1$.

Hence, $\dfrac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times 60}{\underset{1}{\cancel{9}} \times \underset{1}{\cancel{24}}} = \dfrac{1 \times 5 \times 10}{1 \times 1} = 50.$ Ans.

69. **Rule.—I.** *Cancel the common factors from both the dividend and the divisor.*

II. *Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

EXAMPLES FOR PRACTICE.

70. Divide.

(*a*) $14 \times 18 \times 16 \times 40$ by $7 \times 8 \times 6 \times 5 \times 3$.

(*b*) $3 \times 65 \times 50 \times 100 \times 60$ by $30 \times 60 \times 13 \times 10$.

(*c*) $8 \times 4 \times 3 \times 9 \times 11$ by $11 \times 9 \times 4 \times 3 \times 8$.

(*d*) $164 \times 321 \times 6 \times 7 \times 4$ by $82 \times 321 \times 7$.

(*e*) $50 \times 100 \times 200 \times 72$ by $1{,}000 \times 144 \times 100$.

(*f*) $48 \times 63 \times 55 \times 49$ by $7 \times 21 \times 11 \times 48$.

(*g*) $110 \times 150 \times 84 \times 32$ by $11 \times 15 \times 100 \times 64$.

(*h*) $115 \times 120 \times 400 \times 1{,}000$ by $23 \times 1{,}000 \times 60 \times 800$.

Ans. (*a*) 32. (*b*) 250. (*c*) 1. (*d*) 48. (*e*) 5. (*f*) 105. (*g*) 42. (*h*) 5.

ARITHMETIC.

(SECTION 2.)

FRACTIONS.

Remark.—The general term *fractions* embraces both common fractions and decimal fractions. In the older treatises on arithmetic, what are now called common fractions were termed vulgar fractions, but both terms have the same meaning. At the present time it is quite customary to omit the word fraction in speaking or writing the expression decimal fraction and to omit the word common when referring to a common fraction. As the result of this practice, the meaning of the word fraction has become restricted, it being used to designate common fractions only, while the decimal is used to designate the entire term, decimal fraction.

The subjects of fractions and decimals are among the most useful and important treated in arithmetic. As it is impossible in every-day transactions to deal in whole numbers only, it follows that it is very nearly, if not quite, as necessary to have a good knowledge of how to add, subtract, multiply, and divide fractions and decimals as how to perform the same operations on whole numbers. It is natural and easy to pay a quarter of a dollar for an eighth of a pound of some article, but as a rule calculations involving fractions are not nearly as simple as in this instance. The rules governing the operations of addition, subtraction, multiplication, and division of fractions apparently bear little resemblance to the corresponding rules for whole numbers and decimals; hence, fractions appears to be a difficult subject for many. A thorough understanding of the preliminary definitions will assist the student very materially in studying

For notice of copyright, see page immediately following the title page.

this somewhat difficult subject, and he is therefore advised to pay particular attention to the first two or three pages of this section.

DEFINITIONS.

71. **A fraction** is a *part of a unit. One-half, one-third, two-fifths* are fractions.

72. *Two* numbers are required to express a fraction; one is called the **numerator,** and the other, the **denominator.**

73. The numerator is placed above the denominator, with a line between them, as $\frac{2}{3}$. Here, 3 is the denominator, and shows into how many *equal parts* the *unit*, or *one*, is divided. The numerator, 2, shows how many of these equal parts are taken or considered. The denominator also indicates the names of the parts.

$\frac{1}{2}$ is read one-half.
$\frac{3}{4}$ is read three-fourths.
$\frac{3}{8}$ is read three-eighths.
$\frac{5}{16}$ is read five-sixteenths.
$\frac{29}{47}$ is read twenty-nine forty-sevenths.

74. In the expression "$\frac{3}{4}$ of an apple," the *denominator*, 4, shows that the apple is to be (or has been) cut into 4 *equal parts*, and the *numerator*, 3, shows that *three of these parts*, or *fourths*, are taken or considered.

If each of the *parts*, or *fourths*, of the apple were cut into *two equal pieces*, there would then be twice as many pieces as before, or $4 \times 2 = 8$ pieces in all, one of these pieces would be called one-eighth, and would be expressed in figures as $\frac{1}{8}$. Three of these pieces would be called three-eighths, and written $\frac{3}{8}$. The words three-fourths, three-eighths, five-sixteenths, etc. are abbreviations of three one-fourths, three one-eighths, five one-sixteenths, etc. It is evident that the larger the *denominator*, the greater is the number of parts into which anything is divided; consequently, the parts themselves are smaller, and the value of the fraction is less for the same number of parts taken. In other words, $\frac{7}{9}$, for example, is smaller than $\frac{7}{8}$, because if an object be divided into 9 parts, the parts are smaller than if

the same object had been divided into 8 parts; and, since $\frac{1}{9}$ is smaller than $\frac{1}{8}$, it is clear that 7 one-ninths is a smaller amount than 7 one-eighths. Hence, also, $\frac{3}{8}$ is less than $\frac{3}{4}$.

75. The **value** of a fraction is the numerator divided by the denominator, as $\frac{4}{2} = 2$, $\frac{6}{2} = 3$.

76. The line between the numerator and the denominator means *divided by*, or $\div$.

$\frac{3}{4}$ is equivalent to $3 \div 4$.

$\frac{5}{8}$ is equivalent to $5 \div 8$.

77. The numerator and the denominator of a fraction are called the **terms** of a fraction.

78. The *value* of a fraction whose numerator and denominator are equal is 1.

$\frac{4}{4}$, or four-fourths $= 1$.

$\frac{8}{8}$, or eight-eighths $= 1$.

$\frac{64}{64}$, or sixty-four sixty-fourths $= 1$.

79. A **proper fraction** is a fraction whose numerator is *less* than its denominator. Its value is *less* than 1, as $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{16}$.

80. An **improper fraction** is a fraction whose numerator *equals* or is *greater* than the denominator. Its value is 1 or *more than* 1, as $\frac{4}{4}$, $\frac{8}{8}$, $\frac{42}{32}$.

81. A **mixed number** is a whole number and a fraction united. $4\frac{2}{3}$ is a mixed number, and is equivalent to $4 + \frac{2}{3}$. It is read *four and two-thirds.*

REDUCTION OF FRACTIONS.

82. **Reduction of fractions** is the process of changing their form without changing their *value.*

83. *A fraction is reduced to higher terms by multiplying both terms of the fraction by the same number.* Thus, $\frac{3}{4}$ is reduced to $\frac{6}{8}$ by multiplying both terms by 2.

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}.$$

The value is not changed, since $\frac{3}{4} = \frac{6}{8}$. For, suppose that an object, say an apple, is divided into 8 equal parts. If these parts be arranged into 4 piles, each containing 2 parts, it is evident each pile will be composed of the same amount of the entire apple as would have been the case had the apple been originally cut into 4 equal parts. Now, if one of these piles (containing 2 parts) be removed, there will be 3 piles left, each containing 2 equal parts, or 6 equal parts in all, i. e., six-eighths. But, since one pile, or one-quarter, was removed, there are three-quarters left. Hence, $\frac{3}{4} = \frac{6}{8}$. The same course of reasoning may be applied to any similar case. Therefore, multiplying both terms of a fraction by the same number does not alter its value.

84. To reduce a fraction to an equal fraction having a given denominator:

EXAMPLE.—Reduce $\frac{7}{8}$ to an equal fraction having 96 for a denominator.

SOLUTION.—Both the numerator and the denominator must be multiplied by the same number in order not to change the value of the fraction. The denominator must be multiplied by some number which will, in this case, make the product 96; this number is evidently $96 \div 8 = 12$, since $8 \times 12 = 96$. Hence, $\frac{7 \times 12}{8 \times 12} = \frac{84}{96}$. Ans.

85. Rule.—*Divide the given denominator by the denominator of the given fraction, and multiply both terms of the fraction by the result.*

EXAMPLE.—Reduce $\frac{3}{4}$ to 100ths.

SOLUTION.— $100 \div 4 = 25$; hence, $\frac{3 \times 25}{4 \times 25} = \frac{75}{100}$. Ans.

86. *A fraction is reduced to lower terms by dividing both terms by the same number.* Thus, $\frac{8}{10}$ is reduced to $\frac{4}{5}$ by dividing both terms by 2.

$$\frac{8 \div 2}{10 \div 2} = \frac{4}{5}.$$

That $\frac{8}{10} = \frac{4}{5}$ is readily seen from the explanation given in Art. **83**; for, multiplying both terms of the fraction $\frac{4}{5}$ by 2, $\frac{4}{5} \times \frac{2}{2} = \frac{8}{10}$, and, if $\frac{4}{5} = \frac{8}{10}$, $\frac{8}{10}$ must equal $\frac{4}{5}$. Hence, dividing both terms of a fraction by the same number does not alter its value.

87. A fraction is reduced to its *lowest terms* when its numerator and denominator cannot both be divided by the *same* number without a remainder; for example, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{11}{24}$, $\frac{8}{15}$.

EXAMPLES FOR PRACTICE.

88. Reduce the following:

(*a*) $\frac{7}{16}$ to 128ths.
(*b*) $\frac{24}{128}$ to its lowest terms.
(*c*) $\frac{64}{1000}$ to its lowest terms.
(*d*) $\frac{5}{7}$ to 49ths.
(*e*) $\frac{13}{16}$ to 10,000ths.

Ans. (*a*) $\frac{56}{128}$. (*b*) $\frac{3}{16}$. (*c*) $\frac{8}{125}$. (*d*) $\frac{35}{49}$. (*e*) $\frac{8125}{10000}$.

89. To reduce a whole number or a mixed number to an improper fraction:

EXAMPLE.—How many fourths in 5?

SOLUTION.—Since there are 4 fourths in 1 ($\frac{4}{4} = 1$), in 5 there will be 5×4 fourths, or 20 fourths; i. e., $5 \times \frac{4}{4} = \frac{20}{4}$. Ans.

EXAMPLE.—Reduce $8\frac{3}{4}$ to an improper fraction.

SOLUTION.— $8 \times \frac{4}{4} = \frac{32}{4}$. $\frac{32}{4} + \frac{3}{4} = \frac{35}{4}$. Ans.

90. Rule.—*Multiply the whole number by the denominator of the fraction, add the numerator to the product and place the denominator under the result. If it is desired to reduce a whole number to a fraction, multiply the whole number by the denominator of the given fraction, and write the result over the denominator.*

EXAMPLES FOR PRACTICE.

91. Reduce to improper fractions:

(*a*) $4\frac{1}{8}$.
(*b*) $5\frac{1}{9}$.
(*c*) $10\frac{2}{10}$.
(*d*) $37\frac{3}{4}$.
(*e*) $50\frac{4}{5}$.
(*f*) Reduce 7 to a fraction whose denominator is 16.

Ans. (*a*) $\frac{33}{8}$. (*b*) $\frac{46}{9}$. (*c*) $\frac{102}{10}$. (*d*) $\frac{151}{4}$. (*e*) $\frac{254}{5}$. (*f*) $\frac{112}{16}$.

92. To reduce an improper fraction to a whole or a mixed number:

EXAMPLE.—Reduce $\frac{21}{4}$ to a mixed number.

SOLUTION.— 4 is contained in 21, 5 times and 1 remaining (see Art. 75); as this is also divided by 4, its value is $\frac{1}{4}$. Therefore, $5 + \frac{1}{4}$, or $5\frac{1}{4}$, is the number. Ans.

93. **Rule.**—*Divide the numerator by the denominator, the quotient will be the whole number; the remainder, if there be any, will be the numerator of the fractional part of which the denominator is the same as the denominator of the improper fraction.*

EXAMPLES FOR PRACTICE.

94. Reduce to whole or mixed numbers:

(*a*) $\frac{145}{6}$.
(*b*) $\frac{185}{8}$.
(*c*) $\frac{701}{6}$.
(*d*) $\frac{149}{8}$.
(*e*) $\frac{76}{19}$.
(*f*) $\frac{125}{25}$.

Ans. (*a*) $24\frac{1}{6}$. (*b*) $61\frac{2}{8}$. (*c*) $116\frac{5}{6}$. (*d*) $49\frac{2}{8}$. (*e*) 4. (*f*) 5.

---

95. A **common denominator** of two or more fractions is a number which will contain (i. e., which may be divided by) the denominator of each of the given fractions without a remainder. The **least common denominator** is the least number that will contain each denominator of the given fractions without a remainder.

96. **To find the least common denominator:**

EXAMPLE.—Find the least common denominator of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{16}$.

SOLUTION.—We first place the denominators in a row, separated by commas.

2)	4,	3,	9,	16
2)	2,	3,	9,	8
3)	1,	3,	9,	4
3)	1,	1,	3,	4
4)	1,	1,	1,	4
	1,	1,	1,	1

$2 \times 2 \times 3 \times 3 \times 4 = 144$, the least common denominator. Ans.

EXPLANATION.—Divide each of them by some prime number which will divide at least two of them without a remainder (if possible), bringing down those denominators to the row below which will not contain the divisor without a remainder. Dividing each of the numbers by 2, the second row becomes 2, 3, 9, 8, since 2 will not divide 3 and 9 without a remainder. Dividing again by 2, the result is 1, 3, 9, 4. Dividing the third row by 3, the result, is 1, 1,

3, 4. So continue until the last row contains only 1's. The product of all the divisors, or $2 \times 2 \times 3 \times 3 \times 4 = 144$, is the least common denominator.

97. EXAMPLE.—Find the least common denominator of $\frac{4}{9}$, $\frac{5}{12}$, $\frac{7}{18}$.

SOLUTION.—

$$\begin{array}{r|l} 3 & 9,\ 12,\ 18 \\ 3 & 3,\ 4,\ 6 \\ 2 & 1,\ 4,\ 2 \\ 2 & 1,\ 2,\ 1 \\ & 1,\ 1,\ 1 \end{array}$$

$3 \times 3 \times 2 \times 2 = 36$. Ans.

98. To reduce two or more fractions to fractions having a common denominator:

EXAMPLE.—Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ to fractions having a common denominator.

SOLUTION.—The common denominator is a number which will contain 3, 4, and 2. The least common denominator is 12, because it is the smallest number which can be divided by 3, 4, and 2 without a remainder.

$$\frac{2}{3} = \frac{8}{12}, \quad \frac{3}{4} = \frac{9}{12}, \quad \frac{1}{2} = \frac{6}{12}.$$

Reducing $\frac{2}{3}$ (see Art. 84), 3 is contained in 12, 4 times. By multiplying both numerator and denominator of $\frac{2}{3}$ by 4, we find

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}.$$ In the same way we find $\frac{3}{4} = \frac{9}{12}$ and $\frac{1}{2} = \frac{6}{12}$.

99. Rule.—*Divide the common denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

EXAMPLES FOR PRACTICE.

100. Reduce to fractions having a common denominator:

(*a*) $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$.
(*b*) $\frac{8}{16}$, $\frac{3}{4}$, $\frac{7}{32}$.
(*c*) $\frac{7}{8}$, $\frac{7}{88}$, $\frac{10}{11}$.
(*d*) $\frac{3}{5}$, $\frac{5}{8}$, $\frac{11}{40}$.
(*e*) $\frac{4}{10}$, $\frac{6}{40}$, $\frac{9}{20}$.
(*f*) $\frac{7}{15}$, $\frac{17}{30}$, $\frac{21}{30}$.

Ans.

(*a*) $\frac{6}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.
(*b*) $\frac{6}{32}$, $\frac{24}{32}$, $\frac{7}{32}$.
(*c*) $\frac{77}{88}$, $\frac{7}{88}$, $\frac{80}{88}$.
(*d*) $\frac{24}{40}$, $\frac{25}{40}$, $\frac{11}{40}$.
(*e*) $\frac{16}{40}$, $\frac{6}{40}$, $\frac{18}{40}$.
(*f*) $\frac{14}{30}$, $\frac{17}{30}$, $\frac{21}{30}$.

ADDITION OF FRACTIONS.

101. *Fractions cannot be added unless they have a common denominator.* We cannot add $\frac{3}{4}$ to $\frac{7}{8}$ as they now stand, since the denominators represent parts of different sizes. Fourths cannot be added to eighths.

Suppose we divide an apple into 4 equal parts, and then divide 2 of these parts into 2 equal parts. It is evident that we shall have 2 one-fourths and 4 one-eighths. Now, if we add these parts, the result is $2+4=6$ something. But what is this something? It is not fourths, for 6 fourths are $1\frac{1}{2}$, and we had only 1 apple to begin with; neither is it eighths, for 6 eighths are $\frac{3}{4}$, which is less than 1 apple. By reducing the quarters to eighths, we have $\frac{2}{4}=\frac{4}{8}$, and adding the other 4 eighths, $4+4=8$ eighths. This result is correct, since $\frac{8}{8}=1$. Or we can, in this case, reduce the eighths to quarters. Thus, $\frac{4}{8}=\frac{2}{4}$; whence, adding, $2+2=4$ quarters, a correct result, since $\frac{4}{4}=1$.

Before adding, fractions should be reduced to a common denominator, preferably the *least* common denominator.

102. EXAMPLE.—Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$.

SOLUTION.—The *least common denominator*, or the *least number* which will contain all the denominators, is 8.

$$\frac{1}{2}=\frac{4}{8},\quad \frac{3}{4}=\frac{6}{8},\quad \text{and}\quad \frac{5}{8}=\frac{5}{8}.$$

EXPLANATION.—As the denominator tells or indicates the *names* of the parts, the numerators only are added, to obtain the total number of *parts* indicated by the denominator. Thus, 4 one-eighths plus 6 one-eighths plus 5 one-eighths =

$$\frac{4}{8}+\frac{6}{8}+\frac{5}{8}=\frac{4+6+5}{8}=\frac{15}{8}=1\tfrac{7}{8}.\quad \text{Ans.}$$

103. EXAMPLE.—What is the sum of $12\frac{3}{4}$, $14\frac{5}{8}$, and $7\frac{5}{16}$?

SOLUTION.—The least common denominator in this case is 16.

$$\begin{aligned} 12\tfrac{3}{4} &= 12\tfrac{12}{16}\\ 14\tfrac{5}{8} &= 14\tfrac{10}{16}\\ 7\tfrac{5}{16} &= 7\tfrac{5}{16}\\ \textit{sum}\quad & 33+\tfrac{27}{16}=33+1\tfrac{11}{16}=34\tfrac{11}{16}. \end{aligned}$$ Ans.

The sum of the fractions $=\frac{27}{16}$ or $1\frac{11}{16}$, which added to the sum of the whole numbers $=34\frac{11}{16}$.

EXAMPLE.—What is the sum of 17, $13\frac{3}{16}$, $\frac{9}{32}$, and $3\frac{1}{4}$?

SOLUTION.—The least common denominator is 32. $13\frac{3}{16}=13\frac{6}{32}$, $3\frac{1}{4}=3\frac{8}{32}$.

$$\begin{aligned} &17\\ &13\tfrac{6}{32}\\ &\tfrac{9}{32}\\ &3\tfrac{8}{32}\\ \textit{sum}\quad &33\tfrac{23}{32}. \end{aligned}$$ Ans.

104. **Rule.—I.** *Reduce the given fractions to fractions having the least common denominator, and write the sum of the numerators over the common denominator.*

II. *When there are mixed numbers and whole numbers, add the fractions first, and if their sum is an improper fraction, reduce it to a mixed number and add the whole number with the other whole numbers.*

EXAMPLES FOR PRACTICE.

105. Find the sum of:

(*a*) $\frac{4}{5}$, $\frac{7}{24}$, $\frac{5}{8}$.
(*b*) $\frac{2}{3}$, $\frac{5}{15}$, $\frac{24}{45}$.
(*c*) $\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$.
(*d*) $\frac{5}{6}$, $\frac{11}{12}$, $\frac{13}{18}$.
(*e*) $\frac{10}{11}$, $\frac{6}{33}$, $\frac{23}{66}$.
(*f*) $\frac{23}{45}$, $\frac{11}{15}$, $\frac{14}{45}$.
(*g*) $\frac{4}{11}$, $\frac{7}{22}$, $\frac{14}{23}$.
(*h*) $\frac{3}{7}$, $\frac{14}{49}$, $\frac{2}{7}$.

Ans. (*a*) $1\frac{7}{12}$. (*b*) $1\frac{8}{15}$. (*c*) $1\frac{3}{16}$. (*d*) $1\frac{51}{72}$. (*e*) $1\frac{29}{66}$. (*f*) $1\frac{5}{9}$. (*g*) $1\frac{7}{23}$. (*h*) **1.**

SUBTRACTION OF FRACTIONS.

106. *Fractions cannot be subtracted without first reducing them to a common denominator.* This can be shown in the same manner as in the case of addition of fractions.

EXAMPLE.—Subtract $\frac{3}{8}$ from $\frac{13}{16}$.

SOLUTION.—The common denominator is 16.

$$\frac{3}{8} = \frac{6}{16}. \quad \frac{13}{16} - \frac{6}{16} = \frac{13-6}{16} = \frac{7}{16}. \quad \text{Ans.}$$

107. EXAMPLE.—From 7 take $\frac{5}{8}$.

SOLUTION.— $1 = \frac{8}{8}$; therefore, since $7 = 6+1$, $7 = 6+\frac{8}{8} = 6\frac{8}{8}$, or $6\frac{8}{8} - \frac{5}{8} = 6\frac{3}{8}$. Ans.

108. EXAMPLE.—What is the difference between $17\frac{9}{16}$ and $9\frac{15}{32}$?

SOLUTION.—The common denominator of the fractions is 32. $17\frac{9}{16} = 17\frac{18}{32}$.

minuend	$17\frac{18}{32}$
subtrahend	$9\frac{15}{32}$
difference	$8\frac{3}{32}$. Ans.

109. EXAMPLE.—From $9\frac{1}{4}$ take $4\frac{7}{16}$.

SOLUTION.—The common denominator of the fractions is 16. $9\frac{1}{4} = 9\frac{4}{16}$.

minuend	$9\frac{4}{16}$ or $8\frac{20}{16}$	
subtrahend	$4\frac{7}{16}$	$4\frac{7}{16}$
difference	$4\frac{13}{16}$	$4\frac{13}{16}$. Ans.

EXPLANATION.—As the fraction in the subtrahend is greater than the fraction in the minuend, it cannot be subtracted; therefore, *borrow* 1, or $\frac{16}{16}$, from the 9 in the minuend and add it to the $\frac{4}{16}$; $\frac{4}{16} + \frac{16}{16} = \frac{20}{16}$. $\frac{7}{16}$ from $\frac{20}{16} = \frac{13}{16}$. Since 1 was borrowed from 9, 8 remains; 4 from $8 = 4$; $4 + \frac{13}{16} = 4\frac{13}{16}$.

110. EXAMPLE.—From 9 take $8\frac{3}{16}$.

SOLUTION.—

minuend	9	or $8\frac{16}{16}$
subtrahend	$8\frac{3}{16}$	$8\frac{3}{16}$
difference	$\frac{13}{16}$	$\frac{13}{16}$. Ans.

EXPLANATION.—As there is no fraction in the minuend from which to take the fraction in the subtrahend, borrow 1, or $\frac{16}{16}$, from 9. $\frac{3}{16}$ from $\frac{16}{16} = \frac{13}{16}$. Since 1 was borrowed from 9, only 8 is left. 8 from $8 = 0$.

111. **Rule.—I.** *Reduce the fractions to fractions having a common denominator. Subtract one numerator from the other and place the remainder over the common denominator.*

II. *When there are mixed numbers, subtract the fractions and whole numbers separately, and place the remainders side by side.*

III. *When the fraction in the subtrahend is greater than the fraction in the minuend, borrow 1 from the whole number in the minuend and add it to the fraction in the minuend, from which subtract the fraction in the subtrahend.*

IV. *When the minuend is a whole number, borrow 1; reduce it to a fraction whose denominator is the same as the denominator of the fraction in the subtrahend, and place it over that fraction for subtraction.*

EXAMPLES FOR PRACTICE.

112. Subtract:

(*a*) $\frac{10}{24}$ from $\frac{11}{18}$.
(*b*) $\frac{7}{14}$ from $\frac{17}{28}$.
(*c*) $\frac{4}{30}$ from $\frac{5}{16}$.
(*d*) $\frac{16}{35}$ from $\frac{45}{70}$.
(*e*) $\frac{15}{16}$ from $\frac{57}{48}$.
(*f*) $13\frac{1}{4}$ from $30\frac{1}{2}$.
(*g*) $12\frac{1}{8}$ from 27.
(*h*) $5\frac{1}{4}$ from 30.

Ans. (*a*) $\frac{1}{9}$. (*b*) $\frac{3}{28}$. (*c*) $\frac{11}{80}$. (*d*) $\frac{3}{14}$. (*e*) $\frac{1}{8}$. (*f*) $17\frac{1}{4}$. (*g*) $14\frac{7}{8}$. (*h*) $24\frac{3}{4}$.

MULTIPLICATION OF FRACTIONS.

113. *In multiplication of fractions it is not necessary to reduce the fractions to fractions having a common denominator.*

114. *Multiplying the numerator or dividing the denominator multiplies the fraction.*

EXAMPLE.—Multiply $\frac{3}{4}$ by 4.

SOLUTION.— $\frac{3}{4} \times 4 = \frac{3}{4} \times 4 = \frac{12}{4} = 3$. Ans.

Or, $\frac{3}{4} \times 4 = \frac{3}{4 \div 4} = \frac{3}{1} = 3$. Ans.

The word "of," when placed between two fractions, or between a fraction and a whole number, means the same as $\times$, or times. Thus,

$$\tfrac{3}{4} \text{ of } 4 = \tfrac{3}{4} \times 4 = 3.$$
$$\tfrac{1}{8} \text{ of } \tfrac{5}{16} = \tfrac{1}{8} \times \tfrac{5}{16} = \tfrac{5}{128}.$$

EXAMPLE.—Multiply $\frac{3}{8}$ by 2.

SOLUTION.— $2 \times \frac{3}{8} = \frac{3}{8} \times 2 = \frac{6}{8} = \frac{3}{4}$. Ans.

Or, $2 \times \frac{3}{8} = \frac{3}{8 \div 2} = \frac{3}{4}$. Ans.

115. EXAMPLE.—What is the product of $\frac{4}{16}$ and $\frac{7}{8}$?

SOLUTION.— $\frac{4}{16} \times \frac{7}{8} = \frac{4 \times 7}{16 \times 8} = \frac{28}{128} = \frac{7}{32}$. Ans.

Or, by cancelation, $\frac{\cancel{4} \times 7}{\cancel{16} \times 8} = \frac{7}{4 \times 8} = \frac{7}{32}$. Ans.
$\quad\quad\quad

117. EXAMPLE.—What is the product of $9\frac{3}{4}$ and $5\frac{5}{8}$?

SOLUTION.— $9\frac{3}{4} = \frac{39}{4}$; $5\frac{5}{8} = \frac{45}{8}$.

$$\frac{39}{4} \times \frac{45}{8} = \frac{39 \times 45}{4 \times 8} = \frac{1755}{32} = 54\frac{27}{32}. \quad \text{Ans.}$$

118. EXAMPLE.—Multiply $15\frac{7}{8}$ by 3.

SOLUTION.—

$$\begin{array}{r} 15\frac{7}{8} \\ \underline{\quad 3 \quad} \\ 47\frac{5}{8} \end{array} \quad \text{or} \quad \begin{array}{r} 15\frac{7}{8} \\ \underline{\quad 3 \quad} \\ 45 + \frac{21}{8} = 45 + 2\frac{5}{8} = 47\frac{5}{8}. \end{array} \quad \text{Ans.}$$

119. Rule.—I. *Divide the product of the numerators by the product of the denominators. All factors common to the numerators and denominators should first be cast out by cancelation.*

II. *To multiply one mixed number by another, reduce them both to improper fractions.*

III. *To multiply a mixed number by a whole number, first multiply the fractional part by the multiplier, and if the product is an improper fraction, reduce it to a mixed number and add the whole-number part to the product of the multiplier and the whole number.*

EXAMPLES FOR PRACTICE.

120. Find the product of:

(*a*)	$7 \times \frac{8}{19}$.	(*a*)	$1\frac{8}{19}$.
(*b*)	$14 \times \frac{6}{16}$.	(*b*)	$4\frac{3}{8}$.
(*c*)	$\frac{21}{32} \times \frac{5}{14}$.	(*c*)	$\frac{15}{64}$.
(*d*)	$\frac{16}{27} \times 4$.	(*d*)	$2\frac{10}{27}$.
(*e*)	$\frac{10}{18} \times 7$.	(*e*)	$7\frac{7}{18}$.
(*f*)	$17\frac{18}{21} \times 7$.	(*f*)	125.
(*g*)	$\frac{105}{224} \times 32$.	(*g*)	15.
(*h*)	$\frac{15}{28} \times 14$.	(*h*)	$7\frac{1}{2}$.

Ans.

DIVISION OF FRACTIONS.

121. *In division of fractions it is not necessary to reduce the fractions to fractions having a common denominator.*

122. *Dividing the numerator or multiplying the denominator divides the fraction.*

EXAMPLE.—Divide $\frac{6}{8}$ by 3.

SOLUTION.—When *dividing the numerator*, we have

$$\frac{6}{8} \div 3 = \frac{6 \div 3}{8} = \frac{2}{8} = \frac{1}{4}. \quad \text{Ans.}$$

When *multiplying the denominator*, we have

$$\tfrac{6}{8} \div 3 = \frac{6}{8 \times 3} = \tfrac{6}{24} = \tfrac{1}{4}. \quad \text{Ans.}$$

EXAMPLE.—Divide $\frac{3}{16}$ by 2.

SOLUTION.— $$\tfrac{3}{16} \div 2 = \frac{3}{16 \times 2} = \tfrac{3}{32}. \quad \text{Ans.}$$

EXAMPLE.—Divide $\frac{14}{32}$ by 7.

SOLUTION.— $$\tfrac{14}{32} \div 7 = \frac{14 \div 7}{32} = \tfrac{2}{32} = \tfrac{1}{16}. \quad \text{Ans.}$$

123. To **invert** a fraction is to *turn it upside down;* that is, make the numerator and denominator *change places.*

Invert $\frac{3}{4}$ and it becomes $\frac{4}{3}$.

124. EXAMPLE.—Divide $\frac{9}{16}$ by $\frac{3}{16}$.

SOLUTION.—1. The fraction $\frac{3}{16}$ is contained in $\frac{9}{16}$, 3 times, for the denominators are the same, and one numerator is contained in the other 3 times. 2. If we now invert the divisor, $\frac{3}{16}$, and multiply, the solution is

$$\frac{9}{16} \times \frac{16}{3} = \frac{\overset{3}{\cancel{9}} \times \cancel{16}}{\cancel{16} \times \cancel{3}} = 3. \quad \text{Ans.}$$

This brings the same quotient as in the first case.

125. EXAMPLE.—Divide $\frac{3}{8}$ by $\frac{1}{4}$.

SOLUTION.—We cannot divide $\frac{3}{8}$ by $\frac{1}{4}$, as in the first case above, for the denominators are not the same; therefore, we must solve as in the second case.

$$\tfrac{3}{8} \div \tfrac{1}{4} = \tfrac{3}{8} \times \tfrac{4}{1} = \frac{3 \times \overset{1}{\cancel{4}}}{\underset{2}{\cancel{8}} \times 1} = \tfrac{3}{2} \text{ or } 1\tfrac{1}{2}. \quad \text{Ans.}$$

126. EXAMPLE.—Divide 5 by $\frac{10}{16}$.

SOLUTION.— $\frac{10}{16}$ inverted becomes $\frac{16}{10}$.

$$5 \times \frac{16}{10} = \frac{\overset{1}{\cancel{5}} \times \overset{8}{\cancel{16}}}{\underset{2}{\cancel{10}}} = 8. \quad \text{Ans.}$$

127. EXAMPLE.—How many times is $3\frac{3}{4}$ contained in $7\frac{7}{16}$?

SOLUTION.— $3\frac{3}{4} = \frac{15}{4}$; $7\frac{7}{16} = \frac{119}{16}$.

$\frac{15}{4}$ inverted equals $\frac{4}{15}$.

$$\frac{119}{16} \times \frac{4}{15} = \frac{119 \times \overset{1}{\cancel{4}}}{\underset{4}{\cancel{16}} \times 15} = \frac{119}{60} = 1\tfrac{59}{60}. \quad \text{Ans.}$$

128. **Rule.**—*Invert the divisor and proceed as in multiplication.*

129. We have learned that a line placed between two numbers indicates that the number above the line is to be divided by the number below it. Thus, $\frac{18}{3}$ shows that 18 is to be divided by 3. This is also true if a fraction or a fractional expression be placed above or below a line.

$\frac{9}{\frac{3}{8}}$ means that 9 is to be divided by $\frac{3}{8}$; $\frac{3\times 7}{\frac{8+4}{16}}$ means that 3×7 is to be divided by the value of $\frac{8+4}{16}$.

$\frac{\frac{1}{4}}{\frac{3}{8}}$ is the same as $\frac{1}{4}\div\frac{3}{8}$.

It will be noticed that there is a heavy line between the 9 and the $\frac{3}{8}$. This is necessary, since otherwise there would be nothing to show as to whether 9 was to be divided by $\frac{3}{8}$, or $\frac{9}{3}$ was to be divided by 8. Whenever a heavy line is used, as shown here, it indicates that *all above the line is to be divided by all below it.*

EXAMPLES FOR PRACTICE.

130. Divide:

(*a*) 15 by $6\frac{3}{7}$.
(*b*) 30 by $\frac{5}{8}$.
(*c*) 172 by $\frac{4}{5}$.
(*d*) $\frac{14}{15}$ by $1\frac{7}{16}$.
(*e*) $\frac{102}{6}$ by $14\frac{2}{3}$.
(*f*) $\frac{142}{27}$ by $17\frac{1}{9}$.
(*g*) $\frac{14}{15}$ by $\frac{145}{72}$.
(*h*) $\frac{128}{15}$ by $72\frac{1}{8}$.

Ans. (*a*) $2\frac{1}{3}$. (*b*) 40. (*c*) 215. (*d*) $\frac{112}{207}$. (*e*) $1\frac{15}{88}$. (*f*) $\frac{71}{281}$. (*g*) $\frac{56}{145}$. (*h*) $\frac{64}{661}$.

131. Whenever an expression like one of the three following ones is obtained, it may always be simplified by transposing the denominator from *above* to *below* the line, or from *below* to *above*, as the case may be, taking care, however, to indicate that the denominator when so transferred is a multiplier.

1. $\frac{\frac{3}{4}}{9}=\frac{3}{9\times 4}=\frac{3}{36}=\frac{1}{12}$; for, regarding the fraction above the heavy line as the numerator of a fraction whose denominator is 9, $\frac{\frac{3}{4}\times 4}{9\times 4}=\frac{3}{9\times 4}$, as before.

2. $\frac{9}{\frac{3}{4}} = \frac{9\times4}{3} = 12$. The proof is the same as in the first case.

3. $\frac{\frac{5}{9}}{\frac{3}{4}} = \frac{5\times4}{3\times9} = \frac{20}{27}$. For, regarding $\frac{5}{9}$ as the numerator of a fraction whose denominator is $\frac{3}{4}$, $\frac{\frac{5}{9}\times9}{\frac{3}{4}\times9} = \frac{5}{\frac{3\times9}{4}}$; and $\frac{\frac{5}{3\times9}\times4}{\frac{4}{}\times4} = \frac{5\times4}{3\times9} = \frac{20}{27}$, as above.

This principle may be used to great advantage in cases like $\frac{\frac{1}{4}\times310\times\frac{27}{\frac{1}{2}}\times72}{40\times4\frac{1}{2}\times5\frac{1}{6}}$. Reducing the mixed numbers to fractions, the expression becomes $\frac{\frac{1}{4}\times310\times\frac{27}{\frac{1}{2}}\times72}{40\times\frac{9}{2}\times\frac{31}{6}}$. Now transferring the denominators of the fractions and canceling,

$$\frac{1\times310\times27\times72\times2\times6}{40\times9\times31\times4\times12} = \frac{1\times\cancel{310}^{10}\times27\times\cancel{72}^{3}\times\cancel{2}\times\cancel{6}^{3}}{\cancel{40}^{10}\times\cancel{9}^{3}\times\cancel{31}\times\cancel{4}^{2}\times\cancel{12}^{3}_{4}_{2}}$$

$$= \frac{27}{2} = 13\tfrac{1}{2}.$$

Greater exactness in results can usually be obtained by using this principle than by reducing the fractions to decimals. The principle, however, should not be employed if a sign of addition or subtraction occurs either above or below the dividing line.

ARITHMETIC.

(SECTION 3.)

DECIMALS.

Remark.—A knowledge of decimals is of the utmost importance to all who are required to make calculations of any kind. The subject is easy to learn, and for this reason the student is somewhat inclined to study it too hastily, the result being that he afterwards has trouble that might have been entirely avoided had he given the text the proper attention in the beginning. Decimals are much easier to use than common fractions, which they replace; at the same time it is frequently more expedient to use common fractions in certain operations, and, hence, they cannot be wholly dispensed with. Particular attention should be paid to the rules for multiplication and division—especially to the locating of the decimal point—and to the operations of changing a common fraction to a decimal and vice versa.

132. **Decimals** are *tenth* fractions; that is, the parts of a unit are expressed on the scale of ten, as *tenths*, *hundredths*, *thousandths*, etc.

133. The *denominator*, which is always ten or a multiple of ten, as 10, 100, 1,000, etc., is not expressed, as it would be in common fractions, by writing it under the

For notice of copyright, see page immediately following the title page.

numerator with a line between them, as $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, but is expressed by placing a *period* (.), which is called a **decimal point,** to the *left* of the *figures of the numerator*, so as to indicate that the number on the right is the numerator of a fraction whose denominator is 10, 100, 1,000, etc.

134. The *reading* of a decimal number depends upon the number of decimal places in it, or the number of figures to the right of the decimal point.

One decimal place expresses *tenths.*
Two decimal places express *hundredths.*
Three decimal places express *thousandths.*
Four decimal places express *ten-thousandths.*
Five decimal places express *hundred-thousandths.*
Six decimal places express *millionths.*

Thus:

.3	=	$\frac{3}{10}$	= 3 tenths.
.03	=	$\frac{3}{100}$	= 3 hundredths.
.003	=	$\frac{3}{1000}$	= 3 thousandths.
.0003	=	$\frac{3}{10000}$	= 3 ten-thousandths.
.00003	=	$\frac{3}{100000}$	= 3 hundred-thousandths.
.000003	=	$\frac{3}{1000000}$	= 3 millionths.

We see in the above that the *number of decimal places in a decimal equals the number of ciphers to the right of the figure 1 in the denominator of its equivalent fraction.* This fact kept in mind will be of much assistance in reading and writing decimals.

Whatever may be written to the *left* of a decimal point is a whole number. The *decimal point merely separates the fraction on the right from the whole number on the left.*

When a whole number and decimal are written together, the expression is a *mixed number.* Thus, 8.12 and 17.25 are mixed numbers.

The relation of decimals and whole numbers to each other is clearly shown by the following table:

9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9
hundreds of millions.	tens of millions.	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.	decimal point.	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	ten-millionths.	hundred-millionths.

The figures to the *left* of the decimal point represent *whole numbers;* those to the *right* are *decimals.*

In both the decimals and whole numbers, the *units* place is made the *starting point* of notation and numeration. Both whole numbers and decimals *decrease* on the scale of ten to the *right*, and both *increase* on the scale of ten to the *left*. The *first* figure to the *left* of units is *tens*, and the *first* figure to the *right* of units is *tenths*. The *second* figure to the *left* of units is *hundreds*, and the *second* figure to the *right* is *hundredths*. The *third* figure to the *left* is *thousands*, and the *third* to the *right* is *thousandths*, and so on; the *whole* numbers on the *left* and the *decimals* on the *right*. The figures equally distant from units place correspond in name, the *decimals* having the ending *ths*, to distinguish them from *whole* numbers. The following is the numeration of the number in the above table: nine hundred eighty-seven million, six hundred fifty-four thousand, three hundred twenty-one and twenty-three million, four hundred fifty-six thousand, seven hundred eighty-nine hundred-millionths.

The decimals increase to the *left*, on the scale of *ten*, the same as whole numbers; for, if you begin at the 4 in thousandths place in the above table, the next figure to the left is *hundredths*, which is ten times as great, and the next *tenths*, or ten times the *hundredths*, and so on through both decimals and whole numbers.

135. *Annexing, or taking away, a cipher at the right of a decimal, does not affect its value.*

.5 is $\frac{5}{10}$; .50 is $\frac{50}{100}$, but $\frac{5}{10} = \frac{50}{100}$; therefore, .5 = .50.

136. *Inserting a cipher between a decimal and the decimal point, divides the decimal by 10.*

$.5 = \frac{5}{10}$; $\frac{5}{10} \div 10 = \frac{5}{100} = .05$.

137. *Taking away a cipher from the left of a decimal, multiplies the decimal by 10.*

$.05 = \frac{5}{100}$; $\frac{5}{100} \times 10 = \frac{5}{10} = .5$.

138. In some cases it is convenient to express a mixed decimal fraction in the form of a common (improper) fraction. To do so it is only necessary to write the entire number, omitting the decimal point, as the numerator of the fraction, and the denominator of the decimal part as the denominator of the fraction. Thus, $127.483 = \frac{127483}{1000}$; for, $127.483 = 127\frac{483}{1000} = \frac{127000+483}{1000} = \frac{127483}{1000}$.

ADDITION OF DECIMALS.

139. Addition of decimals is similar in all respects to addition of whole numbers—units are placed under units, tens under tens, etc.; this, of course, brings the decimal points in line, directly under one another. Hence, in placing the numbers to be added, it is only necessary to take care that the *decimal points are in line.* In adding whole numbers, the right-hand figures are always in line; but in adding decimals, the right-hand figures will not be in line unless each decimal contains the same number of figures.

whole numbers	*decimals*	*mixed numbers*
342	.342	342.032
4234	.4234	4234.5
26	.26	26.6782
3	.03	3.06
sum 4605 Ans.	*sum* 1.0554 Ans.	*sum* 4606.2702 Ans.

140. EXAMPLE.—What is the sum of 242, .36, 118.725, 1.005, 6, and 100.1?

SOLUTION.—

242.
.36
118.725
1.005
6.
100.1
sum 468.190 Ans.

141. **Rule.**—*Place the numbers to be added so that the decimal points will be directly under each other. Add as in whole numbers, and place the decimal point in the sum directly under the decimal points above.*

EXAMPLES FOR PRACTICE.

142. Find the sum of:

(*a*) .2143, .105, 2.3042, and 1.1417.
(*b*) 783.5, 21.473, .2101, and .7816.
(*c*) 21.781, 138.72, 41.8738, .72, and 1.413.
(*d*) .3724, 104.15, 21.417, and 100.042.
(*e*) 200.172, 14.105, 12.1465, .705, and 7.2.
(*f*) 1,427.16, .244, .32, .032, and 10.0041.
(*g*) 2,473.1, 41.65, .7243, 104.067, and 21.073.
(*h*) 4,107.2, .00375, 21.716, 410.072, and .0345.

Ans. (*a*) 3.7652. (*b*) 805.9647. (*c*) 204.5078. (*d*) 225.9814. (*e*) 234.3285. (*f*) 1,437.7601. (*g*) 2,640.6143. (*h*) 4,539.02625.

SUBTRACTION OF DECIMALS.

143. As in subtraction of whole numbers, units are placed under units, tens under tens, etc., bringing the decimal points under each other, as in addition of decimals.

EXAMPLE.—Subtract .132 from .3063.

SOLUTION.—

minuend .3063
subtrahend .132
difference .1743 Ans.

144. EXAMPLE.—What is the difference between 7.895 and .725?

SOLUTION.—

minuend 7.895
subtrahend .725
difference 7.170 or 7.17. Ans.

145. EXAMPLE.—Subtract .625 from 11.

SOLUTION.—

minuend	11.000	
subtrahend	.625	
difference	10.375	Ans.

146. **Rule.**—*Place the subtrahend under the minuend, so that the decimal points will be directly under each other. Subtract as in whole numbers, and place the decimal point in the remainder directly under the decimal points above.*

When the figures in the decimal part of the subtrahend extend beyond those in the minuend, place ciphers in the minuend above them and subtract as before.

EXAMPLES FOR PRACTICE.

147. From:

(*a*)	407.385 take 235.0004.	(*a*)	172.3846.
(*b*)	22.718 take 1.7042.	(*b*)	21.0138.
(*c*)	1,368.17 take 13.6817.	(*c*)	1,354.4883.
(*d*)	70.00017 take 7.000017.	(*d*)	63.000153.
(*e*)	630.630 take .6304.	(*e*)	629.9996.
(*f*)	421.73 take 217.162.	(*f*)	204.568.
(*g*)	1.000014 take .00001.	(*g*)	1.000004.
(*h*)	.783652 take .542314.	(*h*)	.241338.

Ans.

MULTIPLICATION OF DECIMALS.

148. In multiplication of decimals we do not place the decimal points directly under each other as in addition and subtraction. We pay no attention for the time being to the decimal points. Place the multiplier under the multiplicand, so that the *right-hand* figure of the one is under the *right-hand* figure of the other, and proceed exactly as in multiplication of whole numbers. After multiplying, *count the number of decimal places in both multiplicand and multiplier, and point off the same number in the product.*

EXAMPLE.—Multiply .825 by 13.

SOLUTION.—

multiplicand	.825	
multiplier	13	
	2475	
	825	
product	10.725	Ans.

In this example there are 3 decimal places in the multiplicand and none in the multiplier; therefore, 3 decimal places are pointed off in the product.

149. EXAMPLE.—What is the product of 426 and the decimal .005?

SOLUTION.—

multiplicand	426
multiplier	.005
product	2.130 or 2.13. Ans.

In this example there are 3 decimal places in the multiplier and none in the multiplicand; therefore, 3 decimal places are pointed off in the product.

150. It is not necessary to multiply by the ciphers on the *left* of a decimal; they merely determine the *number of decimal places.* Ciphers to the *right* of a decimal should be omitted, as they only make more figures to deal with, and do not change the value.

151. EXAMPLE.—Multiply 1.205 by 1.15.

SOLUTION.—

multiplicand	1.205
multiplier	1.15
	6025
	1205
	1205
product	1.38575 Ans.

In this example there are 3 decimal places in the multiplicand and 2 in the multiplier; therefore, 3+2, or 5, decimal places must be pointed off in the product.

152. EXAMPLE.—Multiply .232 by .001.

SOLUTION.—

multiplicand	.232
multiplier	.001
product	.000232 Ans.

In this example we multiply the multiplicand by the digit in the multiplier, which gives 232 for the product; but since there are 3 decimal places each in the multiplier and multiplicand, we must prefix 3 ciphers to the 232 to make 3+3, or 6, decimal places in the product.

153. **Rule.**—*Place the multiplier under the multiplicand, disregarding the position of the decimal points. Multiply*

as in whole numbers, and in the product point off as many decimal places as there are decimal places in both multiplier and multiplicand, prefixing ciphers if necessary.

EXAMPLES FOR PRACTICE.

154. Find the product of:

(*a*)	.000492 × 4.1418.	(*a*)	.0020377656.
(*b*)	4,003.2 × 1.2.	(*b*)	4,803.84.
(*c*)	78.6531 × 1.03.	(*c*)	81.012693.
(*d*)	.3685 × .042.	(*d*)	.015477.
(*e*)	178,352 × .01.	(*e*)	1,783.52.
(*f*)	.00045 × .0045.	(*f*)	.000002025.
(*g*)	.714 × .00002.	(*g*)	.00001428.
(*h*)	.00004 × .008.	(*h*)	.00000032.

Ans.

DIVISION OF DECIMALS.

155. In division of decimals we pay no attention to the decimal point until after the division has been performed. The *number of decimal places in the dividend must equal* (*or be made to equal by annexing ciphers*) *the number of decimal places in the divisor.* Divide exactly as in whole numbers. *Subtract the number of decimal places in the divisor from the number of decimal places in the dividend, and point off as many decimal places in the quotient as there are units in the remainder thus found.*

EXAMPLE.—Divide .625 by 25.

SOLUTION.—

divisor dividend quotient

25).625 (.025 Ans.

50

125

125

remainder 0

In this example there are no decimal places in the divisor, and three decimal places in the dividend; therefore, there are 3 minus 0, or 3, decimal places in the quotient. One cipher has to be prefixed to the 25 to make the three decimal places.

156. EXAMPLE.—Divide 6.035 by .05.

divisor dividend quotient

SOLUTION.— .05)6.035(120.7 Ans.

5

10

10

35

35

remainder 0

In this example we divide by 5, as if the cipher were not before it. There is one more decimal place in the dividend than in the divisor; therefore, one decimal place is pointed off in the quotient.

157. EXAMPLE.—Divide .125 by .005.

divisor dividend quotient

SOLUTION.— .005).125(25 Ans.

10

25

25

remainder 0

In this example there are the same number of decimal places in the dividend as in the divisor; therefore, the quotient has no decimal places, and is a whole number.

158. EXAMPLE.—Divide 326 by .25.

divisor dividend quotient

SOLUTION.— .25)326.00(1304 Ans.

25

76

75

100

100

remainder 0

In this problem two ciphers were annexed to the dividend, to make the number of decimal places equal to the number in the divisor. The quotient is a whole number.

159. EXAMPLE.—Divide .0025 by 1.25.

divisor dividend quotient

SOLUTION.— 1.25).00250(.002 Ans.

250

remainder 0

EXPLANATION.—In this example we are to divide .0025 by 1.25. Consider the dividend as a whole number, i. e., as 25 (disregarding the two ciphers at its left, for the present); also, consider the divisor as a whole number, i. e., as 125. It is clearly evident that the dividend, 25, will not contain the divisor, 125; we must, therefore, annex one cipher to the 25, thus making the dividend 250. 125 is contained twice in 250, so we place the figure 2 in the quotient. In pointing off the decimal places in the quotient, it must be remembered that there were only four decimal places in the dividend; but one cipher was annexed, thereby making $4+1$, or 5, decimal places. Since there are five decimal places in the dividend and two decimal places in the divisor, we must point off $5-2$, or 3, decimal places in the quotient. In order to point off three decimal places, two ciphers must be prefixed to the figure 2, thereby making .002 the quotient. It is not necessary to consider the ciphers at the left of a decimal when dividing, except when determining the position of the decimal point in the quotient.

160. RULE.—I. *Place the divisor to the left of the dividend, and proceed as in division of whole numbers; in the quotient, point off as many decimal places as the number of decimal places in the dividend exceed those in the divisor, prefixing ciphers to the quotient, if necessary.*

II. *If in dividing one number by another there be a remainder, the remainder can be placed over the divisor, as a fractional part of the quotient, but it is generally better to annex ciphers to the remainder, and continue dividing until there are 3 or 4 decimal places in the quotient, and then if there still be a remainder, terminate the quotient by the plus sign* (+), *which shows that it can be carried further.*

161. EXAMPLE.—What is the quotient of 199 divided by 15?

divisor dividend quotient

SOLUTION.— $15)199(13+\frac{4}{15}$ Ans.

$$\begin{array}{r} 15 \\ \hline 49 \\ 45 \\ \hline \textit{remainder}\ \ 4 \end{array}$$

Or, $15)199.000(13.266+$ Ans.

$$\begin{array}{r} 15 \\ \hline 49 \\ 45 \\ \hline 40 \\ 30 \\ \hline 100 \\ 90 \\ \hline 100 \\ 90 \\ \hline \textit{remainder}\ \ 10 \end{array}$$

$13\frac{4}{15} = 13.266+$

$\frac{4}{15} = .266+$

162. It frequently happens, as in the above example, that the division will never terminate. In such cases, decide to how many decimal places the division is to be carried, and carry the work one place further. If the last figure of the quotient thus obtained is 5 or a greater number, increase the preceding figure by 1, and write after it the minus sign (−), thus indicating that the quotient is not quite as large as indicated; if the figure thus obtained is less than 5, write the plus sign (+) after the quotient, thus indicating that the number is slightly greater than as indicated. In the last example, had it been desired to obtain the answer correct to four decimal places, the work would have been carried to five places, obtaining 13.26666, and the answer would have been given as 13.2667 −. This remark applies to any other calculation involving decimals, when it is desired to omit some of the figures in the decimal. Thus, if it was desired to retain three decimal places in the number .2471253, it would be expressed as .247 +; if it was desired to retain five decimal places, it would be expressed as .24713 −. Both the + and − signs are frequently omitted; they are

seldom used outside of arithmetic, except in exact calculations, when it is desired to call particular attention to the fact that the result obtained is not *quite* exact.

EXAMPLES FOR PRACTICE.

163. Divide:

(*a*)	101.6688 by 2.36.	Ans. (*a*)	43.08.
(*b*)	187.12264 by 123.107.	(*b*)	1.52.
(*c*)	.08 by .008.	(*c*)	10.
(*d*)	.0003 by 3.75.	(*d*)	.00008.
(*e*)	.0144 by .024.	(*e*)	.6.
(*f*)	.00375 by 1.25.	(*f*)	.003.
(*g*)	.004 by 400.	(*g*)	.00001.
(*h*)	.4 by .008.	(*h*)	50.

REDUCTION OF DECIMALS.

TO REDUCE A FRACTION TO A DECIMAL.

164. EXAMPLE.— $\frac{3}{4}$ equals what decimal?

SOLUTION.—

$$\begin{array}{r} 4\,)\,3.0\,0 \\ \hline .7\,5 \end{array} \quad \text{or } \tfrac{3}{4} = .75. \quad \text{Ans.}$$

EXAMPLE.—What decimal is equivalent to $\frac{7}{8}$?

SOLUTION.—

$$\begin{array}{l} 8\,)\,7.0\,0\,0\,(\,.8\,7\,5 \\ \quad\;\; \underline{6\,4} \\ \quad\;\;\; 6\,0 \\ \quad\;\;\; \underline{5\,6} \\ \quad\;\;\;\;\; 4\,0 \\ \quad\;\;\;\;\; \underline{4\,0} \\ \quad\;\;\;\;\;\;\; 0 \end{array} \quad \text{or } \tfrac{7}{8} = .875. \quad \text{Ans.}$$

165. **Rule.**—*Annex ciphers to the numerator and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed.*

EXAMPLES FOR PRACTICE.

166. Reduce the following common fractions to decimals:

(*a*)	$\frac{15}{32}$.	Ans. (*a*)	.46875.
(*b*)	$\frac{7}{8}$.	(*b*)	.875.
(*c*)	$\frac{21}{32}$.	(*c*)	.65625.
(*d*)	$\frac{51}{64}$.	(*d*)	.796875.
(*e*)	$\frac{4}{25}$.	(*e*)	.16.
(*f*)	$\frac{5}{8}$.	(*f*)	.625.
(*g*)	$\frac{10}{200}$.	(*g*)	.05.
(*h*)	$\frac{4}{1000}$.	(*h*)	.004.

167. To reduce inches to decimal parts of a foot:

EXAMPLE.—What decimal part of a foot is 9 inches?

SOLUTION.—Since there are 12 inches in one foot, 1 inch is $\frac{1}{12}$ of a foot, and 9 inches is $9 \times \frac{1}{12}$, or $\frac{9}{12}$ of a foot. This reduced to a decimal by the above rule shows what decimal part of a foot 9 inches is.

12)9.00(.75 of a foot. Ans.
84
60
60
0

168. Rule.—I. *To reduce inches to a decimal part of a foot, divide the number of inches by 12.*

II. *Should the resulting decimal be an unending one, and it is desired to terminate the division at some point, say the fourth decimal place, carry the division one place further, and if the fifth figure is 5 or greater increase the fourth figure by 1, omitting the signs + and −.*

EXAMPLES FOR PRACTICE.

169. Reduce to the decimal part of a foot:

(*a*)	3 in.	Ans.	(*a*) .25 ft.
(*b*)	$4\frac{1}{2}$ in.		(*b*) .375 ft.
(*c*)	5 in.		(*c*) .4167 ft.
(*d*)	$6\frac{5}{6}$ in.		(*d*) .5521 ft.
(*e*)	11 in.		(*e*) .9167 ft.

TO REDUCE A DECIMAL TO A FRACTION.

170. EXAMPLE.—Reduce .125 to a fraction.

SOLUTION.— $.125 = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8}$. Ans.

EXAMPLE.—Reduce .875 to a fraction.

SOLUTION.— $.875 = \frac{875}{1000} = \frac{35}{40} = \frac{7}{8}$. Ans.

171. Rule.—*Under the figures of the decimal, place 1 with as many ciphers at its right as there are decimal places in the decimal, and reduce the resulting fraction to its lowest terms by dividing both numerator and denominator by the same number.*

EXAMPLES FOR PRACTICE.

172. Reduce the following to common fractions:

(*a*) .125.
(*b*) .625.
(*c*) .3125.
(*d*) .04.
(*e*) .06.
(*f*) .75.
(*g*) .15625.
(*h*) .875.

Ans. (*a*) $\frac{1}{8}$. (*b*) $\frac{5}{8}$. (*c*) $\frac{5}{16}$. (*d*) $\frac{1}{25}$. (*e*) $\frac{3}{50}$. (*f*) $\frac{3}{4}$. (*g*) $\frac{5}{32}$. (*h*) $\frac{7}{8}$.

---

173. To express a decimal approximatively as a fraction having a given denominator:

174. EXAMPLE.—Express .5827 in 64ths.

SOLUTION.— $.5827 \times \frac{64}{64} = \frac{37.2928}{64}$, say $\frac{37}{64}$.

Hence, $.5827 = \frac{37}{64}$, nearly. Ans.

EXAMPLE.—Express .3917 in 12ths.

SOLUTION.— $.3917 \times \frac{12}{12} = \frac{4.7004}{12}$, say $\frac{5}{12}$.

Hence, $.3917 = \frac{5}{12}$, nearly. Ans.

175. Rule.—*Reduce 1 to a fraction having the given denominator. Multiply the given decimal by the fraction so obtained, and the result will be the fraction required.*

EXAMPLES FOR PRACTICE.

176. Express:

(*a*) .625 in 8ths.
(*b*) .3125 in 16ths.
(*c*) .15625 in 32ds.
(*d*) .77 in 64ths.
(*e*) .81 in 48ths.
(*f*) .923 in 96ths.

Ans. (*a*) $\frac{5}{8}$. (*b*) $\frac{5}{16}$. (*c*) $\frac{5}{32}$. (*d*) $\frac{49}{64}$. (*e*) $\frac{39}{48}$. (*f*) $\frac{89}{96}$.

---

177. The sign for dollars is \$. It is read *dollars*. \$25 is read *25 dollars*.

Since there are 100 cents in a dollar, 1 cent is 1 one-hundredth of a dollar; the first two figures of a decimal part of

a dollar represent *cents*. Since a mill is $\frac{1}{10}$ of a cent, or $\frac{1}{1000}$ of a dollar, the third figure represents *mills*.

Thus, \$25.16 is read *twenty-five dollars and sixteen cents;* \$25.168 is read *twenty-five dollars sixteen cents and eight mills.*

SYMBOLS OF AGGREGATION.

178. The **vinculum** ——, **parenthesis** ( ), **brackets** [ ], and **brace** { } are called **symbols of aggregation,** and are used to include numbers which are to be considered together; thus, $13 \times \overline{8-3}$, or $13 \times (8-3)$, shows that 3 is to be taken from 8 before multiplying by 13.

$$13 \times (8-3) = 13 \times 5 = 65.$$

$$13 \times \overline{8-3} = 13 \times 5 = 65.$$

When the vinculum or parenthesis is not used, we have

$$13 \times 8 - 3 = 104 - 3 = 101.$$

179. In any series of numbers connected by the signs $+$, $-$, $\times$, and $\div$, the operations indicated by the signs must be performed in order from left to right, *except* that no addition or subtraction may be performed if a sign of multiplication or division *follows* the number on the *right* of a sign of addition or subtraction until the indicated multiplication or division has been performed. In all cases the sign of multiplication takes the precedence, the reason being that when two or more numbers or expressions are connected by the sign of multiplication the numbers thus connected are regarded as factors of the product indicated, and not as separate numbers.

EXAMPLE.—What is the value of $4 \times 24 - 8 + 17$?

SOLUTION.—Performing the operations in order from left to right, $4 \times 24 = 96$; $96 - 8 = 88$; $88 + 17 = 105$. Ans.

180. EXAMPLE.—What is the value of the following expression: $1{,}296 \div 12 + 160 - 22 \times 3\frac{1}{2} = ?$

SOLUTION.— $1{,}296 \div 12 = 108$; $108 + 160 = 268$; here we cannot subtract 22 from 268 because the sign of multiplication follows 22; hence, multiplying 22 by $3\frac{1}{2}$, we get 77, and $268 - 77 = 191$. Ans.

Had the above expression been written $1{,}296 \div 12 + 160 - 22 \times 3\frac{1}{2} \div 7 + 25$, it would have been necessary to have divided $22 \times 3\frac{1}{2}$ by 7 before subtracting, and the final result would have been $22 \times 3\frac{1}{2} = 77$; $77 \div 7 = 11$; $268 - 11 = 257$; $257 + 25 = 282$. Ans. In other words, it is necessary to perform all the indicated multiplication or division included between the signs + and −, or − and +, before adding or subtracting. Also, had the expression been written $1{,}296 \div 12 + 160 - 24\frac{1}{2} \div 7 \times 3\frac{1}{2} + 25$, it would have been necessary to have multiplied $3\frac{1}{2}$ by 7 before dividing $24\frac{1}{2}$, since the sign of multiplication takes the precedence, and the final result would have been $3\frac{1}{2} \times 7 = 24\frac{1}{2}$; $24\frac{1}{2} \div 24\frac{1}{2} = 1$; $268 - 1 = 267$; $267 + 25 = 292$. Ans.

It likewise follows that if a succession of multiplication and division signs occur, the indicated operations must not be performed in order, from left to right—the multiplication must be performed first. Thus, $24 \times 3 \div 4 \times 2 \div 9 \times 5 = \frac{1}{5}$. Ans. In order to obtain the same result that would be obtained by performing the indicated operations in order, from left to right, symbols of aggregation must be used. Thus, by using two vinculums the last expression becomes $24 \times \overline{3 \div 4} \times \overline{2 \div 9} \times 5 = 20$, the same result that would be obtained by performing the indicated operations in order, from left to right.

EXAMPLES FOR PRACTICE.

181. Find the values of the following expressions:

(*a*)	$(8 + 5 - 1) \div 4$.	Ans.	(*a*) 3.
(*b*)	$5 \times 24 - 32$.		(*b*) 88.
(*c*)	$5 \times 24 \div 15$.		(*c*) 8.
(*d*)	$144 - 5 \times 24$.		(*d*) 24.
(*e*)	$(1{,}691 - 540 + 559) \div 3 \times 57$.		(*e*) 10.
(*f*)	$2{,}080 + 120 - 80 \times 4 - 1{,}670$.		(*f*) 210.
(*g*)	$\overline{(90 + 60 \div 25)} \times 5 - 29$.		(*g*) 1.
(*h*)	$\overline{90 + 60 \div 25} \times 5$.		(*h*) 1.2.

ARITHMETIC.

(SECTION 4.)

PERCENTAGE.

1. **Percentage** is the process of calculating by *hundredths.*

2. The *term* **per cent.** is an abbreviation of the Latin words *per centum*, which mean *by the hundred.* A certain per cent. of a number is the number of hundredths of that number which is indicated by the number of units in the per cent. Thus, 6 per cent. of 125 is $125 \times \frac{6}{100} = 7.5$; 25 per cent. of 80 is $80 \times \frac{25}{100} = 20$; 43 per cent. of 432 pounds is $432 \times \frac{43}{100} = 185.76$ pounds.

3. The **sign** of per cent. is %, and is read *per cent.* Thus, 6% is read *six per cent.*; $12\frac{1}{2}\%$ is read *twelve and one-half per cent.*, etc.

When expressing the per cent. of a number to use in calculations, it is customary to express it decimally instead of fractionally. Thus, instead of expressing 6%, 25%, and 43% as $\frac{6}{100}$, $\frac{25}{100}$, and $\frac{43}{100}$, it is usual to express them as .06, .25, and .43.

The following table will show how any per cent. can be expressed either as a decimal or as a fraction:

Per Cent.	Decimal.	Fraction.	Per Cent.	Decimal.	Fraction.
1%.....	.01	$\frac{1}{100}$	150%.....	1.50	$\frac{150}{100}$ or $1\frac{1}{2}$
2%.....	.02	$\frac{2}{100}$ or $\frac{1}{50}$	500%.....	5.00	$\frac{500}{100}$ or 5
5%.....	.05	$\frac{5}{100}$ or $\frac{1}{20}$	$\frac{1}{4}\%$.....	.0025	$\frac{1}{100}$ or $\frac{1}{400}$
10%.....	.10	$\frac{10}{100}$ or $\frac{1}{10}$	$\frac{1}{2}\%$.....	.005	$\frac{1}{100}$ or $\frac{1}{200}$
25%.....	.25	$\frac{25}{100}$ or $\frac{1}{4}$	$1\frac{1}{2}\%$.....	.015	$\frac{1\frac{1}{2}}{100}$ or $\frac{3}{200}$
50%.....	.50	$\frac{50}{100}$ or $\frac{1}{2}$	$8\frac{1}{3}\%$.....	.08$\frac{1}{3}$	$\frac{8\frac{1}{3}}{100}$ or $\frac{1}{12}$
75%.....	.75	$\frac{75}{100}$ or $\frac{3}{4}$	$12\frac{1}{2}\%$.....	.125	$\frac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$
100%.....	1.00	$\frac{100}{100}$ or 1	$16\frac{2}{3}\%$.....	.16$\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}$ or $\frac{1}{6}$
125%.....	1.25	$\frac{125}{100}$ or $1\frac{1}{4}$	$62\frac{1}{2}\%$.....	.625	$\frac{62\frac{1}{2}}{100}$ or $\frac{5}{8}$

For notice of copyright, see page immediately following the title page.

4. The names of the different elements used in percentage are: the *base*, the *rate per cent.*, the *percentage*, the *amount*, and the *difference*.

5. The **base** is the number on which the per cent. is computed.

6. The **rate** is the number of hundredths of the base to be taken.

7. The **percentage** is the part, or number of *hundredths*, of the base indicated by the rate; or, the percentage is the result obtained by multiplying the base by the rate.

Thus, when it is stated that 7% of \$25 is \$1.75, \$25 is the base, 7% is the rate, and \$1.75 is the percentage.

8. The **amount** is the sum of the base and percentage.

9. The **difference** is the remainder obtained by subtracting the percentage from the base.

Thus, if a man has \$180, and he earns 6% more, he will have altogether $\$180 + \$180 \times .06$, or $\$180 + \$10.80 = \$190.80$. Here \$180 is the base; 6%, the rate; \$10.80, the percentage; and \$190.80, the *amount*.

Again, if an engine of 125 horsepower uses 16% of it in overcoming friction and other resistances, the amount left for obtaining useful work is $125 - 125 \times .16 = 125 - 20 = 105$ horsepower. Here 125 is the base; 16%, the rate; 20, the percentage; and 105, the *difference*.

10. From the foregoing it is evident that to find the percentage, the base must be multiplied by the rate. Hence, the following

Rule.—*To find the percentage, multiply the base by the rate expressed decimally.*

EXAMPLE.—Out of a lot of 300 bushels of apples 76% were sold how many bushels were sold?

SOLUTION.— 76%, the rate, expressed decimally, is .76; the base is 300; hence, the number of bushels sold, or the percentage, is, by the above rule,

$$300 \times .76 = 228 \text{ bushels.} \quad \text{Ans.}$$

Expressing the rule as a

Formula, $percentage = base \times rate$.

11. When the percentage and rate are given, the base may be found by dividing the percentage by the rate. For, suppose that 12 is 6%, or $\frac{6}{100}$, of some number; then 1%, or $\frac{1}{100}$, of the number, is $12 \div 6$, or 2. Consequently, if $2 = 1\%$, or $\frac{1}{100}$, 100%, or $\frac{100}{100} = 2 \times 100 = 200$. But, since the same result may be arrived at by dividing 12 by .06, for $12 \div .06 = 200$, it follows that:

Rule.—*When the percentage and rate are given, to find the base, divide the percentage by the rate expressed decimally.*

Formula, *base = percentage ÷ rate.*

EXAMPLE.—Bought a certain number of bushels of apples and sold 76% of them. If I sold 228 bushels, how many bushels did I buy?

SOLUTION.—Here 228 is the percentage, and 76%, or .76, is the rate; hence, applying the rule,

$228 \div .76 = 300$ bushels. Ans.

12. When the base and percentage are given, to find the rate, the rate may be found, expressed decimally, by dividing the percentage by the base. For, suppose that it is desired to find what per cent. 12 is of 200. 1% of 200 is $200 \times .01 = 2$. Now, if 1% is 2, 12 is evidently as many per cent. as the number of times that 2 is contained in 12, or $12 \div 2 = 6\%$. But the same result may be obtained by dividing 12, the percentage, by 200, the base, since $12 \div 200 = .06 = 6\%$. Hence,

Rule.—*When the percentage and base are given, to find the rate, divide the percentage by the base, and the result will be the rate expressed decimally.*

Formula, *rate = percentage ÷ base.*

EXAMPLE.—Bought 300 bushels of apples and sold 228 bushels. What per cent. of the total number of bushels was sold?

SOLUTION.—Here 300 is the base and 228 is the percentage; hence, applying rule, rate $= 228 \div 300 = .76 = 76\%$. Ans.

EXAMPLE.—What per cent. of 875 is 25?

SOLUTION.—Here 875 is the base, and 25 is the percentage; hence, applying rule, $25 \div 875 = .02\frac{6}{7} = 2\frac{6}{7}\%$. Ans.

PROOF.— $875 \times .02\frac{6}{7} = 25$.

EXAMPLES FOR PRACTICE.

13. What per cent. of:

(*a*)	360 is 90?	Ans.	(*a*) 25%.
(*b*)	900 is 360?		(*b*) 40%.
(*c*)	125 is 25?		(*c*) 20%.
(*d*)	150 is 750?		(*d*) 500%.
(*e*)	280 is 112?		(*e*) 40%.
(*f*)	400 is 200?		(*f*) 50%.
(*g*)	47 is 94?		(*g*) 200%.
(*h*)	500 is 250?		(*h*) 50%.

14. The amount may be found, when the base and rate are given, by multiplying the base by 1 plus the rate, expressed decimally. For, suppose that it is desired to find the amount when 200 is the base and 6% is the rate. The percentage is $200 \times .06 = 12$, and, according to definition, Art. **8**, the amount is $200 + 12 = 212$. But, the same result may be obtained by multiplying 200 by $1 + .06$, or 1.06, since $200 \times 1.06 = 212$. Hence,

Rule.—*When the base and rate are given, to find the amount, multiply the base by 1 plus the rate expressed decimally.*

Formula, *amount* = *base* $\times (1 +$ *rate*$)$.

EXAMPLE.—If a man earned $725 in a year, and the next year 10% more, how much did he earn the second year?

SOLUTION.—Here 725 is the base and 10% is the rate, and the amount is required. Hence, applying the rule,

$$725 \times 1.10 = \$797.50. \quad \text{Ans.}$$

15. When the base and rate are given, the difference may be found by multiplying the base by 1 minus the rate expressed decimally. For, suppose that it is desired to find the difference when the base is 200 and the rate is 6%. The percentage is $200 \times .06 = 12$; and, according to definition, Art. **9**, the difference $= 200 - 12 = 188$. But, the same result may be obtained by multiplying 200 by $1 - .06$, or .94, since $200 \times .94 = 188$. Hence,

Rule.—*When the base and rate are given, to find the difference, multiply the base by 1 minus the rate expressed decimally.*

Formula, *difference* = *base* $\times (1 -$ *rate*$)$.

EXAMPLE.—Bought 300 bushels of apples and sold all but 24% of them. How many bushels were sold?

SOLUTION.—Here 300 is the base, 24% is the rate, and it is desired to find the difference. Hence, applying the rule,

$$300 \times (1 - .24) = 228 \text{ bushels. Ans.}$$

16. When the amount and rate are given, the base may be found by dividing the amount by 1 plus the rate. For, suppose that it is known that 212 equals some number increased by 6% of itself. Then, it is evident that 212 equals 106% of the number (base) that it is desired to find. Consequently, if $212 = 106\%$, $1\% = \frac{212}{106} = 2$. and $100\% = 2 \times 100 = 200 =$ the base. But the same result may be obtained by dividing 212 by $1 + .06$, or 1.06, since $212 \div 1.06 = 200$. Hence,

Rule.—*When the amount and rate are given, to find the base, divide the amount by 1 plus the rate expressed decimally.*

Formula, *base* = *amount* $\div (1 +$ *rate*).

EXAMPLE.—The theoretical discharge of a certain pump when running at a piston speed of 100 feet per minute is 278,910 gallons per day of 10 hours. Owing to leakage and other defects, this value is 25% greater than the actual discharge. What is the actual discharge?

SOLUTION.—Here 278,910 equals the actual discharge (base) increased by 25% of itself. Consequently, 278,910 is the amount, and 25% is the rate. Applying rule,

$$\text{actual discharge} = 278{,}910 \div 1.25 = 223{,}128 \text{ gallons. Ans.}$$

17. When the difference and rate are given, the base may be found by dividing the difference by 1 minus the rate. For, suppose that 188 equals some number less 6% of itself. Then, 188 evidently equals $100 - 6 = 94\%$ of some number. Consequently, if $188 = 94\%$, $1\% = 188 \div 94 = 2$, and $100\% = 2 \times 100 = 200$. But the same result may be obtained by dividing 188 by $1 - .06$, or .94, since $188 \div .94 = 200$. Hence,

Rule.—*When the difference and rate are given, to find the base, divide the difference by 1 minus the rate expressed decimally.*

Formula, *base* = *difference* $\div (1 -$ *rate*).

EXAMPLE.—Bought a certain number of bushels of apples and sold 76% of them. If there were 72 bushels left unsold, how many bushels did I buy?

SOLUTION.—Here 72 is the difference and 76% is the rate. Applying rule,

$$72 \div (1 - .76) = 300 \text{ bushels.} \quad \text{Ans.}$$

EXAMPLE.—The theoretical number of foot-pounds of work per minute required to operate a boiler feed-pump is 127,344. If 30% of the total number actually required be allowed for friction, leakage, etc., how many foot-pounds are actually required to work the pump?

SOLUTION.—Here the number actually required is the base; hence, 127,344 is the difference, and 30% is the rate. Applying the rule,

$$127{,}344 \div (1 - .30) = 181{,}920 \text{ foot-pounds.} \quad \text{Ans.}$$

18. EXAMPLE.—A certain chimney gives a draft of 2.76 inches of water. By increasing the height 20 feet, the draft was increased to 3 inches of water. What was the gain per cent.?

SOLUTION.—Here it is evident that 3 inches is the amount, and that 2.76 inches is the base. Consequently, $3 - 2.76 = .24$ inch is the percentage, and it is required to find the rate. Hence, applying the rule given in Art. **12**,

$$\text{gain per cent.} = .24 \div 2.76 = .087 = 8.7\%. \quad \text{Ans.}$$

19. EXAMPLE.—A certain chimney gave a draft of 3 inches of water. After an economizer had been put in, the draft was reduced to 1.2 inches of water. What was the loss per cent.?

SOLUTION.—Here it is evident that 1.2 inches is the difference (since it equals 3 inches diminished by a certain per cent. loss of itself), and 3 inches is the base. Consequently, $3 - 1.2 = 1.8$ inches is the percentage. Hence, applying the rule given in Art. **12**,

$$\text{loss per cent.} = 1.8 \div 3 = .60 = 60\%. \quad \text{Ans.}$$

20. To find the gain or loss per cent.:

Rule.—*Find the difference between the initial and the final value; divide this difference by the initial value.*

EXAMPLE.—If a man buys a house for $1,860, and some time afterwards builds a barn for 25% of the cost of the house, does he gain or lose, and how much per cent., if he sells both house and barn for $2,100?

SOLUTION.—The cost of the barn was $\$1{,}860 \times .25 = \$465$; consequently, the initial value, or total cost, was $\$1{,}860 + \$465 = \$2{,}325$. Since he sold them for $2,100 he lost $\$2{,}325 - \$2{,}100 = \$225$. Hence, applying rule,

$$225 \div 2{,}325 = .0968 = 9.68\% \text{ loss.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

21. Solve the following:

(*a*)	What is $12\frac{1}{2}\%$ of \$900?	(*a*)	\$112.50.
(*b*)	What is $\frac{4}{5}\%$ of 627?	(*b*)	5.016.
(*c*)	What is $33\frac{1}{3}\%$ of 54?	(*c*)	18.
(*d*)	101 is $68\frac{3}{4}\%$ of what number?	(*d*)	$146\frac{10}{11}$.
(*e*)	784 is $83\frac{1}{3}\%$ of what number?	(*e*)	940.8.
(*f*)	What % of 960 is 160?	(*f*)	$16\frac{2}{3}\%$.
(*g*)	What % of \$3,606 is $\$450\frac{3}{4}$?	(*g*)	$12\frac{1}{2}\%$.
(*h*)	What % of 280 is 112?	(*h*)	40%.

Ans.

1. A steam plant consumed an average of 3,640 pounds of coal per day. The engineer made certain alterations which resulted in a saving of 250 pounds per day. What was the per cent. of coal saved?

Ans. 7%, nearly.

2. If the speed of an engine running at 126 revolutions per minute should be increased $6\frac{1}{2}\%$, how many revolutions per minute would it then make? Ans. 134.19 rev.

3. The list price of a lot of silk goods is \$1,400, of some laces \$1,150, and of some calico \$340. If 25% discount was allowed on the silk, 22% on the laces, and $12\frac{1}{2}\%$ on the calico, what was the actual cost of the purchase? Ans. \$2,244.50.

4. If I loan a man \$1,100, and this is $18\frac{1}{2}\%$ of the amount that I have on interest, how much money have I on interest? Ans. \$5,945.95.

5. A test showed that an engine developed 190.4 horsepower, 15% of which was consumed in friction. How much power was available for use? Ans. 161.84 H. P.

6. By adding a condenser to a steam engine, the power was increased 14% and the consumption of coal per horsepower per hour was decreased 20%. If the engine could originally develop 50 horsepower, and required $3\frac{1}{2}$ pounds of coal per horsepower per hour, what would be the total weight of coal used in an hour, with the condenser, assuming the engine to run full power? Ans. 159.6 pounds.

DENOMINATE NUMBERS.

22. A **denominate number** is a concrete number, and may be either simple or compound; as, 8 quarts; 5 feet; 10 inches, etc.

23. A **simple denominate number** consists of units of but one denomination; as, 16 cents; 10 hours; 5 dollars, etc.

24. A **compound denominate number** consists of units of two or more denominations of a similar kind; as, 3 yards, 2 feet, 1 inch; 34 square feet, 57 square inches.

25. In **whole numbers** and in **decimals** the law of increase and decrease is on the scale of 10, but in **compound** or **denominate numbers** the scale varies.

26. A **measure** is a *standard unit*, established by *law* or *custom*, by which quantity of any kind is measured. The standard unit of **dry measure** is the Winchester bushel; of **weight,** the pound; of **liquid measure,** the gallon, etc.

27. Measures are of six kinds:

1. Extension.
2. Weight.
3. Capacity.
4. Time.
5. Angles.
6. Money or value.

MEASURES OF EXTENSION.

28. **Measures of extension** are used in measuring lengths, distances, surfaces, and solids.

LINEAR MEASURE.

TABLE.

Abbreviation.								
12 inches (in.) = 1 foot . .	ft.	in.	ft.	yd.	rd.	fur.	mi.
3 feet . . . = 1 yard .	yd.	36 =	3	= 1			
5.5 yards . . = 1 rod . .	rd.	198 =	$16\frac{1}{2}$	= 5.5	= 1		
40 rods . . . = 1 furlong	fur.	7,920 =	660	= 220	= 40	= 1	
8 furlongs . = 1 mile .	mi.	63,360 =	5,280	= 1,760	= 320	= 8	= 1

SURVEYOR'S LINEAR MEASURE.

TABLE.

7.92 inches	= 1 link	li.
25 links	= 1 rod	rd.
4 rods 100 links	= 1 chain	ch.
80 chains	= 1 mile	mi.

mi.	ch.	rd.	li.	in.
1 =	80 =	320 =	8,000 =	63,360

29. The linear unit, generally used by surveyors, is **Gunter's chain,** which is equal to 4 rods, or 66 feet.

30. An **engineer's chain,** used by civil engineers, is 100 feet long, and consists of 100 links. In computations, the links are written as so many hundredths of a chain.

SQUARE MEASURE.

TABLE.

144 square inches (sq. in.) . . . = 1 square foot . . . . sq. ft.
9 square feet . . . . . = 1 square yard . . . . sq. yd.
30¼ square yards . . . . . = 1 square rod . . . . sq. rd.
160 square rods . . . . . = 1 acre . . . . . A.
640 acres . . . . . = 1 square mile . . . . sq. mi.

sq. mi.	A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
1	= 640	= 102,400	= 3,097,600	= 27,878,400	= 4,014,489,600

SURVEYOR'S SQUARE MEASURE.

TABLE.

625 square links (sq. li.) . . . . = 1 square rod . . . . sq. rd.
16 square rods . . . . . = 1 square chain . . . sq. ch.
10 square chains . . . . . = 1 acre . . . . . A.
640 acres. . . . . = 1 square mile . . . . sq. mi.
36 square miles (6 mi. square) . . = 1 township . . . . . Tp.

sq. mi.	A.	sq. ch.	sq. rd.	sq. li.
1	= 640	= 6,400	= 102,400	= 64,000,000

CUBIC MEASURE.

TABLE.

1,728 cubic inches (cu. in.) . . . = 1 cubic foot . . . . cu. ft.
27 cubic feet . . . . . = 1 cubic yard . . . . cu. yd.
128 cubic feet . . . . . = 1 cord . . . . . cd.
24¾ cubic feet . . . . . = 1 perch . . . . . P.

cu. yd.	cu. ft.	cu. in.
1	= 27	= 46,656

MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT.

TABLE.

16 ounces (oz.) . . . . . = 1 pound . . . . . lb.
100 pounds . . . . . = 1 hundredweight . . . cwt.
20 cwt., or 2,000 lb. . . . . = 1 ton . . . . . T.

T.	cwt.	lb.	oz.
1	= 20	= 2.000	= 32,000

31. The ounce is divided into halves, quarters, etc. Avoirdupois weight is used for weighing coarse and heavy articles. One avoirdupois pound contains 7,000 grains.

LONG TON TABLE.

16 ounces	=	1 pound	lb.
112 pounds	=	1 hundredweight	cwt.
20 cwt., or 2,240 lb.	=	1 ton	T.

32. In all the calculations throughout this and the following sections, 2,000 pounds will be considered 1 ton, unless the long ton (2,240 pounds) is especially mentioned.

TROY WEIGHT.

TABLE.

24 grains (gr.)	=	1 pennyweight	pwt.
20 pennyweights	=	1 ounce	oz.
12 ounces	=	1 pound	lb.

lb. oz. pwt. gr.
1 = 12 = 240 = 5,760

33. Troy weight is used in weighing gold and silverware, jewels, etc. It is used by jewelers.

MEASURES OF CAPACITY.

LIQUID MEASURE.

TABLE.

4 gills (gi.)	=	1 pint	pt.
2 pints	=	1 quart	qt.
4 quarts	=	1 gallon	gal.
31½ gallons	=	1 barrel	bbl.
2 barrels, or 63 gallons	=	1 hogshead	hhd.

hhd. bbl. gal. qt. pt. gi.
1 = 2 = 63 = 252 = 504 = 2,016

DRY MEASURE.

TABLE.

2 pints (pt.)	=	1 quart	qt.
8 quarts	=	1 peck	pk.
4 pecks	=	1 bushel	bu.

bu. pk. qt. pt.
1 = 4 = 32 = 64

MEASURE OF TIME.

TABLE.

60 seconds (sec.)	=	1 minute	min.
60 minutes	=	1 hour	hr.
24 hours	=	1 day	da
7 days	=	1 week	wk.
365 days, 12 months	=	1 common year	yr
366 days	=	1 leap year.	
100 years	=	1 century.	

NOTE.—It is customary to consider one month as 30 days.

MEASURE OF ANGLES OR ARCS.

TABLE.

60 seconds (″)	=	1 minute	′
60 minutes	=	1 degree	°
90 degrees	=	1 right angle or quadrant	∟
360 degrees	=	1 circle	cir.

1 cir. = 360° = 21,600′ = 1,296,000″

MEASURE OF MONEY.

UNITED STATES MONEY.

TABLE.

10 mills (m.)	=	1 cent	ct.
10 cents	=	1 dime	d.
10 dimes	=	1 dollar	$.
10 dollars	=	1 eagle	E.

E. $ d. ct. m.
1 = 10 = 100 = 1,000 = 10,000

MISCELLANEOUS TABLE.

12 things are 1 dozen.
12 dozen are 1 gross.
12 gross are 1 great gross.
2 things are 1 pair.
20 things are 1 score.
1 league is 3 miles.
1 fathom is 6 feet.
1 meter is nearly 39.37 inches.
1 hand is 4 inches.
1 palm is 3 inches.
1 span is 9 inches.
24 sheets are 1 quire.
20 quires, or 480 sheets, are 1 ream
1 bushel contains 2,150.4 cubic in

1 U. S. standard gallon (also called a wine gallon) contains 231 cubic in
1 U. S. standard gallon of water weighs 8.355 pounds, nearly.
1 cubic foot of water contains 7.481 U. S. standard gallons, nearly.
1 British imperial gallon weighs 10 pounds.

It will be of great advantage to the student to carefully memorize all the above tables.

REDUCTION OF DENOMINATE NUMBERS.

34. **Reduction** of denominate numbers is the process of changing their denomination without changing their value. They may be changed from a higher to a lower denomination, or from a lower to a higher—either is reduction. As

2 hours = 120 minutes.
32 ounces = 2 pounds.

35. **Principle.**—Denominate numbers are changed to lower denominations by *multiplying*, and to higher denominations by *dividing*.

To reduce denominate numbers to lower denominations:

36. EXAMPLE.—Reduce 5 yd. 2 ft. 7 in. to inches.

SOLUTION.—

yd.	ft.	in.
5	2	7
3		
15 ft.		
2 ft.		
17 ft.		
12		
34		
17		
204 in.		
7 in.		
211 inches. Ans.		

EXPLANATION.—Since there are 3 feet in 1 yard, in 5 yards there are 5×3 or 15 feet, and 15 feet plus 2 feet = 17 feet. There are 12 inches in a foot; therefore, $12 \times 17 = 204$ inches, and 204 inches plus 7 inches = 211 inches = number of inches in 5 yards 2 feet and 7 inches.

37. EXAMPLE.—Reduce 6 hours to seconds.

SOLUTION.—

6	hours.
60	
360	minutes.
60	
21600	seconds. Ans.

EXPLANATION.—As there are 60 minutes in 1 hour, in 6 hours there are 6×60, or 360, minutes; as there are no minutes to add, we multiply 360 minutes by 60, to get the number of seconds.

38. In order to avoid mistakes, if any denomination be omitted, represent it by a cipher. Thus, before reducing 3 rods 6 inches to inches, insert a cipher for yards and a cipher for feet, as

rd.	yd.	ft.	in.
3	0	0	6

39. **Rule.**—*Multiply the number representing the highest denomination by the number of units in the next lower required to make one of the higher denomination, and to the product add the number of given units of that lower denomination. Proceed in this manner until the number is reduced to the required denomination.*

EXAMPLES FOR PRACTICE.

40. Reduce:

(*a*) 4 rd. 2 yd. 2 ft. to ft.
(*b*) 4 bu. 3 pk. 2 qt. to qt.
(*c*) 13 rd. 5 yd. 2 ft. to ft.
(*d*) 5 mi. 100 rd. 10 ft. to ft.
(*e*) 8 lb. 4 oz. 6 pwt. to gr.
(*f*) 52 hhd. 24 gal. 1 pt. to pt.
(*g*) 5 cir. 16° 20′ to minutes.
(*h*) 14 bu. to qt.

Ans. (*a*) 74 ft. (*b*) 154 qt. (*c*) 231.5 ft. (*d*) 28,060 ft. (*e*) 48,144 gr. (*f*) 26,401 pt. (*g*) 108,980′. (*h*) 448 qt.

To reduce lower to higher denominations:

41. EXAMPLE.—Reduce 211 inches to higher denominations.

SOLUTION.—

$$\begin{array}{r} 12\,)\,2\,1\,1 \text{ in.} \\ \hline 3\,)\,1\,7 \text{ ft.} + 7 \text{ in.} \\ \hline 5 \text{ yd.} + 2 \text{ ft.} \end{array}$$ Ans.

EXPLANATION.—There are 12 inches in 1 foot; therefore, 211 divided by 12 = 17 feet and 7 inches over. There are 3 feet in 1 yard; therefore, 17 feet divided by 3 = 5 yards

and 2 feet over. The last quotient and the two remainders constitute the answer, 5 yards 2 feet 7 inches.

42. EXAMPLE.—Reduce 15,735 grains Troy weight to higher denominations.

SOLUTION.—

```
24)15735 gr. (655 pwt.
   144
   ---
    133
    120
    ---
     135
     120
     ---
      15 gr.
20)655 pwt. (32 oz.
   60
   --
    55
    40
    --
    15 pwt.
12)32 oz. (2 lb.
   24
   --
    8 oz.
```

EXPLANATION.—There are 24 grains in 1 pennyweight, and in 15,735 grains there are as many pennyweights as 24 is contained in 15,735, or 655 pennyweights and 15 grains remaining. There are 20 pennyweights in 1 ounce, and in 655 pennyweights there are 32 ounces and 15 pennyweights remaining. There are 12 ounces in 1 pound, and in 32 ounces there are 2 pounds and 8 ounces remaining. The last quotient and the three remainders constitute the answer, 2 pounds 8 ounces 15 pennyweights 15 grains.

The above problem is worked out by long division, because the numbers are too large to solve easily by short division. The student may use either method.

43. Rule.—*Divide the number representing the denomination given by the number of units of this denomination required to make one unit of the next higher denomination. The remainder will be of the same denomination, but the quotient will be of the next higher. Divide this quotient by the number of units of its denomination required to make one unit of the next higher. Continue until the highest*

denomination is reached, or until there is not enough of a denomination left to make one of the next higher. The last quotient and the remainders constitute the required result.

EXAMPLES FOR PRACTICE.

44. Reduce to units of higher denominations:

(*a*) 7,460 sq. in.; (*b*) 7,580 sq. yd.; (*c*) 148,760 cu. in.; (*d*) 7,896 cu. ft. to cd.; (*e*) 17,651″; (*f*) 1,120 cu. ft. to cd.; (*g*) 8,000 gi.; (*h*) 36,450 lb.

Ans.
- (*a*) 5 sq. yd. 6 sq. ft. 116 sq. in.
- (*b*) 1 A. 90 sq. rd. 17 sq. yd. 4 sq. ft. 72 sq. in.
- (*c*) 3 cu. yd. 5 cu. ft. 152 cu. in.
- (*d*) 61 cd. 88 cu. ft.
- (*e*) 4° 54′ 11″.
- (*f*) 8 cd. 96 cu. ft.
- (*g*) 3 hhd. 61 gal.
- (*h*) 18 T. 4 cwt. 50 lb.

ADDITION OF DENOMINATE NUMBERS.

45. EXAMPLE.—Find the sum of 3 cwt. 46 lb. 12 oz.; 8 cwt. 12 lb. 13 oz.; 12 cwt. 50 lb. 13 oz.; 27 lb. 4 oz.

SOLUTION.—

T.	cwt.	lb.	oz.	
0	3	46	12	
0	8	12	13	
0	12	50	13	
0	0	27	4	
1	4	37	10	Ans.

EXPLANATION.—Begin to add at the right-hand column: $4+13+13+12 = 42$ ounces; as 16 ounces make 1 pound, 42 ounces $\div 16 = 2$ and a remainder of 10 ounces, or 2 pounds and 10 ounces. Place 10 ounces under ounce column and add 2 pounds to the next or pound column. Then, $2+27+50+12+46 = 137$ pounds; as 100 pounds make a hundredweight, $137 \div 100 = 1$ hundredweight and a remainder of 37 pounds. Place the 37 under the pounds column, and add 1 hundredweight to the next or hundredweight column. Next, $1+12+8+3 = 24$ hundredweight.

20 hundredweight make a ton; therefore $24 \div 20 = 1$ ton and 4 hundredweight remaining. Hence, the sum is 1 ton 4 hundredweight 37 pounds 10 ounces. Ans.

46. EXAMPLE.—What is the sum of 2 rd. 3 yd. 2 ft. 5 in.; 6 rd. 1 ft. 10 in.; 17 rd. 11 in.; 4 yd. 1 ft.?

SOLUTION.—

	rd.	yd.	ft.	in.	
	2	3	2	5	
	6	0	1	10	
	17	0	0	11	
	0	4	1	0	
	26	$3\frac{1}{2}$	0	2	
or	26	3	1	8	Ans.

EXPLANATION.—The sum of the numbers in the first column = 26 inches, or 2 feet and 2 inches remaining. The sum of the numbers in the next column plus 2 feet = 6 feet, or 2 yards and 0 feet remaining. The sum of the next column plus 2 yards = 9 yards, or $9 \div 5\frac{1}{2} = 1$ rod and $3\frac{1}{2}$ yards remaining. The sum of the next column plus 1 rod = 26 rods. To avoid fractions in the sum, the $\frac{1}{2}$ yard is reduced to 1 foot and 6 inches, which added to 26 rods 3 yards 0 feet and 2 inches = 26 rods 3 yards 1 foot 8 inches. Ans.

47. EXAMPLE.—What is the sum of 47 ft. and 3 rd. 2 yd. 2 ft. 10 in.?

SOLUTION.—When 47 ft. is reduced it equals 2 rd. 4 yd. 2 ft. which can be added to 3 rd. 2 yd. 2 ft. 10 in. Thus,

	rd.	yd.	ft.	in.	
	3	2	2	10	
	2	4	2	0	
	6	$1\frac{1}{2}$	1	10	
or	6	2	0	4	Ans.

48. Rule.—*Place the numbers so that like denominations are under each other. Begin at the right-hand column, and add. Divide the sum by the number of units of this denomination required to make one unit of the next higher. Place the remainder under the column added, and carry the quotient to the next column. Continue in this manner until the highest denomination given is reached.*

EXAMPLES FOR PRACTICE.

49. What is the sum of:

(*a*) 25 lb. 7 oz. 15 pwt. 23 gr.; 17 lb. 16 pwt.; 15 lb. 4 oz. 12 pwt.; 18 lb. 16 gr.; 10 lb. 2 oz. 11 pwt. 16 gr.?

(*b*) 9 mi. 13 rd. 4 yd. 2 ft.; 16 rd. 5 yd. 1 ft. 5 in.; 16 mi. 2 rd. 3 in.; 14 rd. 1 yd. 9 in.?

(*c*) 3 cwt. 46 lb. 12 oz.; 12 cwt. $9\frac{1}{2}$ lb.; $2\frac{1}{4}$ cwt. $21\frac{3}{8}$ lb.?

(*d*) 10 yr. 8 mo. 5 wk. 3 da.; 42 yr. 6 mo. 7 da.; 7 yr. 5 mo. 18 wk. 4 da.; 17 yr. 17 da.?

(*e*) 17 T. 11 cwt. 49 lb. 14 oz.; 16 T. 47 lb. 13 oz.; 20 T. 13 cwt. 14 lb. 6 oz.; 11 T. 4 cwt. 16 lb. 12 oz.?

(*f*) 14 sq. yd. 8 sq. ft. 19 sq. in.; 105 sq. yd. 16 sq. ft. 240 sq. in.; 42 sq. yd. 28 sq. ft. 165 sq. in.?

Ans. (*a*) 86 lb. 3 oz. 16 pwt. 7 gr.
(*b*) 25 mi. 47 rd. 1 ft. 5 in.
(*c*) 18 cwt. 2 lb. 14 oz.
(*d*) 78 yr. 1 mo. 3 wk. 3 da.
(*e*) 65 T. 9 cwt. 28 lb. 13 oz.
(*f*) 167 sq. yd. 136 sq. in.

SUBTRACTION OF DENOMINATE NUMBERS.

50. EXAMPLE.—From 21 rd. 2 yd. 2 ft. $6\frac{1}{2}$ in. take 9 rd. 4 yd. $10\frac{1}{4}$ in.

SOLUTION.—

rd.	yd.	ft.	in.	
21	2	2	$6\frac{1}{2}$	
9	4	0	$10\frac{1}{4}$	
11	$3\frac{1}{2}$	1	$8\frac{1}{4}$	Ans.

EXPLANATION.—Since $10\frac{1}{4}$ inches cannot be taken from $6\frac{1}{2}$ inches, we must borrow 1 foot or 12 inches from the 2 feet in the next column and add it to the $6\frac{1}{2}$. $6\frac{1}{2}+12=18\frac{1}{2}$. $18\frac{1}{2}$ inches $-10\frac{1}{4}$ inches $=8\frac{1}{4}$ inches. Then, 0 from the 1 remaining foot = 1 foot. 4 yards cannot be taken from 2 yards; therefore, we borrow 1 rod, or $5\frac{1}{2}$ yards, from 21 rods and add it to 2. $2+5\frac{1}{2}=7\frac{1}{2}$; $7\frac{1}{2}-4=3\frac{1}{2}$ yards. 9 rods from 20 rods = 11 rods. Hence, the remainder is 11 rods $3\frac{1}{2}$ yards 1 foot $8\frac{1}{4}$ inches. Ans

To avoid fractions as much as possible, we reduce the $\frac{1}{2}$ yard to inches, obtaining 18 inches; this added to $8\frac{1}{4}$ inches gives $26\frac{1}{4}$ inches, which equals 2 feet $2\frac{1}{4}$ inches. Then, 2 feet + 1 foot = 3 feet = 1 yard, and 3 yards + 1 yard = 4

yards. Hence, the above answer becomes 11 rods 4 yards 0 feet $2\frac{1}{4}$ inches.

51. EXAMPLE.—What is the difference between 3 rd. 2 yd. 2 ft. 10 in. and 47 ft. ?

SOLUTION.— 47 ft. = 2 rd. 4 yd. 2 ft.

	rd.	yd.	ft.	in.	
	3	2	2	10	
	2	4	2	0	
	0	$3\frac{1}{3}$	0	10	
or		3	2	4	Ans.

To find (approximately) the interval of time between two dates:

52. EXAMPLE.—How many years, months, days, and hours between 4 o'clock P. M. of June 16, 1868, and 10 o'clock A. M., September 29, 1891?

SOLUTION.—

yr.	mo.	da.	hr.	
1891	8	28	10	
1868	5	15	16	
23	3	12	18	Ans.

EXPLANATION.—Counting 24 hours in 1 day, 4 o'clock P. M. is the 16th hour from the beginning of the day, or midnight. On September 29, 8 months and 28 days have elapsed, and on June 16, 5 months and 15 days. After placing the earlier date under the later date, subtract as in the previous problems. Count 30 days as 1 month.

53. **Rule.**—*Place the smaller quantity under the larger quantity, with like denominations under each other. Beginning at the right, subtract successively the number in the subtrahend in each denomination from the one above, and place the differences underneath. If the number in the minuend of any denomination is less than the number under it in the subtrahend, one must be borrowed from the minuend of the next higher denomination, reduced, and added to it.*

EXAMPLES FOR PRACTICE.

54. From:

(*a*) 125 lb. 8 oz. 14 pwt. 18 gr. take 96 lb. 9 oz. 10 pwt. 4 gr.

(*b*) 126 hhd. 27 gal. take 104 hhd. 14 gal. 1 qt. 1 pt.

(*c*) 65 T. 14 cwt. 64 lb. 10 oz. take 16 T. 11 cwt. 14 oz.

(*d*) 148 sq. yd. 16 sq. ft. 142 sq. in. take 132 sq. yd. 136 sq. in.

(*e*) 100 bu. take 28 bu. 2 pk. 5 qt. 1 pt.

(*f*) 14 mi. 34 rd. 16 yd. 13 ft. 11 in. take 3 mi. 27 rd. 11 yd. 4 ft. 10 in.

Ans. (*a*) 28 lb. 11 oz. 4 pwt. 14 gr.
(*b*) 22 hhd. 12 gal. 2 qt. 1 pt.
(*c*) 49 T. 3 cwt. 63 lb. 12 oz.
(*d*) 16 sq. yd. 16 sq. ft. 6 sq. in.
(*e*) 71 bu. 1 pk. 2 qt. 1 pt.
(*f*) 11 mi. 7 rd. 5 yd. 9 ft. 1 in.

MULTIPLICATION OF DENOMINATE NUMBERS.

55. EXAMPLE.—Multiply 7 lb. 5 oz. 13 pwt. 15 gr. by 12.

SOLUTION.—

lb.	oz.	pwt.	gr.	
7	5	13	15	
			12	
89	8	3	12	Ans.

EXPLANATION.— 15 grains $\times 12 = 180$ grains. $180 \div 24 = 7$ pennyweights and 12 grains remaining. Place the 12 in the grain column and carry the 7 pennyweights to the next. Now, $13 \times 12 + 7 = 163$ pennyweights; $163 \div 20 = 8$ ounces and 3 pennyweights remaining. Then, $5 \times 12 + 8 = 68$ ounces; $68 \div 12 = 5$ pounds and 8 ounces remaining. Then, $7 \times 12 + 5 = 89$ pounds. The entire product is 89 pounds 8 ounces 3 pennyweights 12 grains. Ans.

56. Rule.—*Multiply the number representing each denomination by the multiplier and reduce each product to the next higher denomination, writing the remainders under each denomination, and carry the quotient to the next, as in Addition of Denominate Numbers.*

57. In multiplication and division of denominate numbers, it is sometimes easier to reduce the number to the lowest denomination given before multiplying or dividing, especially if the multiplier or divisor is a decimal. Thus, in the example of Art. **55**, had the multiplier been 1.2, the easiest way to multiply would have been to reduce the number to grains; then, multiply by 1.2, and reduce the product to higher denominations. For example, 7 lb. 5 oz. 13 pwt. 15 gr. = 43,047 gr. $43{,}047 \times 1.2 = 51{,}656.4$ gr. = 8 lb. 11 oz. 12 pwt. 8.4 gr. Also, $43{,}047 \times 12 = 516{,}564$ gr. = 89 lb. 8 oz. 3 pwt. 12 gr., as above. Either method may be used.

EXAMPLES FOR PRACTICE.

58. Multiply:

(*a*) 15 cwt. 90 lb. by 5; (*b*) 12 yr. 10 mo. 4 wk. 3 da. by 14; (*c*) 11 mi. 145 rd. by 20; (*d*) 12 gal. 4 pt. by 9; (*e*) 8 cd. 76 cu. ft. by 15; (*f*) 4 hhd. 3 gal. 1 qt. 1 pt. by 12.

Ans. (*a*) 79 cwt. 50 lb.
(*b*) 180 yr. 11 mo. 2 wk.
(*c*) 229 mi. 20 rd.
(*d*) 112 gal. 2 qt.
(*e*) 128 cd. 113 cu. ft.
(*f*) 48 hhd. 40 gal. 2 qt.

DIVISION OF DENOMINATE NUMBERS.

59. EXAMPLE.—Divide 48 lb. 11 oz. 6 pwt. by 8.

SOLUTION.—

	lb.	oz.	pwt.	gr.
8)	48	11	6	0
	6 lb.	1 oz.	8 pwt.	6 gr. Ans.

EXPLANATION.—After placing the quantities as above, proceed as follows: 8 is contained in 48 six times without a remainder. 8 is contained in 11 ounces once, with 3 ounces remaining. $3 \times 20 = 60$; $60 + 6 = 66$ pennyweights; 66 pennyweights $\div 8 = 8$ pennyweights and 2 remaining; 2×24 grains $= 48$ grains; 48 grains $\div 8 = 6$ grains. Therefore, the entire quotient is 6 pounds 1 ounce 8 pennyweights 6 grains. Ans.

EXAMPLE.—A silversmith melted up 2 lb. 8 oz. 10 pwt. of silver, which he made into 6 spoons; what was the weight of each spoon?

SOLUTION.—

	lb.	oz.	pwt.
6)	2	8	10
		5 oz.	8 pwt. 8 gr. Ans.

EXPLANATION.—Since we cannot divide 2 pounds by 6, we reduce it to ounces. 2 pounds = 24 ounces, and 24 ounces + 8 ounces = 32 ounces; 32 ounces $\div 6 = 5$ ounces and 2 ounces over. 2 ounces = 40 pennyweights; 40 pennyweights + 10 pennyweights = 50 pennyweights, and 50 pennyweights $\div 6 = 8$ pennyweights and 2 pennyweights over. 2 pennyweights = 48 grains, and 48 grains $\div 6 = 8$ grains. Hence, each spoon contains 5 ounces 8 pennyweights 8 grains. Ans.

60. Example.—Divide 820 rd. 4 yd. 2 ft. by 112.

Solution.—

```
                    rd.  yd. ft.  rd. yd. ft.  in.
112)820  4  2(  7  1  2  5.143 Ans.
    784
     36 rd. rem.
     5.5
    180
   180
   198.0 yd.
      4
112)202 yd. ( 1 yd.
    112
     90 yd. rem.
      3
    270 ft.
      2 ft.
112)272 ft. ( 2 ft.
    224
     48 ft. rem.
     12
     96
    48
112)576 in. ( 5.1428+ in. or 5.143 in.
    560
     160
     112
      480
      448
       320
       224
        960
        896
         64
```

Explanation.—The first quotient is 7 rods with 36 rods remaining. $5.5 \times 36 = 198$ yards; 198 yards + 4 yards = 202 yards; 202 yards $\div 112 = 1$ yard and 90 yards remaining. $90 \times 3 = 270$ feet; 270 feet + 2 feet = 272 feet; 272 feet $\div 112 = 2$ feet, and 48 feet remaining; $48 \times 12 = 576$ inches; 576 inches $\div 112 = 5.143$ inches, nearly. Ans.

The preceding example is solved by long division, because

the numbers are too large to deal with mentally. Instead of expressing the last result as a decimal, it might have been expressed as a common fraction. Thus, $576 \div 112 = 5\frac{16}{112} = 5\frac{1}{7}$ inches. The chief advantage of using a common fraction is that if the quotient be multiplied by the divisor, the result will always be the same as the original dividend.

61. Rule.—*Find how many times the divisor is contained in the first or highest denomination of the dividend. Reduce the remainder (if any) to the next lower denomination, and add to it the number in the given dividend expressing that denomination. Divide this new dividend by the divisor. The quotient will be the next denomination in the quotient required. Continue in this manner until the lowest denomination is reached. The successive quotients will constitute the entire quotient.*

EXAMPLES FOR PRACTICE.

62. Divide:

(*a*) 376 mi. 276 rd. by 22; (*b*) 1,137 bu. 3 pk. 4 qt. 1 pt. by 10; (*c*) 84 cwt. 48 lb. 49 oz. by 16; (*d*) 78 sq. yd. 18 sq. ft. 41 sq. in. by 18; (*e*) 148 mi. 64 rd. 24 yd. by 12; (*f*) 100 T. 16 cwt. 18 lb. 11 oz. by 15; (*g*) 36 lb. 18 oz. 18 pwt. 14 gr. by 8; (*h*) 112 mi. 48 rd. by 100.

Ans.
(*a*) 17 mi. $41\frac{7}{11}$ rd.
(*b*) 113 bu. 3 pk. 1 qt. $\frac{1}{2}$ pt.
(*c*) 5 cwt. 28 lb. $3\frac{1}{16}$ oz.
(*d*) 4 sq. yd. 4 sq. ft. $2\frac{5}{18}$ sq. in
(*e*) 12 mi. 112 rd. 2 yd.
(*f*) 6 T. 14 cwt. 41 lb. $3\frac{14}{15}$ oz.
(*g*) 4 lb. 8 oz. 7 pwt. $7\frac{3}{4}$ gr.
(*h*) 1 mi. $38\frac{22}{25}$ rd.

ARITHMETIC.

(SECTION 5.)

INVOLUTION.

63. If a product consists of equal factors, it is called a **power** of one of those equal factors, and one of the equal factors is called a **root** of the product. The power and the root are named according to the number of equal factors in the product. Thus, 3×3, or 9, is the *second power*, or **square,** of 3; $3\times3\times3$, or 27, is the *third power*, or **cube,** of 3; $3\times3\times3\times3$, or 81, is the **fourth power** of 3. Also, 3 is the **second root,** or **square root,** of 9; 3 is the **third root,** or **cube root,** of 27; 3 is the **fourth root** of 81.

64. For the sake of brevity,

3×3 is written 3^2, and read **three square,**
or *three exponent two;*

$3\times3\times3$ is written 3^3, and read **three cube,**
or *three exponent three;*

$3\times3\times3\times3$ is written 3^4, and read **three fourth,**
or *three exponent four;*

and so on.

A number written above and to the right of another number, to show how often the latter number is used as a factor, is called an **exponent.** Thus, in 3^{12}, the number ^{12} is the exponent, and shows that 3 is to be used as a factor twelve times; so that 3^{12} is a contraction for

$$3\times3\times3\times3\times3\times3\times3\times3\times3\times3\times3\times3.$$

In an expression like 3^5, the exponent ^{5} shows how often

For notice of copyright, see page immediately following the title page.

3 is used as a factor. Hence, if the exponent of a number is unity, the number is used once as a factor; thus, $3^1 = 3$, $4^1 = 4$, $5^1 = 5$.

65. If the side of a square contains 5 inches, the area of the square contains 5×5, or 5^2, square inches. If the edge of a cube contains 5 inches, the volume of the cube contains $5 \times 5 \times 5$, or 5^3, cubic inches. It is for this reason that 5^2 and 5^3 are called the square and cube of 5, respectively.

66. To find any power of a number:

EXAMPLE 1.—What is the third power, or cube, of 35?

SOLUTION.— $35 \times 35 \times 35$

or

$$\begin{array}{r} 35 \\ 35 \\ \hline 175 \\ 105 \\ \hline 1225 \\ 35 \\ \hline 6125 \\ 3675 \\ \hline \end{array}$$

cube = 42875 Ans.

EXAMPLE 2.—What is the fourth power of 15?

SOLUTION.— $15 \times 15 \times 15 \times 15$

or

$$\begin{array}{r} 15 \\ 15 \\ \hline 75 \\ 15 \\ \hline 225 \\ 15 \\ \hline 1125 \\ 225 \\ \hline 3375 \\ 15 \\ \hline 16875 \\ 3375 \\ \hline \end{array}$$

fourth power = 50625 Ans.

EXAMPLE 3.— 1.2^3 = what?

SOLUTION.—

$$1.2 \times 1.2 \times 1.2$$

or

$$\begin{array}{r} 1.2 \\ 1.2 \\ \hline 1.44 \\ 1.2 \\ \hline 288 \\ 144 \\ \hline \end{array}$$

cube = 1.728 Ans.

EXAMPLE 4.—What is the third power, or cube, of $\frac{3}{8}$?

SOLUTION.— $\left(\frac{3}{8}\right)^3 = \frac{3^3}{8^3} = \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} = \frac{3 \times 3 \times 3}{8 \times 8 \times 8} = \frac{27}{512}$. Ans.

67. Rule.—I. *To raise a whole number or a decimal to any power, use it as a factor as many times as there are units in the exponent.*

II. *To raise a fraction to any power, raise both the numerator and denominator to the power indicated by the exponent.*

EXAMPLES FOR PRACTICE.

Raise the following to the powers indicated:

(*a*) 85^2.
(*b*) $(\frac{12}{13})^2$.
(*c*) 6.5^2.
(*d*) 14^4.
(*e*) $(\frac{3}{4})^3$.
(*f*) $(\frac{5}{6})^3$.
(*g*) $(\frac{7}{2})^3$.
(*h*) 1.4^5.

Ans. (*a*) 7,225. (*b*) $\frac{144}{169}$. (*c*) 42.25. (*d*) 38,416. (*e*) $\frac{27}{64}$. (*f*) $\frac{125}{216}$. (*g*) $\frac{343}{8}$. (*h*) 5.37824.

EVOLUTION.

68. Evolution is the reverse of involution. It is the process of finding the root of a number that is considered as a power.

69. The **square root** of a number is that number which, when used twice as a factor, produces the number.

Thus, 2 is the square root of 4, since 2×2, or $2^2 = 4$.

70. The **cube root** of a number is that number which, when used three times as a factor, produces the number.

Thus, 3 is the cube root of 27, since $3 \times 3 \times 3$, or $3^3 = 27$.

71. The **radical sign** $\surd$, when placed before a number, indicates that some root of that number is to be found. The vinculum is almost always used in connection with the radical sign, as shown in Art. **72.**

72. The **index** of the root is a *small figure* placed *over* and to the *left* of the *radical sign*, to show what root is to be found.

Thus, $\sqrt[2]{100}$ denotes the *square root* of 100.

$\sqrt[3]{125}$ denotes the *cube root* of 125.

$\sqrt[4]{256}$ denotes the *fourth root* of 256, and so on.

73. When the square root is to be extracted, the index is generally omitted. Thus, $\surd\overline{100}$ indicates the square root of 100. Also, $\surd 225$ indicates the square root of 225.

74. In any number, the figures beginning with the first digit* at the left and ending with the last digit at the right, are called the **significant figures** of the number. Thus, the number 405,800 has the four significant figures 4, 0, 5, 8; and the number .000090067 has the five significant figures 9, 0, 0, 6, and 7.

The part of a number consisting of its significant figures is called the **significant part** of the number. Thus, in the number 28,070, the significant part is 2807; in the number .00812, the significant part is 812; and in the number 170.3, the significant part is 1703.

In speaking of the significant figures or of the significant part of a number, we consider the figures, in their proper order, from the first digit at the left to the last digit at the right, but we pay no attention to the position of the decimal point. Hence, *all numbers that differ only in the position of the decimal point have the same significant part.* For example, .002103, 21.03, 21,030, and 210,300 have the same significant figures 2, 1, 0, and 3, and the same significant part 2103.

* A cipher is not a digit.

SQUARE ROOT.

75. The *largest* number that can be written with *one* figure is 9, and $9^2 = 81$; the *largest* number that can be written with *two* figures is 99, and $99^2 = 9{,}801$; with *three* figures 999, and $999^2 = 998{,}001$; with *four* figures 9,999, and $9{,}999^2 = 99{,}980{,}001$, etc.

In *each* of the above it will be noticed that the square of the number contains just *twice* as many figures as the number.

In order to find the square root of a number, the first step is to find how many figures there will be in the root. This is done by pointing off the number into *periods* of *two* figures each, *beginning at the right.* The number of periods will indicate the number of figures in the root.

Thus, the square root of 83,740,801 must contain four figures, since, pointing off the periods, we get 83′74′08′01, or four periods; consequently, there must be four figures in the root. In like manner, the square root of 50,625 must contain three figures, since there are (5′06′25) three periods. The extreme left-hand period may contain either one or two figures, according to the size of the number squared.

76. The square of any number wholly decimal always contains twice as many figures as the number squared. For example, $.1^2 = .01$, $.13^2 = .0169$, $.751^2 = .564001$, etc.

77. It will also be noticed that the square of a decimal is always less than the decimal. Hence, the square root of a number wholly decimal is greater than the number itself. If it be required to find the square root of a decimal, and the decimal has not an even number of figures in it, annex a cipher. The best way to point off a decimal is to begin at the decimal point, and, going toward the *right*, point off the decimal into periods of two figures each. Then, if the last period contains but one figure, annex a cipher to complete the period.

78. There are comparatively few numbers that can be separated into exactly equal factors; these numbers are called

perfect powers, and the factors are called *rational factors.* Numbers that cannot be separated into exactly equal factors are called **imperfect powers,** and the factors are called **surds** or *irrational factors.* In the numbers from 1 to 1,000, inclusive, there are only 42 perfect powers, not counting 1, and of these only 30 are perfect squares and 9 perfect cubes.

The root of any number that cannot be divided into as many equal factors as there are units in the index of the root contains an interminable decimal. For example, the number 20 lies between 16 ($= 4^2$) and 25 ($= 5^2$); hence, the square root of 20, or $\sqrt{20}$, is greater than 4 and less than 5, and is therefore equal to 4 plus an interminable decimal. In other words, no matter to how many figures the square root of 20 may be calculated, the root will never be found exactly.

79. Although the root of an imperfect power cannot be found exactly, as close an approximation may be obtained as is desired. In practice, five significant figures are all that are likely to be required, and four are generally sufficient. In the following examples, all roots will be calculated to five figures, unless the given number is a perfect power whose root contains less than five figures.

80. The student will find the following principles of value, both in connection with the extraction of roots and in other arithmetical calculations:

a. In general, if any two numbers are multiplied together —no matter how many significant figures they contain—the first five significant figures of the product will be the same as the first five significant figures of the product obtained by multiplying the same two numbers when limited to five significant figures.

For example, the product of 4,562,357 and 6,421,849 is 29,298,767,738,093; limiting the numbers to five significant figures, the product of 45,624 and 64,218 is 2,929,882,032; and the value of both these products to five significant figures is 29,299. In other words, if only five significant figures are required in the product, it is not

necessary to use more than five significant figures in the multiplier and multiplicand, the remaining figures, if any, being replaced by ciphers, and the fifth figure being increased by 1 if the sixth figure is 5 or a larger digit. In some cases, however, the fifth figure may be one unit too large or one unit too small; hence, if it is necessary that the fifth figure be absolutely exact, it is better to limit the multiplier and multiplicand to six figures instead of five.

For example, $4,562,347 \times 6,421,849 = 29,298,703,519,603$, or 29,299,000,000,000 to five significant figures; $4,562,300 \times 6,421,800 = 29,298,178,140,000 = 29,298,000,000,000$ to five significant figures, the fifth figure being 1 less than it should be; but $4,562,350 \times 6,421,850 = 29,298,727,347,500 = 29,299,000,000,000$ to five significant figures.

b. If the divisor and dividend are limited to six significant figures, the quotient will always be correct to five (usually to six) significant figures, regardless of how many significant figures there may have been in the dividend and divisor.

For example, $6,421,849 \div 4,562,357 = 1.407572+ = 1.4076$ to five significant figures; also, $642,185 \div 456,236 = 1.407571+ = 1.4076$ to five significant figures.

c. If the number whose root is to be extracted be limited to six significant figures, the root will be correct to five (usually to six) significant figures.

81. These principles may be summed up in the following general statement: *In any series of arithmetical operations—addition, subtraction, multiplication, division, involution, and evolution—if it be desired to have the final result limited to a certain number of significant figures, it is unnecessary to use more significant figures in any of the numbers operated on than the desired number in the result plus 1.* For example, if only four significant figures are desired in the final result, all the numbers used in the various operations may be limited to $4+1 = 5$ significant figures, the fifth figure being increased by 1 in all cases if the sixth figure is 5 or a greater digit.

From the foregoing, it follows that any method that will give five significant figures of the root correctly will be

sufficiently exact for all practical purposes. Such a method will now be explained for extracting square root.

82. Suppose it is desired to find the square root of 20; that is, $\sqrt{20} = ?$ The problem is to divide 20 into two equal factors, or into two factors, the first five significant figures of which shall be equal. Since 20 is not a perfect square, inspection shows that one of the equal factors is 4 plus an interminable decimal, since 20 lies between $4^2 = 16$ and $5^2 = 25$. Dividing 20 by 4, the result is 5; i. e., $4 \times 5 = 20$. Now, by taking the average of these unequal factors, a new factor will be obtained, which will be nearer the correct value of the root than either of the two unequal factors, viz., $\frac{4+5}{2} = 4.5$, the square of which is $4.5^2 = 20.25$.

Assuming 4.5 for a new factor and dividing 20 by it, the result is $20 \div 4.5 = 4.444+$; that is, $4.444 \times 4.5 = 20$, nearly, the product not being exactly equal to 20 because 4.444 was used as one factor, instead of $4\frac{4}{9}$, the exact value. Again, taking the average of the two factors, $\frac{4.444 + 4.5}{2} = 4.472$, which is the root correct to at least three figures.

Assuming 4.47 to be one of the factors and dividing 20 by it to obtain the other, the result is $20 \div 4.47 = 4.474272+$; that is, $4.47 \times 4.474273 = 20$, very nearly. The average of these two factors is $\frac{4.47 + 4.474272}{2} = 4.472136+ = 4.4721$ to five significant figures. The exact root to thirteen figures is 4.472135954999+.

That 4.4721 is the square root of 20 correct to five figures may easily be proved by squaring it; thus, $4.4721^2 = 19.99967841$, or 20.000 to five figures. Since the square agrees with the given number to five figures, the root is correct to five figures.

83. A close examination of the foregoing results reveals some remarkable facts. (1) The value of the first average 4.5 is correct to two figures of the root. (2) The value of the second average 4.472+ is correct to four figures of the root. (3) The value of the third average 4.472136 is correct

to seven figures of the root. (4) All these averages are somewhat greater than the correct value of the root. (5) Of the two factors used in finding the average, one is a little greater and the other a little less than the correct value of the root. (6) Each step of the process gives a result approaching more and more nearly to the correct value of the root.

84. Calling the first average value the **first approximation,** the second average value the **second approximation,** and the third average value the **third approximation,** the following general method of procedure may be adopted: *Calculate the first approximation to two significant figures; the second approximation to three significant figures; and the third approximation to five significant figures. It is not safe to calculate the second approximation to more than three significant figures, because the fourth figure cannot, as a rule, be depended on. If the second significant figure of the first approximation be determined correctly, the third approximation will always be correct to at least five significaut figures.*

85. The method will now be applied to numbers in general, and the best manner of explaining it is by means of examples.

EXAMPLE.— $\sqrt{714,627} = ?$

SOLUTION.—The first step is to point off the number into periods of two figures each, obtaining 71'46'27. To find the first approximation, only the first two significant figures are necessary; in this case, the first period, 71. The first figure of the root is evidently 8, since $8^2 = 64$ and $9^2 = 81$. The two factors then are 8 and $71 \div 8 = 8.87+$. The first approximation is $\frac{8 + 8.87}{2} = 8.43+ = 8.4$ to two figures.

To find the second approximation, use the first two periods, or 7146, and drop the decimal point in the first approximation. One factor is then 84 and the other $7146 \div 84 = 85.07+$. The second approximation is therefore $\frac{84 + 85.07}{2} = 84.53+$, or 84.5 to three figures.

To find the third approximation, use the first three periods, or 714627, and drop the decimal point in the second approximation. One factor is then 845 and the other is $714627 \div 845 = 845.712+$. The third approximation is therefore $\frac{845 + 845.712}{2} = 845.356$, or 845.36 to five significant figures. Ans.

REMARK.—It will be noticed in the last example, and also in those that follow, that when finding the unknown factor to be used in determining the value of the first, second, or third approximation, the division is carried one place further than the number of figures desired in the approximation and that no attention is paid to the succeeding figures. Thus, in the last example, $71 \div 8 = 8.875$, or 8.88, correct to three figures, while the number used was 8.87. The reason for this is that the value obtained for the approximation would be the same in either case, and it saves time to calculate as here shown. For instance, using 8.88 for the second factor, the first approximation is $\frac{8 + 8.88}{2} = 8.44$, or 8.4 to two figures.

86. The decimal point is located by employing the following principle: *There must be as many figures in the integral* part of the root as there are periods in the integral part of the given number whose root is to be found. If the given number is wholly decimal and there are two or more ciphers between the decimal point and the first significant figure, there will be as many ciphers between the decimal point and the first significant figure of the root as there are entirely cipher periods between the decimal point and the first significant figure of the given number.* Had the number in the last example been 71.4627, the root would have been 8.4536; had it been .714627, the root would have been .84536; had it been .0000714627, the root would have been .0084536. In the latter case, the number would have been pointed off thus, .00′00′71′46′27.

87. In all cases, numbers having the same significant parts and the same number of significant figures in the first (or left-hand) period of the significant part of the number, have the same significant figures in the root, the roots differing only in the position of the decimal point.

EXAMPLE.— $\sqrt{714.627} = ?$

SOLUTION.—Pointing off into periods, we have 7′14.62′70, adding a cipher to complete the last period. In all cases when pointing off the decimal part of numbers, begin at the decimal point and point off to the right, and add ciphers to the last period when it does not contain enough figures to make up a period. Since the first period contains but one figure and it is necessary to have two figures at least in order that the first approximation may be correct to two figures, regard

* The *integral* part of a number is the part to the left of the decimal point. Thus, the integral part of 1,726.943 is 1,726.

the decimal point as situated between 7 and 1 instead of between 4 and 6, thus obtaining 7.1 for the first two figures of the given number. The first two figures of the square root of 7.1 will be the same as the first two figures of the square root of 714.

It is evident that the first figure of the root is 2, since $2^2 = 4$ and $3^2 = 9$. Using 2 as one factor, the other is $7.1 \div 2 = 3.55$, and the first approximation is $\frac{2+3.55}{2} = 2.77+$, or 2.8 to two figures. Had 3 been used as one factor, the other would have been $7.1 \div 3 = 2.36+$, and the first approximation would have been $\frac{3+2.36}{2} = 2.68$, or 2.7 to two figures. In the first case, the difference between the two factors is $3.55-2 = 1.55$; in the second case, the difference is $3-2.38 = .62$. As the factors are more nearly equal in the second case than in the first, it is evident that 2.7 is more nearly equal to the correct value of the root than 2.8 is; hence, 2.7 will be used for the first approximation.

For the second approximation, use the first two periods and 27 for one factor, the other factor being $715 \div 27 = 26.48+$; hence, the second approximation $= \frac{27+26.48}{2} = 26.74$, or 26.7 to three figures. We used 715 for the first three figures of the given number, instead of 714, because the fourth figure was 6 and the number correct to three figures was 715. In finding the third approximation, the first three periods may be used or all the figures; the result will be the same in either case. Since it is better to use six figures than five, move the decimal point two places to the right, obtaining 71462.7; one factor is 267 and the other $71462.7 \div 267 = 267.650+$. The third approximation is $\frac{267+267.650}{2} = 267.325$, or 267.33 to five figures. Since there are two periods in the integral part of the given number, there are two figures in the integral part of the root, and $\sqrt{714.627} = 26.733$. Ans.

88. When determining the first approximation, that number should always be used for the first factor which will make the less difference between it and the second factor, as was done in the last example. Thus, for 2.5, the factors would be 2 and 1.25, the difference between them being .75. If 1 were selected for the first factor, the other would be $2.5 \div 1 = 2.5$, and the difference between them, 1.5. In one case, the first approximation would be $\frac{2+1.25}{2} = 1.6+$, and in the other case, $\frac{1+2.5}{2} = 1.8-$. Since $1.6^2 = 2.56$ and

$1.8^2 = 3.24$, it is evident 1.6 is very much nearer the correct value of the root than 1.8.

89. If the given number is a perfect square and contains not more than ten significant figures, the exact root will be obtained in all cases. That the number is a perfect square may be suspected by the fact that there are one or more 9's or 0's following the fifth figure of the number expressing the third approximation, and that when the third approximation is expressed correct to five figures, the square of its last digit (or the second figure of this square when the square contains more than one figure) will be the same as the last digit of the given number. This will be illustrated by two examples.

EXAMPLE.— $\sqrt{3,749,602,756} = ?$

SOLUTION.—Pointing off, we obtain 37'49'60'27'56. The first two factors are evidently 6, and $37 \div 6 = 6.16+$, and the first approximation $\frac{6+6.16}{2} = 6.08$, or 6.1 to two figures.

$3749 \div 61 = 61.45+$; $\frac{61+61.45}{2} = 61.23-$, or 61.2 to three figures.

$$374960 \div 612 = 612.67973+; \frac{612+612.67973}{2} = 612.33986+,$$

or 612.34 to five figures. But $4^2 = 16$, and as the last figure of the given number is also 6, and as the sixth and seventh figures are 9 and 8, respectively, we suspect that the given number is a perfect power. It may not be, however, for the reason that the figures 5, 7, and 2, preceding 6, may be different from the ones given without changing the value of the root to five figures. Hence, the only way to ascertain the fact beyond possibility of doubt is to square the root; doing so, it is found that $61,234^2 = 3,749,602,756$, which is therefore a perfect square.

Had all the figures of the given number been used in finding the third approximation, the result would have been as follows: $3,749,602,756 \div 612 = 612.6801+$, and $\frac{612+612.6801}{2} = 612.34005+$, or 612.34 to five figures, as before, or 61,234 after locating the decimal point. Ans.

90. If the given number contains not more than three periods of significant figures—that is, if it contains not more than five or six significant figures—and is a perfect power, the fact will be revealed when finding the second factor in

the third approximation, for the two factors will then be exactly equal.

EXAMPLE.— $\sqrt{.00095481}$.

SOLUTION.— .00095481 = .00'09'54'81 when pointed off into periods of two figures each. The first two significant figures are 9.5. The first factor is evidently 3 and the second factor $9.5 \div 3 = 3.16+$. The first approximation is $\frac{3+3.16}{2} = 3.08$, or 3.1 to two figures.

$955 \div 31 = 30.80+$; the second approximation is $\frac{31+30.80}{2} = 30.90 = 30.9$ to three figures.

$95481 \div 309 = 309$; hence, .00095481 is a perfect power and the significant figures of the root are 309. There being one full cipher period following the decimal point, the root is .0309. Ans.

91. One more example will be given to show the student how to arrange his work when solving examples in square root by this method.

EXAMPLE.— $\sqrt{3,265.47} = ?$

SOLUTION.— 3,265.47 = 32'65.47.

$33 \div 5 = 6.6$; $33 \div 6 = 5.5$; $6.6 - 5 = 1.6$; $6 - 5.5 = .5$; hence, use 6 for first factor.

$$\frac{6+5.5}{2} = 5.75, \text{ or } 5.8.$$

$$3,265 \div 58 = 56.29; \quad \frac{58+56.29}{2} = 57.14, \text{ or } 57.1.$$

$326,547 \div 571 = 571.886$; $\frac{571+571.886}{2} = 571.443$, or 571.44 to five figures. Therefore, $\sqrt{3,265.47} = 57.144$. Ans.

EXAMPLES FOR PRACTICE.

Find the square root of:

(*a*) 186,624.
(*b*) 2,050,624.
(*c*) 29,855,296.
(*d*) .0116964.
(*e*) 198.1369.
(*f*) 994,009.
(*g*) 2.375.
(*h*) 1.625.
(*i*) .3025.
(*j*) .571428.
(*k*) .78125.

Ans. (*a*) 432.
(*b*) 1,432.
(*c*) 5,464.
(*d*) .10815—.
(*e*) 14.076+.
(*f*) 997.
(*g*) 1.5411+.
(*h*) 1.2748—.
(*i*) .55.
(*j*) .75593—.
(*k*) .88388+.

CUBE ROOT.

92. Cube root may be extracted in a manner similar to that just described for square root, the only essential differences being that the given number must be pointed off into periods of *three* figures each; the first period, if integral, may contain one, two, or three figures; and the number must be divided into three equal factors.

93. As might be expected, cube root is a longer operation than square root, but the method is similar and is no more difficult to remember or apply. As in the case of square root, it is unnecessary to use more than six significant figures in order to obtain five significant figures of the root. The method is best illustrated by an example. The student is advised to make a little table, containing the cubes of numbers from 1 to 9, similar to that here given.

$1^3 =$	1
$2^3 =$	8
$3^3 =$	27
$4^3 =$	64
$5^3 =$	125
$6^3 =$	216
$7^3 =$	343
$8^3 =$	512
$9^3 =$	729

EXAMPLE.— $\sqrt[3]{389{,}247} = ?$

SOLUTION.— 389,247 = 389′247 when pointed off into periods of three figures each. As in the case of square root, consider the first period only when finding the first approximation. In other words, divide 389 into three factors as nearly equal as possible. It is readily seen that 389 lies between $7^3 = 343$ and $8^3 = 512$; hence, the first figure of the root is 7. Now, assume that two of the equal factors are each equal to 7 and divide 389 by their product to obtain the third factor; that is, divide 389 by $7^2 = 49$. The result is $389 \div 49 = 7.93+$. Hence, $7 \times 7 \times 7.93+ = 389$, nearly. The average of these factors is $\frac{7+7+7.93}{3} = \frac{2 \times 7 + 7.93}{3} = 7.31$, or 7.3 to two figures, the first approximation.

Assuming 73 to be the value of two of the three equal factors, divide the first two periods of the given number by their product 73×73; that is, by 73^2, or 5,329. The result is $389{,}247 \div 5{,}329 = 73.04+$; that is, $73 \times 73 \times 73.04 = 389{,}247$, nearly. The average of the three factors is $\frac{2 \times 73 + 73.04}{3} = 73.01$, or 73.0 to three figures, the second approximation.

Assuming 730 to be two of the three equal factors, divide 389,247 by 730^2, or by 73^2, since the cipher at the right is not a significant

figure and will not affect the result, obtaining for the third factor 73.0431.

The average of these three factors is $\frac{2 \times 73 + 73.0431}{3} = 73.0143$, or 73.014 to five figures. Ans.

NOTE.—The decimal point is located by applying the principle of Art. **86**; viz., there must be as many figures in the integral part of the root as there are periods in the integral part of the given number.

94. An inspection of the foregoing example shows that about the only respect in which the work of extracting cube root exceeds the work of extracting square root consists in squaring one number of two figures and one number of three figures. The work of division in finding the third factor is a little harder on account of the divisors being a little larger than when finding the second factor in square root.

95. If the given number contains an integral part, it is better to locate the decimal point as soon as possible, in order to prevent confusion, instead of waiting until the third approximation has been found.

EXAMPLE.— $\sqrt[3]{3.274} = ?$

SOLUTION.—The first period contains but one figure; therefore, we operate on three figures in order to have the first approximation correct to two figures (see **c**, Art. **80**). If 1 be chosen for one of the two equal factors, the third factor will be $3.27 \div 1^2 = 3.27$, and the difference between one of the equal factors and the third factor will be $3.27 - 1 = 2.27$. If 2 be chosen for one of the equal factors, the third one will be $3.27 \div 2^2 = .817+$, and the difference between this and one of the equal factors is $2 - .817 = 1.183$. Since 1.18 is less than 2.27, use 2 for one of the two equal factors. The first approximation is $\frac{2 \times 2 + .817}{3} = 1.60+$, or 1.6 to two figures. Since there is but one period in the integral part of the given number, the root is equal to 1 plus an interminable decimal, as the given number is not a perfect cube. Therefore, retain the decimal point in its present position through all the subsequent operations.

Assuming 1.6 to be one of the two equal factors, the third factor is $3.274 \div 1.6^2 = 3.274 \div 2.56 = 1.278+$, and the second approximation is $\frac{2 \times 1.6 + 1.278}{3} = 1.492+$, or 1.49 to three figures.

Assuming 1.49 to be one of the two equal factors, the other factor is $3.274 \div 1.49^2 = 3.274 \div 2.2201 = 1.474708+$; hence, the third approximation is $\frac{2 \times 1.49 + 1.474708}{3} = 1.484902+$, or 1.4849 to five figures. Ans.

The exact root to seven figures is 1.484886—.

96. The only case in cube root that will give any trouble in determining the fifth significant figure correctly is when the difference between the numbers representing the first and second approximations, expressed to two figures, is greater than one unit in the second figure. In the last example, the first approximation was 1.6 and the second 1.49, or 1.5 to two figures; the difference is .1, or one unit in the second figure. For numbers, the significant part of whose first period is 2, the difference between the first and second approximations may differ by more than one unit in the second figure; in such cases, recalculate the second approximation, using for one of the equal factors the value of the second approximation to two figures as first determined. An example will illustrate this.

EXAMPLE.— $\sqrt[3]{.0027} = ?$

SOLUTION.— $.0027 = .002'700$ when pointed off into periods. But, the significant figures in the cube root of 2.7 will be the same as in the cube root of $.002'700$; therefore, find the cube root of 2.7 and locate the decimal point after the operation is finished.

If 1 be chosen as one of the equal factors, the third factor will be $2.7 \div 1^2 = 2.7$, and the first approximation is $\frac{2 \times 1 + 2.7}{3} = 1.56+$, or 1.6 to two figures. If 2 be chosen for one of the equal factors, the third factor is $2.7 \div 2^2 = .675$, and the second approximation is $\frac{2 \times 2 + .675}{3} = 1.55+$, or 1.6 to two figures.

Using 1.6 for one of the equal factors, the third factor is $2.7 \div 1.6^2 = 2.7 \div 2.56 = 1.054+$, and the second approximation is $\frac{2 \times 1.6 + 1.054}{3} = 1.418$, or 1.42 to three figures, or 1.4 to two figures. The difference between the first and second approximations is $1.6 - 1.4 = .2$, or two units in the second figure. Therefore, recalculate the second approximation, using 1.4 for one of the equal factors. The third factor is then equal to $2.7 \div 1.4^2 = 2.7 \div 1.96 = 1.377$, and the second approximation is $\frac{2 \times 1.4 + 1.377}{3} = 1.392+$.

Using 1.39 for one of the equal factors, the third factor is $2.7 \div 1.39^2 = 2.7 \div 1.9321 = 1.39744+$, and the third approximation is $\frac{2 \times 1.39 + 1.39744}{3} = 1.39248$, or 1.3925 to five figures. The root correct to nine figures is 1.39247665. Since the given number is wholly decimal and has no period composed entirely of ciphers, $\sqrt[3]{.0027} = .13925$. Ans.

Had 1.42 been used for one of the equal factors the third approximation would have been 1.3930.

97. The remarks made in Art. **89** regarding the square root of perfect squares apply, with slight modifications, to the cube root of perfect cubes. If the given number is a perfect cube and contains not more than five periods, i. e., not more than $5 \times 3 = 15$ significant figures, the exact root can always be found. That the given number is a perfect cube will be suspected from the fact that the root ends in a string of 9's or 0's; that in one of the approximations the three factors become exactly equal; and that the last digit in the cube of the last figure of the root is the same as the last digit of the given number. An example will illustrate this.

EXAMPLE.— $\sqrt[3]{106,294,343.553} = ?$

SOLUTION.—The number when pointed off becomes 106'294'343.553; hence, there are three figures in the integral part of the root. The first period 106 lies between $4^3 = 64$ and $5^3 = 125$. Trying 4 for one of the equal factors, the third factor is $106 \div 4^2 = 6.62+$. Trying 5, the third factor is $106 \div 5^2 = 4.24$; hence, use 5, and obtain for the first approximation $\frac{2 \times 5 + 4.24}{3} = 4.74+$. Using two periods and 47 for one of the equal factors, the third factor is $106,294 \div 47^2 = 106,294 \div 2,209 = 48.11+$, and the second approximation is $\frac{2 \times 47 + 48.11}{3} = 47.37$.

To find the third approximation, two, or three, or all four periods may be used, since the first two periods contain six significant figures, and hence will give the root correct to five figures (see **c**, Art. **80**). Using the first three periods, to avoid the decimal point, and 474 for one of the equal factors, the third factor is $106,294,343 \div 474^2 = 106,294,343 \div 224,676 = 473.1005+$, and the third approximation is $\frac{2 \times 474 + 473.1005}{3} = 473.7001+$, or 473.70 to five figures. It will be noticed that the results obtained for the second and third approximations are alike and the last digit in $7^3 = 343$ is the same as the last significant figure of the given number; hence, it is at once suspected that the given number is a perfect power, and this is proved by cubing the root. Therefore, $\sqrt[3]{106,294,343.553} = 473.7$. Ans.

98. Square and cube root are two of the most important operations described in Arithmetic, and the student is earnestly advised to thoroughly familiarize himself with the process. Few practical problems involving mensuration arise that do not require the extraction of the square

or cube root. For instance, to find the diameter of a circle that will contain a given area, requires the extraction of square root; to find the diameter of a sphere that will contain a given volume, requires the extraction of cube root.

EXAMPLES FOR PRACTICE.

Find the cube root of:

(*a*) 78,347.809639.
(*b*) 2.
(*c*) 4,180,769,192.462.
(*d*) .696.
(*e*) .375.
(*f*) 513,229.783302144.

Ans. (*a*) 42.79. (*b*) 1.2599+. (*c*) 1,611.0−. (*d*) .88621−. (*e*) .72112+. (*f*) 80.064.

TABLE METHOD OF EXTRACTING SQUARE AND CUBE ROOT.

99. By means of the Table of Squares, Cubes, Fourth and Fifth Powers, which contains the squares and cubes of numbers from 1 to 10, varying by tenths, and the first five figures of the fourth and fifth powers of the same numbers, the first three, and frequently the first four, significant figures of the square root or cube root of any number can be readily determined. The remaining figures can then be easily determined in the same manner as the third approximation in the preceding pages.

The student is advised to use the table in all cases, as it will greatly shorten his work.

100. By the aid of this table the first two significant figures of the root can be obtained directly and one more by a slight calculation. For example, suppose it is desired to find the first three significant figures of $\sqrt{5,269.73}$. Pointing off into periods and moving the decimal point so that it falls between the first and second periods, the number becomes $52.69'73$; in other words, the significant figures of $\sqrt{5,269.73}$ are the same as for $\sqrt{52.6973}$. Since four figures

SQUARES, CUBES, FOURTH AND FIFTH POWERS.

No.	Square.	Cube.	4th Power.	5th Power.	No.	Square.	Cube.	4th Power.	5th Power.
1.0	1.00	1.000	1.0000	1.0000	5.5	30.25	166.375	915.06	5,032.8
1.1	1.21	1.331	1.4641	1.6105	5.6	31.36	175.616	983.45	5,507.3
1.2	1.44	1.728	2.0736	2.4883	5.7	32.49	185.193	1,055.6	6,016.9
1.3	1.69	2.197	2.8561	3 7129	5.8	33.64	195.112	1,131.6	6,563.6
1.4	1.96	2.744	3.8416	5.3782	5.9	34.81	205.379	1,211.7	7,149.2
1.5	2.25	3.375	5.0625	7.5938	6.0	36.00	216.000	1,296.0	7,776.0
1.6	2.56	4.096	6.5536	10.486	6.1	37.21	226.981	1,384.6	8,446.0
1.7	2.89	4.913	8.3521	14.199	6.2	38.44	238.328	1,477.6	9.161.3
1.8	3.24	5.832	10.498	18.896	6.3	39.69	250.047	1,575.3	9,924.4
1.9	3.61	6.859	13.032	24.761	6.4	40.96	262.144	1,677.7	10,737
2.0	4.00	8.000	16.000	32.000	6.5	42.25	274.625	1,785.1	11,603
2.1	4.41	9.261	19.448	40.841	6.6	43.56	287.496	1,897.5	12,523
2.2	4.84	10.648	23.426	51.536	6.7	44.89	300.763	2,015.1	13,501
2.3	5.29	12.167	27.984	64.363	6.8	46.24	314.432	2,138.1	14,539
2.4	5.76	13.824	33.178	79.626	6.9	47.61	328.509	2,266.7	15,640
2.5	6.25	15.625	39.063	97.656	7.0	49.00	343.000	2,401.0	16,807
2.6	6.76	17.576	45.698	118.81	7.1	50.41	357.911	2,541.2	18,042
2.7	7.29	19.683	53.144	143.49	7.2	51.84	373.248	2,687.4	19,349
2.8	7.84	21.952	61.466	172.10	7.3	53.29	389.017	2,839.8	20,731
2.9	8.41	24.389	70.728	205.11	7.4	54.76	405.224	2,998.7	22,190
3.0	9.00	27.000	81.000	243.00	7.5	56.25	421.875	3,164.1	23,730
3.1	9.61	29.791	92.352	286.29	7.6	57.76	438.976	3,336.2	25,355
3.2	10.24	32.768	104.86	335.54	7.7	59.29	456.533	3,515.3	27,068
3.3	10.89	35.937	118.59	391.35	7.8	60.84	474.552	3,701.5	28,872
3.4	11.56	39.304	133.63	454.35	7.9	62.41	493.039	3,895.0	30,771
3.5	12.25	42.875	150.06	525.22	8.0	64.00	512.000	4,096.0	32,768
3.6	12.96	46.656	167.96	604.66	8.1	65.61	531.441	4,304.7	34,868
3.7	13.69	50.653	187.42	693.44	8.2	67.24	551.368	4,521.2	37,074
3.8	14.44	54.872	208.51	792.35	8.3	68.89	571.787	4,745.8	39,390
3.9	15.21	59.319	231.34	902.24	8.4	70.56	592.704	4,978.7	41,821
4.0	16.00	64.000	256.00	1,024.0	8.5	72.25	614.125	5,220.1	44,371
4.1	16.81	68.921	282.58	1,158.6	8.6	73.96	636.056	5,470.1	47,043
4.2	17.64	74.088	311.17	1,306.9	8.7	75.69	658.503	5,729.0	49,842
4.3	18.49	79.507	341.88	1,470.1	8.8	77.44	681.472	5,997.0	52,773
4.4	19.36	85.184	374.81	1,649.2	8.9	79.21	704.969	6,274.2	55,841
4.5	20.25	91.125	410.06	1,845.3	9.0	81.00	729.000	6,561.0	59,049
4.6	21.16	97.336	447.75	2,059.6	9.1	82.81	753.571	6,857.5	62,403
4.7	22.09	103.823	487.97	2,293.5	9.2	84.64	778.688	7,163.9	65,908
4.8	23.04	110.592	530.84	2,548.0	9.3	86.49	804.357	7,480.5	69,569
4.9	24.01	117.649	576.48	2,824.8	9.4	88.36	830.584	7,807.5	73,390
5.0	25.00	125.000	625.00	3,125.0	9.5	90.25	857.375	8,145.1	77,378
5.1	26.01	132.651	676.52	3,450.3	9.6	92.16	884.736	8,493.5	81,537
5.2	27.04	140.608	731.16	3,802.0	9.7	94.09	912.673	8,852.9	85,873
5.3	28.09	148.877	789.05	4,182.0	9.8	96.04	941.192	9,223.7	90,392
5.4	29.16	157.464	850.31	4,591.7	9.9	98.01	970.299	9,606.0	95,099

only are given in the table, reduce the given number to four figures. The problem then becomes: find the first three figures of $\sqrt{52.70}$. Referring to the table 52.70 lies between $51.84 = 7.2^2$ and $53.29 = 7.3^2$; hence, the first two figures of the root are 7.2. Find the difference between the two numbers in the table between which the given number falls and call it the **first difference**; thus, $53.29 - 51.84 = 1.45 =$ the first difference. Find the difference between the lower number in the table and the given number and call it the **second difference**; thus, $52.70 - 51.84 = .86 =$ the second difference. Divide the second difference by the first difference, and the first figure of the quotient, if the quotient is .05 or greater, will be the third figure of the root, when reduced to one figure. If the quotient is less than .05, the third figure of the root is a cipher. Thus, $.86 \div 1.45 = .59+$, or .6 when reduced to one figure. Therefore, the first three figures of $\sqrt{52.70}$ are 7.26. Since the integral part of the given number contains two periods, there are two figures in the integral part of the root; therefore, $\sqrt{5,269.73} = 72.6$, to three figures. Ans.

101. The cube root is found to three significant figures in exactly the same way, as shown in the following example:

EXAMPLE.—Find the first three figures of $\sqrt[3]{.0625}$.

SOLUTION.—Pointing off and placing the decimal point between the first and second significant periods, the result is 62.500. Referring to the table, the first two figures of the root are 3.9; the first difference is $64.000 - 59.319 = 4.681$; the second difference is $62.500 - 59.319 = 3.181$; $3.181 \div 4.681 = .67+$, or .7 to one figure. Therefore, $\sqrt[3]{62.5} = 3.97$, and $\sqrt[3]{.0625} = .397$ to three significant figures. Ans.

102. Having found the first three significant figures by means of the table, find the fourth and fifth figures in the usual manner by using the first three figures in finding the third approximation.

For example, find the cube root of 126.57 to five figures. Referring to the table, the first two figures are 5.0. The first difference is $132.651 - 125.000 = 7.651$; the second

difference is $126.57 - 125.000 = 1.57$; $1.57 \div 7.651 = .20+$. Hence, the first three figures are 5.02. Using 5.02 for one of the equal factors, the third factor is $126.57 \div 5.02^2 = 5.02253+$, and the third approximation is $\frac{2 \times 5.02 + 5.02253}{3} = 5.02084+$, or $\sqrt[3]{126.57} = 5.0208$ to five figures. Ans.

103. If more than five significant figures of the square or cube root are desired, use the five figures of the third approximation for one of the equal factors and calculate the unknown factor to as many figures as are desired plus one; the next approximation will be correct to at least nine figures, if the unknown factor has been calculated to ten figures.

ROOTS OF FRACTIONS.

104. If the given number is in the form of a fraction, and it is required to find some root of it, the simplest and most exact method is to reduce the fraction to a decimal and extract the required root of the decimal. If, however, the numerator and denominator of the fraction are perfect powers, extract the required root of each separately, and write the root of the numerator for a new numerator, and the root of the denominator for a new denominator.

EXAMPLE 1.—What is the square root of $\frac{9}{64}$?

SOLUTION.— $\sqrt{\frac{9}{64}} = \frac{\sqrt{9}}{\sqrt{64}} = \frac{3}{8}$. Ans.

EXAMPLE 2.—What is the square root of $\frac{5}{8}$?

SOLUTION.— $\sqrt{\frac{5}{8}} = \sqrt{.625} = .79057-$, since $\frac{5}{8} = .625$. Ans.

EXAMPLE 3.—What is the cube root of $\frac{27}{64}$?

SOLUTION.— $\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$. Ans.

EXAMPLE 4.—What is the cube root of $\frac{1}{4}$?

SOLUTION.—Since $\frac{1}{4} = .25$, $\sqrt[3]{\frac{1}{4}} = \sqrt[3]{.25} = .62996+$. Ans.

105. **Rule.**—*Extract the required root of the numerator and denominator separately; or, reduce the fraction to a decimal, and extract the root of the decimal.*

EXAMPLES FOR PRACTICE.

(*a*) $\sqrt{\frac{9}{16}} = ?$
(*b*) $\sqrt[3]{\frac{27}{1728}} = ?$
(*c*) $\sqrt[3]{\frac{27}{375}} = ?$
(*d*) $\sqrt[3]{\frac{5}{128}\frac{60}{0}} = ?$

Ans. (*a*) $\frac{3}{4}$. (*b*) $\frac{1}{4}$. (*c*) .41602—. (*d*) 1.6355+.

FOURTH ROOT.

Since fourth and fifth roots are very seldom required, the student may, if he so desires, omit the following articles. No examples relating to fourth and fifth roots are included among the Examination Questions at the end of this Paper; nevertheless, the student is advised to read carefully the following articles, as they contain much that may be of value to him.

106. The fourth root may be found by a method similar to that just described for extracting square and cube roots, dividing the given number into periods of four figures each, and resolving the given number into four equal factors. It is generally easier and shorter, however, to extract the square root and then extract the square root of the result. For example, to extract the fourth root of 5,735,796,283.8016, which is a perfect fourth power (and consequently a perfect square, also), the square root would be extracted in the usual manner, obtaining 75,735.04. The square root of this result would then be extracted, obtaining 275.2. In other words, $\sqrt[4]{5,735,796,283.8016} = \sqrt{\sqrt{5,735,796,283.8016}} = \sqrt{75,735.04} = 275.2$.

The fourth root is very seldom required, and can always be found as just described.

FIFTH ROOT.

107. The fifth root is required oftener than the fourth root, but nevertheless it is seldom necessary to extract it. The method is the same in principle as that explained for cube root. The given number is divided into periods of five figures each and resolved into five equal factors; the first period may contain one, two, three, four, or five figures. As in the case of cube root, it is advisable to construct a little table giving the fifth powers of the nine digits, similar to that here given. An example will illustrate the process.

$1^5 =$	1
$2^5 =$	32
$3^5 =$	243
$4^5 =$	1,024
$5^5 =$	3,125
$6^5 =$	7,776
$7^5 =$	16,807
$8^5 =$	32,768
$9^5 =$	59,049

EXAMPLE.— $\sqrt[5]{5,186.42} = ?$

SOLUTION.—The first period 5186 lies between $5^5 = 3125$ and $6^5 = 7776$; hence, the root is 5 plus an interminable decimal. Trying 5 as one of the four equal factors and dividing the first period by their product to find the fifth factor, we have $\frac{5186}{5 \times 5 \times 5 \times 5} = 5186 \div 5^4 = 5186 \div 625 = 8.29+$. Trying 6 as one of the four equal factors, the fifth factor is $5186 \div 6^4 = 5186 \div 1296 = 4.00+$. Since the difference between 6 and 4 is less than the difference between 5 and 8.29, use 6 as one of the equal factors. Then, $5186 = 6 \times 6 \times 6 \times 6 \times 4$. The average of these factors is $\frac{6+6+6+6+4}{5} = \frac{4 \times 6 + 4}{5} = \frac{28}{5} = 5.6$, the first approximation.

Using 5.6 as one of the equal factors, the fifth factor is $5186.42 \div 5.6^4 = 5186 \div 983.4496 = 5186 \div 983.4000$ (using but four significant figures, since only three figures of the second approximation are required—see **b**, Art. **80**) $= 5.273$, and the second approximation is $\frac{4 \times 5.6 + 5.273}{5} = 5.534+$, or 5.53 to three figures.

Using 5.53 as one of the equal factors, the fifth factor is $5186.42 \div 5.53^4 = 5186.42 \div 935.191 = 5.54584+$, and the third approximation is $5.53316+$, or 5.5332 to five figures. Ans.

The exact root to seven figures is 5.533164.

108. In order that the fifth significant figure of the fifth root of a number may be correct, it is absolutely essential

that the third significant figure of the second approximation be correct. In the last example, it will be noticed that the difference between the first and second approximations is $5.60 - 5.53 = .07$, which is less than one unit in the second figure, but very near to it. Had this difference been as much as 1, or had it exceeded one unit in the second figure, it would have been advisable to recalculate the second approximation.

109. The labor involved in extracting the fifth root is very much greater than that necessary to extract the cube root, chiefly on account of raising numbers to the fourth power. This labor may be shortened considerably in the following manner:

Consider any number, as 4. Now, $4^4 = 4 \times 4 \times 4 \times 4 = (4 \times 4) \times (4 \times 4) = 16 \times 16 = 256$. In other words, to raise a number to the fourth power, square the number and then square the square. Now the square of any number contains twice as many significant figures as the number or twice as many less 1; the cube of any number contains three times as many significant figures as the number or three times as many less one or two; the fourth power will contain four times as many or four times as many less one, two, or three; and so on. Hence, the fourth power of a number containing two figures will contain five, six, seven, or eight figures; and of one containing three figures, nine, ten, eleven, or twelve figures. In determining the fifth factor for the second approximation, only four figures of the fourth power are required, and in determining the fifth factor for the third approximation, only six figures of the fourth power are required. Therefore, any method that will enable us to dispense with unnecessary figures will lessen the work. The following method, which will also apply to any case of multiplication when only a certain definite number of figures are desired in the product, is the best we know of; it is best illustrated by an example.

EXAMPLE.—Multiply 467,295 by 634,137 and obtain six figures of the product correct.

SOLUTION.—

(*a*)		(*b*)
467295		4'6'7'2'9'5
634137		634137
2803770		2803770
140188	5	140188
18691	80	18691
467	295	467
140	1885	140
32	71065	32
2963290	49415	2963288, or 296329 to six figures.

EXPLANATION.—Instead of beginning with the last, or right-hand, digit, as in ordinary multiplication, begin with the first, or left-hand, digit of the multiplier, and multiply in the ordinary manner, the product being 2,803,770. Multiply by the second digit and write the first figure obtained in the second partial product one place to the right of the last figure of the first partial product. So proceed until all the partial products have been found; adding them to find the entire product. The result is shown at (*a*). Now, in order to have six figures of the product correct, seven figures should be obtained (see **a,** Art. **80**). It is therefore evident that all figures to the right of the vertical line in (*a*) are unnecessary. Hence, proceed as shown in (*b*). The first partial product contains seven figures, all that are required; therefore, cut off the figure 5 in the multiplicand when finding the second partial product, but multiply it by 3 in order to determine how much to carry. Thus, say mentally "three times five is fifteen," and carry 1. Then say "three times nine is twenty-seven and one is twenty-eight," etc. When multiplying by the next digit 4, cut off the second figure from right of the multiplicand, but carry the 3 that is obtained by multiplying 9 by 4, and say "four times two is eight and three is eleven." Proceeding in this manner, no figure of any of the partial products will extend beyond the place occupied by the seventh figure of the entire product.

110. The operation of division may be shortened in a similar manner to that just described for multiplication. Perform the division in the usual manner until the number of significant figures in the quotient equals the number obtained by subtracting the number of significant figures in the divisor from the number desired in the quotient plus three; then cut off one figure from the right of the divisor before finding the next figure of the quotient; cut off the second figure from the right of the divisor before finding the succeeding figure of the quotient; and so on until the quotient contains one more than the required number of figures. It is here assumed that the dividend and divisor do not contain more than one significant figure in excess of the number required in the quotient. (See **a** and **b**, Art. **80.**)

EXAMPLE.—Divide 71,346.247 by 27,846.392 and obtain five significant figures of the quotient correct.

SOLUTION.—

```
7 1 3 4 6.2 0 ( 2 7'8'4'6'.4
5 5 6 9 2 8     2.5 6 2 1 3, or 2.5621 to five figures. Ans.
1 5 6 5 3 4 0
1 3 9 2 3 2 0
  1 7 3 0 2 0
  1 6 7 0 7 8
    5 9 4 2
    5 5 6 9
      3 7 3
      2 7 8
        9 5
        8 3
        1 2
```

EXPLANATION.—Since five significant figures are required in the quotient, the dividend and divisor are limited to six significant figures. The number of significant figures required in the quotient before beginning to cut off figures from the divisor is $5 + 3 - 6 = 8 - 6 = 2$; hence, before finding the third figure of the quotient, cut off the figure 4 from the right of the divisor, but multiply 4 by 6 in order to see how much to carry. Thus, say "six times four is twenty-

four," and carry 2; then, say "six times six is thirty-six and two is thirty-eight," and write 8 and carry 3; and so on. Before finding the fourth figure of the quotient, cut off the next figure 6 of the divisor. The student will find it convenient to place the divisor at the right of the dividend with the quotient underneath, as shown. This arrangement saves space and brings each figure of the quotient directly under the divisor, making the multiplication easier.

To locate the decimal point in the quotient, the easiest way is to proceed as follows: Move the decimal point in the divisor to the right until it follows the right-hand figure; that is, make the divisor a whole number; move the decimal point in the dividend as many decimal places to the right as it was moved in the divisor, annexing ciphers if necessary. If the dividend will contain the divisor one or more times, there will be as many figures in the integral part of the quotient as there are figures left in the dividend after finding the first remainder plus one. If the dividend will not contain the divisor, annex ciphers to follow the decimal point until the dividend contains the divisor, and the first significant figure of the quotient will then be located as many decimal places to the right of the decimal point as there were ciphers annexed. For instance, $.046 \div 21.76 = 4.6 \div 2{,}176. = 4.600 \div 2{,}176 = .002+$; $4.6 \div 21.76 = 460. \div 2{,}176. = 460.0 \div 2{,}176 = .2+$; $460 \div 21.76 = 46{,}000. \div 2{,}176 = 21.+$. In the last example, $71{,}346.2 \div 27{,}846.4 = 713{,}462. \div 278{,}464. = 2+$.

111. Having shown how the work of calculation may be greatly reduced, an example will now be given, showing all the figures used in extracting the fifth root, each operation being numbered in the order in which it is performed. If the student will perform the various calculations in the usual manner without shortening the work as just described, and count the number of figures used in both cases, he will readily see what a great saving has been effected by employing the abbreviated methods of multiplication and division.

EXAMPLE.— $\sqrt[5]{8{,}269} = ?$

SOLUTION.—

(1)

6
6
36
36
108
216
1296

(2)

8269 (1296
7776 6.38+
493
388
105
103
2

(3)

$$\frac{4 \times 6 + 6.38}{5} = 6.07+$$

(6)

$$\frac{4 \times 6.1 - 5.974}{5} = \frac{30.374}{5} = 6.074-$$

(4)

6.1
6.1
366
61
37.21
37.21
11163
2604
74
3
13844

(5)

8269.0 (1384
6920 5.974
13490
12456
1034
968
66
55
11

(7)

6.07
6.07
3642
4249
36.8449
36.8445
36.845
110535
22107
2947
147
18
135 7.553

(8)

8269.00 (13577.55
8145 30 6.09 112
123700
122179
1521
1357
164
135
29
27
2

(9)

$$\frac{4 \times 6.07 + 6.09112}{5}$$

$$\frac{30.37112}{5} = 6.07422$$

= 6.0742 to five figures. Ans.

REMARK.—It was unnecessary to try 7 for one of the equal factors, because there was but a very little difference between the fifth factor 6.38 and one of the equal factors 6; in fact, when this difference is not greater than 2.5 units it is unnecessary to try the next higher number for one of the equal factors. In this case the difference is $6.38 - 6 = .38$, or less than one unit.

112. When the sixth significant figure of the third approximation is 5, it is not always advisable to increase the fifth figure by one. To ascertain whether or not the fifth figure should be increased, recalculate the third approximation, using for one of the equal factors the third approximation first found, correct to *four* figures; if the sixth significant figure is then 5+, increase the fifth figure by one.

EXAMPLE 1.— $\sqrt[5]{3.056} = ?$

SOLUTION.—Using the first two significant figures and trying 1 for the equal factors, the fifth factor is $3.1 \div 1^4 = 3.1$, and the first approximation is $\frac{4 \times 1 + 3.1}{5} = 1.42$. Since $3.1 - 1 = 2.1$ is less than 2.5 (see Remark, Art. **111**) it is not necessary to try 2 for one of the equal factors.

$$3.056 \div 1.4^4 = .795+. \quad \frac{4 \times 1.4 + .795}{5} = 1.279+.$$

Since the difference between 1.42, the first approximation, and 1.279, the second approximation, is greater than one unit in the second figure, try 1.3 for one of the equal factors and recalculate the second approximation.

$$3.056 \div 1.3^4 = 1.069+. \quad \frac{4 \times 1.3 + 1.069}{5} = 1.253+.$$

Since the difference, $1.3 - 1.253 = .047$, is less than one unit in the second figure, use 1.25 for one of the equal factors in finding the third approximation.

$$3.056 \div 1.25^4 = 1.25173+. \quad \frac{4 \times 1.25 + 1.25173}{5} = 1.25034+,$$

or 1.2503 to five figures. Ans.

The exact root to seven figures is 1.250347.

EXAMPLE 2.— $\sqrt[5]{3} = ?$

SOLUTION.—Trying 1.3 for one of the equal factors (see last example), $3 \div 1.3^4 = 1.050+$. $\frac{4 \times 1.3 + 1.050}{5} = 1.25$.

Using 1.25 for one of the equal factors, the fifth factor is $3 \div 1.25^4 = 1.22879+$, and the third approximation is $\frac{4 \times 1.25 + 1.22879}{5} = 1.24575+$.

Since the sixth figure is 5, it will be well to recalculate the third approximation, using 1.246 for one of the equal factors. Hence, $3 \div 1.246^4 = 1.244656+$, and the third approximation is

$\frac{4 \times 1.246 + 1.244656}{5} = 1.245731+$, or 1.2457 to five figures. Ans.

The exact root to seven figures is 1.245731.

113. The fifth root is very seldom required; the most prominent case in practice arises in connection with finding the diameter of a pipe that will discharge a required amount of water, the head and length of pipe being known. It is also required in connection with certain problems in mine ventilation. The fourth root is used even less frequently than the fifth root. Roots higher than the fifth are never required.

TABLE METHOD OF EXTRACTING THE FIFTH ROOT.

114. In exactly the same way as in the case of square and cube roots, the first three significant figures of the fifth root may be found by means of the table of the powers of numbers.

EXAMPLE.— $\sqrt[5]{238.75} = ?$

SOLUTION.—Referring to the table, the first two figures are 2.9; the first difference is $243.00 - 205.11 = 37.89$; the second difference is $238.75 - 205.11 = 33.64$; $33.64 \div 37.89 = .88+$, or .9 to one figure. Therefore, $\sqrt[5]{238.75} = 2.99$ to three figures.

115. When finding the fifth root of numbers whose first period contains but one significant figure, carry the quotient obtained by dividing the second difference by the first difference to three decimal places, and if the third figure is 5 or a greater digit, increase the second figure by 1 and add these two figures of the quotient to those previously found for the third and fourth figures of the root. Then use all four figures when finding the third approximation.

EXAMPLE.— $\sqrt[5]{3} = ?$

SOLUTION.—Referring to the table, the first two figures are 1.2; the first difference is $3.7129 - 2.4883 = 1.2246$; the second difference is $3 - 2.4883 = .5117$; $.5117 \div 1.2246 = .417+$, or .42 to two figures. Hence, assume that one of the equal factors is 1.242; the fifth factor is $3 \div 1.242^4 = 1.26076$, and the third approximation is $\frac{4 \times 1.242 + 1.26076}{5}$ $= 1.245752$. Since the sixth figure is 5, recalculate the third approximation, using the result just found correct to four figures for one of the equal factors (see example 2, Art. **112**). The result is 1.2457. Ans.

ARITHMETIC.

(SECTION 6.)

INTRODUCTION.

116. The subject of ratio and proportion is one of the most useful of all the subjects that are taught in Arithmetic. The student will find frequent use for the principles treated of in the following pages, and is requested to study them with great care.

The student should carefully study the definitions, constantly referring to them from time to time as he progresses with the subject; he should note particularly those articles relating to inverse ratio and inverse proportion. The idea of inverse proportion is usually a difficult one for the student to grasp, but a careful study of Art. **149** and of the examples in Arts. **150** and **151** will make the matter clear to him.

Although some of the examples included between Arts. **130** and **153,** inclusive, may be solved by other methods than the use of proportion, all the examples included between the above articles, and those of similar nature included in the Examination Questions, must be solved by applying the principles of proportion; no other method of solution will be accepted. The student should study very carefully Arts. **128, 129, 143,** and **144**; they are very important, and they should be thoroughly understood.

The subject of compound proportion as treated in ordinary textbooks on Arithmetic usually proves of considerable difficulty to the student. The method here given, while not entirely new, presents the matter in a clearer light, we believe, than any other we have ever seen.

For notice of copyright, see page immediately following the title page.

RATIO.

117. Suppose that it is desired to compare two numbers, say 20 and 4. If we wish to know how many times larger 20 is than 4, we divide 20 by 4 and obtain 5 for the quotient; thus, $20 \div 4 = 5$. Hence, we say that 20 is 5 times as large as 4, i. e., 20 contains 5 times as many units as 4. Again, suppose we desire to know what part of 20 is 4. We then divide 4 by 20 and obtain $\frac{1}{5}$; thus, $4 \div 20 = \frac{1}{5}$, or .2. Hence, 4 is $\frac{1}{5}$ or .2 of 20. This operation of comparing two numbers is termed *finding the ratio* of the two numbers. Ratio, then, is a comparison. It is evident that the two numbers to be compared must be expressed in the same unit; in other words, the two numbers must both be abstract numbers or concrete numbers of the same kind. For example, it would be absurd to compare 20 horses with 4 birds, or 20 horses with 4. Hence, **ratio** may be defined as a comparison between two numbers of the same kind.

118. A ratio may be expressed in three ways; thus, if it is desired to compare 20 and 4, and express this comparison as a ratio, it may be done as follows: $20 \div 4$, $20 : 4$, or $\frac{20}{4}$. All three are read *the ratio of 20 to 4.* The ratio of 4 to 20 would be expressed thus: $4 \div 20$, $4 : 20$, or $\frac{4}{20}$. The first method of expressing a ratio, although correct, is seldom or never used; the second form is the one oftenest met with, while the third is rapidly growing in favor, and is likely to supersede the second. The third form, called the fractional form, is preferred by modern mathematicians, and possesses great advantages to students of algebra and of higher mathematical subjects. The second form seems to be better adapted to arithmetical subjects, and is the one we shall ordinarily adopt. There is still another way of expressing a ratio, though seldom or never used in the case of a simple ratio like that given above. Instead of the colon, a straight vertical line is used; thus, $20 \mid 4$.

119. The **terms** of a ratio are the two numbers to be compared; thus, in the above ratio, 20 and 4 are the terms. When both terms are considered together they are called a **couplet**; when considered separately, the first term is called the **antecedent**, and the second term, the **consequent**. Thus, in the ratio 20 : 4, 20 and 4 form a couplet, and 20 is the antecedent, and 4, the consequent.

120. A ratio may be **direct** or **inverse**. The *direct ratio* of 20 to 4 is 20 : 4, while the *inverse ratio* of 20 to 4 is 4 : 20. The direct ratio of 4 to 20 is 4 : 20, and the inverse ratio is 20 : 4. An inverse ratio is sometimes called a **reciprocal** ratio. The **reciprocal** of a number is 1 divided by the number. Thus, the reciprocal of 17 is $\frac{1}{17}$; of $\frac{3}{8}$ is $1 \div \frac{3}{8} = \frac{8}{3}$; i. e., the reciprocal of a fraction is the fraction inverted. Hence, the inverse ratio of 20 to 4 may be expressed as 4 : 20 or as $\frac{1}{20} : \frac{1}{4}$. Both have equal values; for, $4 \div 20 = \frac{1}{5}$, and $\frac{1}{20} \div \frac{1}{4} = \frac{1}{20} \times \frac{4}{1} = \frac{1}{5}$.

121. The term **vary** implies a ratio. When we say that two numbers vary as some other two numbers, we mean that the ratio between the first two numbers is the same as the ratio between the other two numbers.

122. The **value** of a ratio is the result obtained by performing the division indicated. Thus, the value of the ratio 20 : 4 is 5; it is the quotient obtained by dividing the antecedent by the consequent.

123. By expressing the ratio in the fractional form, for example, the ratio of 20 to 4 as $\frac{20}{4}$, it is easy to see, from the laws of fractions, that if both terms be multiplied or both divided by the same number it will not alter the value of the ratio. Thus,

$$\frac{20}{4} = \frac{20 \times 5}{4 \times 5} = \frac{100}{20}; \text{ and } \frac{20}{4} = \frac{20 \div 4}{4 \div 4} = \frac{5}{1}.$$

124. It is also evident, from the laws of fractions, that multiplying the antecedent or dividing the consequent multiplies the ratio, and dividing the antecedent or multiplying the consequent divides the ratio.

125. When a ratio is expressed in words, as the ratio of 20 to 4, the first number named is always regarded as the antecedent and the second as the consequent, without regard to whether the ratio itself is direct or inverse. *When not otherwise specified, all ratios are understood to be direct.* To express an inverse ratio the simplest way of doing it is to express it as if it were a direct ratio, with the first number named as the antecedent, and then transpose the antecedent to the place occupied by the consequent and the consequent to the place occupied by the antecedent; or if expressed in the fractional form, invert the fraction. Thus, to express the inverse ratio of 20 to 4, first write it 20 : 4, and then, transposing the terms, as 4 : 20; or as $\frac{20}{4}$, and then inverting, as $\frac{4}{20}$. Or, the reciprocals of the numbers may be taken, as explained above. To **invert** a ratio is to transpose its terms.

EXAMPLES FOR PRACTICE.

126. What is the value of the ratio of:

(*a*)	98 : 49?	Ans.	(*a*) 2.
(*b*)	\$45 : \$9?		(*b*) 5.
(*c*)	$6\frac{1}{4} : \frac{3}{4}$?		(*c*) $12\frac{1}{2}$.
(*d*)	3.5 : 4.5?		(*d*) $.77\frac{7}{9}$.
(*e*)	The inverse ratio of 76 to 19?		(*e*) $\frac{1}{4}$.
(*f*)	The inverse ratio of 49 to 98?		(*f*) 2.
(*g*)	The inverse ratio of 18 to 24?		(*g*) $1\frac{1}{3}$.
(*h*)	The inverse ratio of 9 to 15?		(*h*) $1\frac{2}{3}$.
(*i*)	The ratio of 10 to 3, multiplied by 3?		(*i*) 10.
(*j*)	The ratio of 35 to 49, multiplied by 7?		(*j*) 5.
(*k*)	The ratio of 18 to 64, divided by 9?		(*k*) $\frac{1}{32}$.
(*l*)	The ratio of 14 to 28, divided by 5?		(*l*) $\frac{1}{10}$.

---

127. Instead of expressing the value of a ratio by a single number as above, it is customary to express it by

means of another ratio in which the consequent is 1. Thus, suppose that it is desired to find the ratio of the weights of two pieces of iron, one weighing 45 pounds and the other weighing 30 pounds. The ratio of the heavier to the lighter is then 45 : 30, an inconvenient expression. Using the fractional form, we have $\frac{45}{30}$. Dividing both terms by 30, the consequent, we obtain $\frac{1\frac{1}{2}}{1}$ or $1\frac{1}{2} : 1$. This is the same result as obtained above, for $1\frac{1}{2} \div 1 = 1\frac{1}{2}$, and $45 \div 30 = 1\frac{1}{2}$.

128. A ratio may be squared, cubed, or raised to any power, or any root of it may be taken. Thus, if the ratio of two numbers is 105 : 63, and it is desired to cube this ratio, the cube may be expressed as $105^3 : 63^3$. That this is correct is readily seen; for, expressing the ratio in the fractional form, it becomes $\frac{105}{63}$, and the cube is $\left(\frac{105}{63}\right)^3 = \frac{105^3}{63^3}$ $= 105^3 : 63^3$. Also, if it is desired to extract the cube root of the ratio $105^3 : 63^3$, it may be done by simply dividing the exponents by 3, obtaining 105 : 63. This may be proved in the same way as in the case of cubing the ratio. Thus, $105^3 : 63^3 = \left(\frac{105}{63}\right)^3$, and $\sqrt[3]{\left(\frac{105}{63}\right)^3} = \frac{105}{63} = 105 : 63$.

129. Since $\left(\frac{105}{63}\right)^3 = \left(\frac{5}{3}\right)^3$, it follows that $105^3 : 63^3$ $= 5^3 : 3^3$ (this expression is read, the ratio of 105 cubed to 63 cubed equals the ratio of 5 cubed to 3 cubed), and, hence, that the antecedent and consequent may both be multiplied or both divided by the same number, irrespective of any indicated powers or roots, without altering the value of the ratio. Thus, $24^2 : 18^2 = 4^2 : 3^2$. For, performing the operations indicated by the exponents, $24^2 = 576$ and $18^2 = 324$. Hence, $576 : 324 = 1\frac{7}{9}$ or $1\frac{7}{9} : 1$. Also, 4^2 $= 16$ and $3^2 = 9$; hence, $16 : 9 = 1\frac{7}{9}$ or $1\frac{7}{9} : 1$, the same result as before. Also, $24^2 : 18^2 = \frac{24^2}{18^2} = \left(\frac{24}{18}\right)^2 = \left(\frac{4}{3}\right)^2$ $= \frac{4^2}{3^2} = 4^2 : 3^2$

The statement may be proved for roots in the same manner. Thus $\sqrt[3]{24^3} : \sqrt[3]{18^3} = \sqrt[3]{4^3} : \sqrt[3]{3^3}$. For, the $\sqrt[3]{24^3} = 24$ and $\sqrt[3]{18^3} = 18$; and, $24:18 = 1\frac{1}{3}$ or $1\frac{1}{3}:1$. Also, $\sqrt[3]{4^3} = 4$ and $\sqrt[3]{3^3} = 3$; $4:3 = 1\frac{1}{3}$ or $1\frac{1}{3}:1$.

If the numbers composing the antecedent and consequent have different exponents, or if different roots of those numbers are indicated, the operations above described cannot be performed. This is evident; for, consider the ratio of $4^2:8^3$. When expressed in the fractional form it becomes $\frac{4^2}{8^3}$, which cannot be expressed either as $\left(\frac{4}{8}\right)^2$ or as $\left(\frac{4}{8}\right)^3$, and, hence, cannot be reduced as described above.

PROPORTION.

130. **Proportion** is an equality of ratios, the equality being indicated by the double colon (::) or by the sign of equality (=). Thus, to write in the form of a proportion the two equal ratios, 8:4 and 6:3, which both have the same value, 2, we may employ one of the three following forms:

$$8:4 :: 6:3 \qquad (1)$$

$$8:4 = 6:3 \qquad (2)$$

$$\frac{8}{4} = \frac{6}{3} \qquad (3)$$

131. The first form is the one most extensively used, by reason of its having been exclusively employed in all the older works on mathematics. The second and third forms are being adopted by all modern writers on mathematical subjects, and, in time, will probably entirely supersede the first form. In this subject we shall adopt the second form, unless some statement can be made clearer by using the third form.

132. A proportion may be read in two ways. The old way to read the above proportion was—*8 is to 4 as 6 is to 3;* the new way is—*the ratio of 8 to 4 equals the ratio of 6 to 3.* The student may read it either way, but we recommend the latter.

133. Each ratio of a proportion is termed a **couplet.** In the above proportion, 8 : 4 is a couplet, and so is 6 : 3.

134. The numbers forming the proportion are called **terms**; and they are numbered consecutively from left to right, thus:

first second third fourth

8 : 4 = 6 : 3

Hence, in any proportion, the ratio of the first term to the second term equals the ratio of the third term to the fourth term.

135. The first and fourth terms of a proportion are called the **extremes,** and the second and third terms, the **means.** Thus, in the foregoing proportion, 8 and 3 are the extremes and 4 and 6 are the means.

136. A **direct proportion** is one in which both couplets are direct ratios.

137. An **inverse proportion** is one which requires one of the couplets to be expressed as an inverse ratio. Thus, 8 is to 4 inversely as 3 is to 6 must be written 8 : 4 = 6 : 3; i. e., the second ratio (couplet) must be inverted.

138. Proportion forms one of the most useful sections of arithmetic. In our grandfathers' arithmetics, it was called "The rule of three."

139. **Rule I.**—*In any proportion, the product of the extremes equals the product of the means.*

Thus, in the proportion,

17 : 51 = 14 : 42.

17 × 42 = 51 × 14, since both products equal 714.

140. **Rule II.**—*The product of the extremes divided by either mean gives the other mean.*

EXAMPLE.—What is the third term of the proportion 17 : 51 = : 42?

SOLUTION.—Applying rule II, 17 × 42 = 714, and 714 ÷ 51 = 14. Ans.

141. **Rule III.**—*The product of the means divided by either extreme gives the other extreme.*

EXAMPLE.—What is the first term of the proportion : 51 = 14 : 42?

SOLUTION.—Applying rule III, 51 × 14 = 714, and 714 ÷ 42 = 17. Ans

142. When stating a proportion in which one of the terms is unknown, represent the missing term by a letter, as x. Thus, the last example would be written,

$$x : 51 = 14 : 42$$

and for the value of x we have $x = \frac{51 \times 14}{42} = 17$.

143. If the same operations (addition and subtraction excepted) be performed upon *all* the terms of a proportion, the proportion is not thereby destroyed. In other words, if all the terms of a proportion be (1) multiplied or (2) divided by the same number; (3) if all the terms be raised to the same power; (4) if the same root of all the terms be taken, or (5) if both couplets be inverted, the proportion still holds. We will prove these statements by a numerical example, and the student can satisfy himself by other similar ones. The fractional form will be used, as it is better suited to the purpose. Consider the proportion $8 : 4 = 6 : 3$. Expressing it in the third form, it becomes $\frac{8}{4} = \frac{6}{3}$. What we are to prove is that if any of the five operations enumerated above be performed upon all the terms of the proportion, the first fraction will still equal the second fraction.

1. Multiplying all the terms by any number, say 7, $\frac{8 \times 7}{4 \times 7} = \frac{6 \times 7}{3 \times 7}$; or $\frac{56}{28} = \frac{42}{21}$. Now $\frac{56}{28}$ evidently equals $\frac{42}{21}$, since the value of either ratio is 2, and the same is true of the original proportion.

2. Dividing all the terms by any number, say 7, $\frac{8 \div 7}{4 \div 7} = \frac{6 \div 7}{3 \div 7}$; or $\frac{\frac{8}{7}}{\frac{4}{7}} = \frac{\frac{6}{7}}{\frac{3}{7}}$. But $\frac{8}{7} \div \frac{4}{7} = 2$, and $\frac{6}{7} \div \frac{3}{7} = 2$ also, the same as in the original proportion.

3. Raising all the terms to the same power, say the cube, $\frac{8^3}{4^3} = \frac{6^3}{3^3}$. This is evidently true, since $\frac{8^3}{4^3} = \left(\frac{8}{4}\right)^3 = 2^3 = 8$, and $\frac{6^3}{3^3} = \left(\frac{6}{3}\right)^3 = 2^3 = 8$ also.

4. Extracting the same root of all the terms, say the cube root, $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \frac{\sqrt[3]{6}}{\sqrt[3]{3}}$. It is evident that this is likewise true, since $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \sqrt[3]{\frac{8}{4}} = \sqrt[3]{2}$, and $\frac{\sqrt[3]{6}}{\sqrt[3]{3}} = \sqrt[3]{\frac{6}{3}} = \sqrt[3]{2}$ also.

5. Inverting both couplets, $\frac{4}{8} = \frac{3}{6}$, which is true, since both equal $\frac{1}{2}$.

144. If both terms of either couplet be multiplied or both divided by the same number, the proportion is not destroyed. This should be evident from the preceding article, and also from Art. **123.** Hence, in any proportion, equal factors may be canceled from the terms of a couplet, before applying rule II or III. Thus, the proportion $45:9 = x:7.1$, we may divide both terms of the first couplet by 9 (that is, cancel 9 from both terms), obtaining $5:1 = x:7.1$, whence $x = 7.1 \times 5 \div 1 = 35.5$. (See Art. **129.**)

145. The principle of all calculations in proportion is this: *Three of the terms are always given, and the remaining one is to be found.*

146. Example.—If 4 men can earn $25 in one week, how much can 12 men earn in the same time?

Solution.—The required term must bear the same relation to the given term of the same kind, as one of the remaining terms bears to the other remaining term. We can then form a proportion by which the required term may be found.

The first question the student must ask himself in every calculation by proportion is:

"What is it I want to find?"

In this case it is dollars. We have two sets of men, one set earning $25, and we want to know how many dollars the other set earns. It is evident that the amount 12 men earn bears the same relation to the amount 4 men earn as 12 men bear to 4 men. Hence, we have the proportion, the amount 12 men earn is to $25 as 12 men are to 4 men, or, since either extreme equals the product of the means divided by the other extreme, we have

The amount 12 men earn : $25 :: 12 men : 4 men,

or the amount 12 men earn $= \frac{\$25 \times 12}{4} = \$75$. Ans.

Since it matters not which place x, or the required term, occupies, the problem could be stated in any of the following forms, the value of x being the same in each:

(a) \$25 : the amount 12 men earn = 4 men : 12 men; or the amount 12 men earn $= \frac{\$25 \times 12}{4}$, or \$75, since either mean equals the product of the extremes divided by the other mean.

(b) 4 men : 12 men = \$25 : the amount that 12 men earn; or the amount that 12 men earn $= \frac{\$25 \times 12}{4}$, or \$75, since either extreme equals the product of the means divided by the other extreme.

(c) 12 men : 4 men = the amount 12 men earn : \$25; or the amount that 12 men earn $= \frac{\$25 \times 12}{4}$, or \$75, since either mean equals the product of the extremes divided by the other mean.

147. If the proportion is an inverse one, first form it as though it were a direct proportion, and then invert one of the couplets.

EXAMPLES FOR PRACTICE.

148. Find the value of x in each of the following:

(a) \$16 : \$64 :: x : \$4.
(b) x : 85 :: 10 : 17.
(c) 24 : x :: 15 : 40.
(d) 18 : 94 :: 2 : x.
(e) \$75 : \$100 = x : 100.
(f) 15 pwt. : x = 21 : 10.
(g) x : 75 yd. = \$15 : \$5.

Ans. (a) x = \$1. (b) x = 50. (c) x = 64. (d) $x = 10\frac{4}{9}$. (e) x = 75. (f) $x = 7\frac{1}{2}$ pwt. (g) x = 225 yd.

1. If 75 pounds of lead cost \$2.10, what would 125 pounds cost at the same rate? Ans. \$3.50.

2. If A does a piece of work in 4 days and B does it in 7 days, how long will it take A to do what B does in 63 days? Ans. 36 days.

3. The circumferences of any two circles are to each other as their diameters. If the circumference of a circle 7 inches in diameter is 22 inches, what will be the circumference of a circle 31 inches in diameter? Ans. $97\frac{3}{7}$ inches

---

INVERSE PROPORTION.

149. In Art. **137**, an inverse proportion was defined as one which required one of the couplets to be expressed as an inverse ratio. Sometimes the word *inverse* occurs in the statement of the example; in such cases, the proportion can

be written directly, merely inverting one of the couplets. But it frequently happens that only by carefully studying the conditions of the example, can it be ascertained whether the proportion is direct or inverse. When in doubt, the student can always satisfy himself as to whether the proportion is direct or inverse by first ascertaining what is required, and stating the proportion as a direct proportion. Then, in order that the proportion may be true, if the first term is smaller than the second term, the third term must be smaller than the fourth; or if the first term is larger than the second term, the third term must be larger than the fourth term. Keeping this in mind, the student can always tell whether the required term will be larger or smaller than the other term of the couplet to which the required term belongs. Having determined this, the student then refers to the example, and ascertains from its conditions whether the required term is to be larger or smaller than the other term of the same kind. If the two determinations agree, the proportion is direct, otherwise it is inverse, and one of the couplets must be inverted.

150. EXAMPLE.—If A's *rate* of doing work is to B's as **5 : 7**, and A does a piece of work in 42 days, in what time will B do it?

SOLUTION.—The required term is the number of days it will take B to do the work. Hence, stating as a direct proportion,

$$5 : 7 = 42 : x.$$

Now, since 7 is greater than 5, x will be greater than 42. But, referring to the statement of the example, it is easy to see that B works faster than A; hence it will take B a less number of days to do the work than A. Therefore, the proportion is an inverse one, and should be stated

$$5 : 7 = x : 42$$

from which $x = \frac{5 \times 42}{7} = 30$ days. Ans.

Had the example been stated thus: The time that A requires to do a piece of work is to the time that B requires, as 5 : 7; A can do it in 42 days, in what time can B do it? it is evident that it would take B a longer time to do the work than it would A; hence, x would be greater than 42, and the proportion would be direct, the value of x being $\frac{7 \times 42}{5}$ = 58.8 days

EXAMPLES FOR PRACTICE.

151. Solve the following:

1. If a pump which discharges 4 gal. of water per min. can fill a tank in 20 hr., how long will it take a pump discharging 12 gal. per min. to fill it? Ans. $6\frac{2}{3}$ hr.

2. If a pump discharges 90 gal. of water in 20 hr., in what time will it discharge 144 gal.? Ans. 32 hr.

3. The weight of any gas (the volume and pressure remaining the same) varies inversely as the absolute temperature. If a certain quantity of some gas weighs 2.927 lb. when the absolute temperature is 525°, what will the same volume of gas weigh when the absolute temperature is 600°, the pressure remaining the same? Ans. 2.561+ lb.

4. If 50 cu. ft. of air weigh 4.2 pounds when the absolute temperature is 562°, what will be the absolute temperature when the same volume weighs 5.8 pounds, the pressure being the same in both cases? Ans. 407°, very nearly.

POWERS AND ROOTS IN PROPORTION.

152. It was stated in Art. **128** that a ratio could be raised to any power or any root of it might be taken. A proportion is frequently stated in such a manner that one of the couplets must be raised to some power or some root of it must be taken. In all such cases, both terms of the couplet so affected *must be raised to the same power or the same root of both terms must be taken.*

153. Example.—Knowing that the weight of a sphere varies as the cube of its diameter, what is the weight of a sphere 6 inches in diameter if a sphere 8 inches in diameter of the same material weighs 180 pounds?

Solution.—This is evidently a direct proportion. Hence, we write

$$6^3 : 8^3 = x : 180.$$

Dividing both terms of the first couplet by 2^3 (see Art. **129**)

$$3^3 : 4^3 = x : 180, \text{ or } 27 : 64 = x : 180;$$

whence, $x = \frac{27 \times 180}{64} = 75\frac{15}{16}$ pounds. Ans.

Example.—A sphere 8 inches in diameter weighs 180 pounds; what is the diameter of another sphere of the same material which weighs $75\frac{15}{16}$ pounds?

Solution.—Since the weights of any two spheres are to each other as the cubes of their diameters, we have the proportion

$$180 : 75\tfrac{15}{16} = 8^3 : x^3.$$

The required term, x, must be cubed, because the other term of the couplet is cubed (see Art. **152**). But, $8^3 = 512$; hence,

$$180 : 75\tfrac{15}{16} = 512 : x^3, \text{ or } x^3 = \frac{75\frac{15}{16} \times 512}{180} = 216;$$

whence, $x = \sqrt[3]{216} = 6$ inches. Ans.

154. Since taking the same root of all the terms of a proportion does not change its value (Art. **143**), the above example might have been solved by extracting the cube root of all the numbers, thus obtaining $\sqrt[3]{180} : \sqrt[3]{75\frac{15}{16}} = 8 : x$; whence, $x = \dfrac{8 \times \sqrt[3]{75\frac{15}{16}}}{\sqrt[3]{180}} = 8 \times \sqrt[3]{\dfrac{75\frac{15}{16}}{180}} = 8\sqrt[3]{\dfrac{1,215}{2,880}} = 8\sqrt[3]{\dfrac{27}{64}}$ $= 8 \times \frac{3}{4} = 6$ inches. The process, however, is longer and is not so direct, and the first method is to be preferred.

155. If two cylinders have equal volumes, but different diameters, the diameters are to each other inversely as the square roots of their lengths. Hence, if it is desired to find the diameter of a cylinder that is to be 15 inches long, and which shall have the same volume as one that is 9 inches in diameter and 12 inches long, we write the proportion

$$9 : x = \sqrt{15} : \sqrt{12}.$$

Since neither 12 nor 15 are perfect squares, we square all the terms (Arts. **154** and **143**) and obtain

$$81 : x^2 = 15 : 12; \text{ whence, } x^2 = \frac{81 \times 12}{15} = 64.8,$$

and $x = \sqrt{64.8} = 8.05$ inches = diameter of 15-inch cylinder.

EXAMPLES FOR PRACTICE.

156. Solve the following examples:

1. The intensity of light varies inversely as the square of the distance from the source of light. If a gas jet illuminates an object 30 feet away with a certain distinctness, how much brighter will the object be at a distance of 20 feet? Ans. $2\frac{1}{4}$ times as bright.

2. In the last example, suppose that the object had been 40 feet from the gas jet; how bright would it have been, compared with its brightness at 30 feet from the gas jet? Ans. $\frac{9}{16}$ as bright.

3. When comparing one light with another, the intensities of their illuminating powers vary as the squares of their distances from the

source. If a man can just distinguish the time indicated by his watch, 50 feet from a certain light, at what distance could he distinguish the time from a light 3 times as powerful? Ans. 86.6+ feet.

4. The quantity of air flowing through a mine varies directly as the square root of the pressure. If 60,000 cubic feet of air flow per minute when the pressure is 2.8 pounds per square foot, how much will flow when the pressure is 3.6 pounds per square foot?

Ans. 68,034 cu. ft. per min., nearly.

5. In the last example, suppose that 70,000 cubic feet per minute had been required; what would be the pressure necessary for this quantity? Ans. 3.81+ lb. per sq. ft.

CAUSES AND EFFECTS.

157. Many examples in proportion may be more easily solved by using the principle of *cause and effect*. That which may be regarded as producing a change or alteration in something, or as accomplishing something, may be called a **cause,** and the change or alteration, or thing accomplished, as the **effect.**

158. *Like causes produce like effects.* Hence, when two causes of the same kind produce two effects of the same kind, the ratio of the causes equals the ratio of the effects; in other words the first cause is to the second cause as the first effect is to the second effect. Thus, in the question—if 3 men can lift 1,400 pounds, how many pounds can 7 men lift?—we call 3 men and 7 men the *causes* (since they accomplish something, viz., the lifting of the weight), the number of pounds lifted, viz., 1,400 pounds and x pounds, are the effects. If we call 3 men the first cause, 1,400 pounds is the first effect; 7 men is the second cause, and x pounds is the second effect. Hence, we may write

1st cause		*2d cause*		*1st effect*		*2d effect*
3	:	7	=	1,400	:	x

whence $x = \frac{7 \times 1{,}400}{3} = 3{,}266\frac{2}{3}$ pounds.

159. The principle of cause and effect is extremely useful in the solution of examples in compound proportion, as we shall now show.

COMPOUND PROPORTION.

160. All the cases of proportion so far considered have been cases of **simple proportion**; i. e., each term has been composed of but one number. There are many cases, however, in which two or all the terms have more than one number in them; all such cases belong to **compound proportion.** In all examples in compound proportion, both causes or both effects or all four consist of more than two numbers. We will illustrate this by an

EXAMPLE.—If 40 men earn $1,280 in 16 days, how much will 36 men earn in 31 days?

SOLUTION.—Since 40 men earn something, 40 men is a cause, and since they take 16 days in which to earn something, 16 days is also a cause. For the same reason 36 men and 31 days are also causes. The effects, that which is earned, are 1,280 dollars and x dollars. Then, 40 men and 16 days make up the first cause, and 36 men and 31 days make up the second cause. $1,280 is the first effect, and $\$x$ is the second effect. Hence, we write

1st cause		*2d cause*		*1st effect*		*2d effect*
40	:	36	=	1,280	:	x
16		31				

Now, instead of using the colon to express the ratio, we shall use the vertical line (see Art. **118**), and the above becomes

$$\begin{matrix} 40 \\ 16 \end{matrix} \Big| \begin{matrix} 36 \\ 31 \end{matrix} = 1{,}280 \Big| \; x.$$

In the last expression, the product of all the numbers included between the vertical lines must equal the product of all the numbers without them; i. e., $36 \times 31 \times 1{,}280 = 40 \times 16 \times x$.

$$\text{Or } x = \frac{36 \times 31 \times \overset{2}{\cancel{1280}}}{\underset{\cancel{16}}{\cancel{40}}} = \$2{,}232. \quad \text{Ans.}$$

161. The above might have been solved by canceling factors of the numbers in the original proportion. For, if any number within the lines has a factor common to any number without the lines, that factor may be canceled from both numbers. Thus,

$$\begin{matrix} \cancel{40} \\ \cancel{16} \end{matrix} \Big| \begin{matrix} 36 \\ 31 \end{matrix} = \begin{matrix} 2 \\ \cancel{80} \\ \cancel{1280} \end{matrix} \Big| \; x.$$

16 is contained in 1,280, 80 times. Cancel 16 and 1,280, and write 80 above 1,280. 40 is contained in 80, 2 times. Cancel

40 and 80, and write 2 above 80. Now, since there are no more numbers that can be canceled, $x = 36 \times 31 \times 2 = \$2,232$, the same result as was obtained in the preceding article.

162. Rule.—*Write all the numbers forming the first cause in a vertical column, and draw a vertical line; on the other side of this line write in a vertical column all the numbers forming the second cause. Write the sign of equality to the right of the second column, and on the right of this form a third column of the numbers composing the first effect, drawing a vertical line to the right; on the other side of this line, write for a fourth column, the numbers composing the second effect. There must be as many numbers in the second cause as in the first cause, and in the second effect as in the first effect; hence, if any term is wanting, write x in its place. Multiply together all the numbers within the vertical lines, and also all those without the lines (canceling previously, if possible), and divide the product of those numbers which do not contain x by the product of the others in which x occurs, and the result will be the value of x.*

163. Example.—If 40 men can dig a ditch 720 feet long, 5 feet wide, and 4 feet deep in a certain time, how long a ditch 6 feet deep and 3 feet wide could 24 men dig in the same time?

Solution.—Here 40 men and 24 men are the causes, and the two ditches are the effects. Hence,

$$\begin{array}{c|c|c} & \cancel{3} & \\ & \cancel{18} & \\ & \cancel{720} & x \\ \cancel{40} & 24 = \quad 5 & \cancel{3} \\ & 4 & \cancel{6} \end{array} \text{ whence, } x = 24 \times 5 \times 4 = 480 \text{ feet. Ans.}$$

164. Example.—The volume of a cylinder varies directly as its length and directly as the square of its diameter. If the volume of a cylinder 10 inches in diameter and 20 inches long is 1,570.8 cubic inches, what is the volume of another cylinder 16 inches in diameter and 24 inches long?

Solution.—In this example, either the dimensions or the volumes may be considered the causes; say we take the dimensions for the causes. Then, squaring the diameters,

$$\begin{array}{c|c|c|c|c} 10^2 & 16^2 & & 100 & 256 \\ 20 & 24 & = 1,570.8 & x, \text{ or } \cancel{20} & \cancel{24} = 1,570.8 & x; \\ & & & 5 & 6 \end{array}$$

whence, $x = \dfrac{256 \times 6 \times 1,570.8}{5 \times 100} = 4,825.4976$ cubic inches. Ans.

165. EXAMPLE.—If a block of granite 8 ft. long, 5 ft. wide, and 3 ft. thick weighs 7,200 lb., what will be the weight of a block of granite 12 ft. long, 8 ft. wide, and 5 ft. thick?

SOLUTION.—Taking the weights as the effects, we have

$$\begin{array}{c|c|c} 8 & 4 & \\ 5 & 12 & \\ 3 & 8 & = 7{,}200 \\ & 5 & \end{array} \quad \Big| \quad x, \text{ or } x = 4 \times 7{,}200 = 28{,}800 \text{ pounds.} \quad \textbf{Ans.}$$

166. EXAMPLE.—If 12 compositors in 30 days of 10 hours each set up 25 sheets of 16 pages each, 32 lines to the page, in how many days 8 hours long can 18 compositors set up, in the same type, 64 sheets of 12 pages each, 40 lines to the page?

SOLUTION.—Here compositors, days, and hours compose the causes, and sheets, pages, and lines the effects. Hence,

$$\begin{array}{c|c|c|c} 3 & 3 & 5 & 2 \\ 12 & 18 & 25 & 64 \\ & & 4 & \\ 30 & x & = 16 & 12, \text{ or } x = 3 \times 10 \times 2 = 60 \text{ days.} \quad \textbf{Ans.} \\ 6 & & & 4 \\ 10 & 8 & 32 & 40 \\ & & & 5 \end{array}$$

167. In examples stated like that in Art. **164**, should an inverse proportion occur, write the various numbers as in the preceding examples, and then transpose from one side of the vertical line to the other side those numbers which are said to vary inversely.

EXAMPLE.—The centrifugal force of a revolving body varies directly as its weight, as the square of its velocity, and inversely as the radius of the circle described by the center of the body. If the centrifugal force of a body weighing 15 pounds is 187 pounds when the body revolves in a circle having a radius of 12 inches, with a velocity of 20 feet per second, what will be the centrifugal force of the same body when the radius is increased to 18 inches and the speed is increased to 24 feet per second?

SOLUTION.—Calling the centrifugal force the effect, we have

$$\begin{array}{c|c|c} 15 & 15 & \\ 20^2 & 24^2 = 187 & \\ 12 & 18 & \end{array} \quad \Big| \quad x.$$

Transposing 12 and 18 (since the radii are to vary inversely) and squaring 20 and 24,

$$\begin{array}{c|c|c} 15 & 15 & \\ & 2 & \\ 25 & 30 & = 187 \\ 400 & 576 & \\ 18 & 12 & \end{array} \quad \Big| \quad x, \text{ or } x = \frac{12 \times 2 \times 187}{25} = 179.52 \text{ pounds.} \quad \textbf{Ans.}$$

EXAMPLES FOR PRACTICE.

168. Solve the following by compound proportion:

1. If 12 men dig a trench 40 rods long in 24 days of 10 hours each, how many rods can 16 men dig in 18 days of 9 hours each?

Ans. 36 rods.

2. If a piece of iron 7 feet long, 4 inches wide, and 6 inches thick weighs 600 pounds, how much will a piece of iron weigh that is 16 feet long, 8 inches wide, and 4 inches thick? Ans. 1,828$\frac{4}{7}$ lb.

3. If 24 men can build a wall 72 rods long, 6 feet wide, and 5 feet high in 60 days of 10 hours each, how many days will it take 32 men to build a wall 96 rods long, 4 feet wide, and 8 feet high, working 8 hours a day? Ans. 80 days.

4. The horsepower of an engine varies as the mean effective pressure, as the piston speed, and as the square of the diameter of the cylinder. If an engine having a cylinder 14 inches in diameter develops 112 horsepower when the mean effective pressure is 48 pounds per square inch and the piston speed is 500 feet per minute, what horsepower will another engine develop if the cylinder is 16 inches in diameter, piston speed is 600 feet per minute, and mean effective pressure is 56 pounds per square inch? Ans. 204.8 horsepower.

5. Referring to the example in Art. **164**, what will be the volume of a cylinder 20 inches in diameter and 24 inches long?

Ans. 7,539.84 cubic inches.

6. Knowing that the product of $3 \times 5 \times 7 \times 9$ is 945, what is the product of $6 \times 15 \times 14 \times 36$? Ans 45,360

FORMULAS.

1. The term **formula,** as used in mathematics and in technical books, may be defined as *a rule in which symbols are used instead of words;* in fact, a formula may be regarded as a shorthand method of expressing a rule. Any formula can be expressed in words, and when so expressed it becomes a rule.

2. Formulas are much more convenient than rules; they show at a glance all the operations that are to be performed; they do not require to be read three or four times, as is the case with most rules, to enable one to understand their meaning; they take up much less space, both in the printed book and in one's note book, than rules; in short, wherever a rule can be expressed as a formula, the formula is to be preferred.

3. As the term "quantity" is a very convenient one to use, we will define it. In mathematics, the word **quantity** is applied to anything that it is desired to subject to the ordinary operations of addition, subtraction, multiplication, etc., when we do not wish to be more specific and state exactly what the thing is. Thus, we can say "two or more numbers," or "two or more quantities"; the word quantity is more general in its meaning than the word number.

4. The signs used in formulas are the ordinary signs indicative of operations, and the signs of aggregation. All these signs are explained in arithmetic, but some of them will here be explained in order to refresh the student's memory.

For notice of the copyright, see page immediately following the title page.

5. The signs indicative of operations are six in number; viz., $+$, $-$, $\times$, $\div$, $|$, $\sqrt{\ }$.

Division is indicated by the sign $\div$, or by placing a straight line between the two quantities. Thus, $25 \mid 17$, $25 / 17$, and $\frac{25}{17}$ all indicate that 25 is to be divided by 17. When both quantities are placed on the same horizontal line, the straight line indicates that the quantity on the left is to be divided by that on the right. When one quantity is below the other, the straight line between indicates that the quantity above the line is to be divided by the one below it.

The sign ($\sqrt{\ }$) indicates that some root of the quantity to the right is to be taken; it is called the **radical sign.** To indicate what root is to be taken, a small figure, called the **index,** is placed within the sign, this being always omitted when the square root is to be indicated. Thus, $\sqrt{25}$ indicates that the square root of 25 is to be taken; $\sqrt[3]{25}$ indicates that the cube root of 25 is to be taken; etc.

6. The signs of aggregation are four in number; viz., $\overline{\ \ }$, $(\ )$, $[\ ]$, $\{\ \}$, respectively called the **vinculum,** the **parenthesis,** the **brackets,** and the **brace;** they are used when it is desired to indicate that all the quantities included by them are to be subjected to the same operation. Thus, if we desire to indicate that the sum of 5 and 8 is to be multiplied by 7, and we do not wish to actually add 5 and 8 before indicating the multiplication, we may employ any one of the four signs of aggregation as here shown: $\overline{5+8}\times 7$, $(5+8)\times 7$, $[5+8]\times 7$, $\{5+8\}\times 7$. The vinculum is placed above those quantities which are to be treated as one quantity and subjected to the same operation.

7. While any one of the four signs may be used as shown above, custom has restricted their use somewhat. The vinculum is rarely used except in connection with the radical sign. Thus, instead of writing $\sqrt[3]{(5+8)}$, $\sqrt[3]{[5+8]}$, or $\sqrt[3]{\{5+8\}}$ for the cube root of 5 plus 8, all of which would be correct, the vinculum is nearly always used, $\sqrt[3]{5+8}$.

In cases where but one sign of aggregation is needed

(except, of course, when a root is to be indicated), the parenthesis is always used. Hence, $(5+8)\times 7$ would be the usual way of expressing the product of 5 plus 8, and 7.

If two signs of aggregation are needed, the brackets and parenthesis are used, so as to avoid having a parenthesis within a parenthesis, the brackets being placed outside. For example, $[(20-5)\div 3]\times 9$ means that the difference between 20 and 5 is to be divided by 3, and this result multiplied by 9.

If three signs of aggregation are required, the brace, brackets, and parenthesis are used, the brace being placed outside, the brackets next, and the parenthesis inside. For example, $\{[(20-5)\div 3]\times 9-21\}\div 8$ means that the quotient obtained by dividing the difference between 20 and 5 by 3 is to be multiplied by 9, and that after 21 has been subtracted from the product thus obtained, the result is to be divided by 8.

Should it be necessary to use all four of the signs of aggregation, the brace would be put outside, the brackets next, the parenthesis next, and the vinculum inside. For example, $\{[(20-\overline{5\div 3})\times 9-21]\div 8\}\times 12$.

8. As stated in arithmetic, when several quantities are connected by the various signs indicating addition, subtraction, multiplication, and division, the operation indicated by the sign of multiplication must always be performed first. Thus, $2+3\times 4$ is equal to 14, 3 being multiplied by 4, before adding to 2. Similarly, $10\div 2\times 5$ is equal to 1, since 2×5 equals 10, and $10\div 10$ is equal to 1. Hence, in the above case, if the brace were omitted, the result would be $\frac{1}{4}$, whereas, by inserting the brace, the result is 36.

Following the sign of multiplication comes the sign of division in order of importance. For example, $5-9\div 3$ is equal to 2, 9 being divided by 3 before subtracting from 5. The signs of addition and subtraction are of equal value; that is, if several quantities are connected by plus and minus signs, the indicated operations may be performed in the order in which the quantities are placed.

9. There is one other sign used, which is neither a sign of aggregation nor a sign indicative of an operation to be

performed ; it is (=), and is called the sign of *equality ;* it means that all on one side of it is exactly equal to all on the other side. For example, $2 = 2$, $5-3 = 2$, $5\times(14-9) = 25$.

10. Having called particular attention to certain signs used in formulas, the formulas themselves will now be explained. First, consider the well known rule for finding the horsepower of a steam engine, which may be stated as follows:

Divide the continued product of the mean effective pressure in pounds per square inch, the length of the stroke in feet, the area of the piston in square inches, and the number of strokes per minute, by 33,000; the result will be the horsepower.

This is a very simple rule, and very little, if anything, will be saved by expressing it as a formula, so far as clearness is concerned. The formula, however, will occupy a great deal less space, as we shall show.

An examination of the rule will show that four quantities (viz., the mean effective pressure, the length of the stroke, the area of the piston, and the number of strokes) are multiplied together, and the result is divided by 33,000. Hence, the rule might be expressed as follows:

$$\text{Horsepower} = \text{mean effective pressure (in pounds per square inch)} \times \text{stroke (in feet)} \times \text{area of piston (in square inches)} \times \text{number of strokes (per minute)} \div 33{,}000.$$

This expression could be shortened by representing each quantity by a single letter; thus, representing horsepower by the letter "H," the mean effective pressure in pounds per square inch by "P," the length of stroke in feet by "L," the area of the piston in square inches by "A," the number of strokes per minute by "N," and substituting these letters for the quantities that they represent, the above expression would reduce to

$$H = \frac{P\times L\times A\times N}{33{,}000},$$

a much simpler and shorter expression. The last expression is called a *formula.*

11. The formula just given shows, as we stated in the beginning, that a formula is really a shorthand method of expressing a rule. It is customary, however, to omit the sign of multiplication between two or more quantities when they are to be multiplied together, or between a number and a letter representing a quantity, it being always understood that, when two letters are adjacent, with no sign between them, the quantities represented by these letters are to be multiplied. Bearing this fact in mind, the formula just given can be further simplified to

$$H = \frac{PLAN}{33,000}.$$

The sign of multiplication, evidently, cannot be omitted between two or more numbers, as it would then be impossible to distinguish the numbers. A near approach to this, however, may be attained by placing a dot between the numbers which are to be multiplied together, and this is frequently done in works on mathematics when it is desired to economize space. In such cases it is usual to put the dot higher than the position occupied by the decimal point. Thus $2 \cdot 3$ means the same as 2×3; $542 \cdot 749 \cdot 1,006$ indicates that the numbers 542, 749, and 1,006 are to be multiplied together.

It is also customary to omit the sign of multiplication in expressions similar to the following: $a \times \sqrt{b+c}$, $3 \times (b+c)$, $(b+c) \times a$, etc., writing them $a\sqrt{b+c}$, $3\,(b+c)$, $(b+c)\,a$, etc. The sign is not omitted when several quantities are included by a vinculum, and it is desired to indicate that the quantities so included are to be multiplied by another quantity. For example, $3 \times \overline{b+c}$, $\overline{b+c} \times a$, $\sqrt{\overline{b+c}} \times a$, etc. are always written as here printed.

12. Before proceeding further, we will explain one other device that is used by formula makers, and which is likely to puzzle one who encounters it for the first time—it is the use of what mathematicians call *primes* and *subs.*, and what printers call *superior* and *inferior* characters. As a rule, formula makers designate quantities by the initial letters of the names

of the quantities. For example, they represent volume by v, pressure by p, height by h, etc. This practice is to be commended, as the letter itself serves in many cases to identify the quantity which it represents. Some authors carry the practice a little further, and represent all quantities of the same nature by the same letter throughout the book, always having the same letter represent the same thing. Now, this practice necessitates the use of the primes and subs. above mentioned, when two quantities have the same name but represent different things. Thus, consider the word *pressure* as applied to steam, at different stages between the boiler and the condenser. First, there is *absolute* pressure, which is equal to the gauge pressure in pounds per square inch plus the pressure indicated by the barometer reading (usually assumed in practice to be 14.7 pounds per square inch, when a barometer is not at hand). If this be represented by p, how shall we represent the gauge pressure? Since the absolute pressure is always greater than the gauge pressure, suppose we decide to represent it by a capital letter, and the gauge pressure by a small (lower-case) letter. Doing so, P represents absolute pressure, and p, gauge pressure. Further, there is usually a "drop" in pressure between the boiler and the engine, so that the initial pressure, or pressure at the beginning of the stroke, is less than the pressure at the boiler. How shall we represent the initial pressure? We may do this in one of three ways and still retain the letter p or P to represent the word pressure: First, by the use of the prime mark; thus, p' or P' (read *p prime* and *P major prime*) may be considered to represent the initial gauge pressure, or the initial absolute pressure. Second, by the use of sub. figures; thus, p_1 or P_1 (read *p sub. one*, and *P major sub. one*). Third, by the use of sub. letters; thus, p_i or P_i (read *p sub. i* and *P major sub. i*). In the same manner p'' (read *p second*), p_2, or p_r might be used to represent the gauge pressure at release, etc. The sub. letters have the advantage of still further identifying the quantity represented; in many instances, however, it is not convenient to use them, in which case primes and subs. are used instead.

The prime notation may be continued as follows: p''', p^{iv}, p^{v}, etc.; it is inadvisable to use superior figures, for example, p^1, p^2, p^3, p^a, etc., as they are liable to be mistaken for exponents.

13. The main thing to be remembered by the student is that *when a formula is given in which the same letters occur several times, all like letters having the same primes or subs. represent the same quantities, while those which differ in any respect represent different quantities.* Thus, in the formula

$$t = \frac{w_1 s_1 t_1 + w_2 s_2 t_2 + w_3 s_3 t_3}{w_1 s_1 + w_2 s_2 + w_3 s_3},$$

w_1, w_2, and w_3 represent the weights of three different bodies; s_1, s_2, and s_3, their specific heats; and t_1, t_2, and t_3, their temperatures; while t represents the final temperature after the bodies have been mixed together. It should be noted that those letters having the *same* subs. refer to the same bodies. Thus, w_1, s_1, and t_1 all refer to one of the three bodies; w_2, s_2, t_2, to another body; etc.

14. It is very easy to apply the above formula when the values of the quantities represented by the different letters are known. All that is required is to substitute the numerical values of the letters, and then perform the indicated operations. Thus, suppose that the values of w_1, s_1, t_1 are, respectively, 2 pounds, .0951, and 80°; of w_2, s_2, and t_2, 7.8 pounds, 1, and 80°; and of w_3, s_3, and t_3, $3\frac{1}{4}$ pounds, .1138, and 780°; then, the final temperature t is, substituting these values for their respective letters in the formula,

$$t = \frac{2\times.0951\times80 + 7.8\times1\times80 + 3\frac{1}{4}\times.1138\times780}{2\times.0951 + 7.8\times1 + 3\frac{1}{4}\times.1138}$$

$$= \frac{15.216 + 624 + 288.483}{.1902 + 7.8 + .36985} = \frac{927.699}{8.36005} = 110.97°.$$

In substituting the numerical values, the signs of multiplication are, of course, written in their proper places; all the multiplications are performed before adding, according to the rule previously given.

15. The student should now be able to apply any formula involving only algebraic expressions that he may meet with, and which do not require the use of logarithms for their solution. We will, however, call his attention to one or two other facts that he may have forgotten.

Expressions similar to $\dfrac{160}{\frac{660}{25}}$ sometimes occur, the heavy line indicating that 160 is to be divided by the quotient obtained by dividing 660 by 25. If both lines were light it would be impossible to tell whether 160 was to be divided by $\frac{660}{25}$, or whether $\frac{160}{660}$ was to be divided by 25. If this latter result were desired, the expression would be written $\dfrac{\frac{160}{660}}{25}$. In every case, the heavy line indicates that all above it is to be divided by all below it.

In an expression like the following, $\dfrac{160}{7+\frac{660}{25}}$, the heavy line is not necessary, since it is impossible to mistake the operation that is required to be performed. But, since $7+\frac{660}{25}=\frac{175+660}{25}$, if we substitute $\frac{175+660}{25}$ for $7+\frac{660}{25}$, the heavy line becomes necessary in order to make the resulting expression clear. Thus,

$$\frac{160}{7+\frac{660}{25}}=\frac{160}{\frac{175+660}{25}}=\frac{160}{\frac{835}{25}}.$$

16. Fractional exponents are sometimes used instead of the radical sign. That is, instead of indicating the square, cube, fourth root, etc. of some quantity, as 37 by $\sqrt{37}$, $\sqrt[3]{37}$, $\sqrt[4]{37}$, etc., these roots are indicated by $37^{\frac{1}{2}}$, $37^{\frac{1}{3}}$, $37^{\frac{1}{4}}$, etc. Should the numerator of the fractional exponent be some quantity other than 1, this quantity, whatever it may

be, indicates that the quantity affected by the exponent is to be raised to the power indicated by the numerator; the denominator is *always* the index of the root. Hence, instead of writing $\sqrt[3]{37^2}$ for the cube root of the square of 37, it may be written $37^{\frac{2}{3}}$, the denominator being the index of the root; in other words, $\sqrt[3]{37^2} = 37^{\frac{2}{3}}$. Likewise, $\sqrt[5]{(1+a^2b)^3}$ may also be written $(1+a^2b)^{\frac{3}{5}}$, a much simpler expression.

17. We will now give several examples showing how to apply some of the more difficult formulas that the student may encounter.

The area of any segment of a circle that is less than (or equal to) a semicircle is expressed by the formula

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r-h),$$

in which A = area of segment;
π = 3.1416;
r = radius;
E = angle obtained by drawing lines from the center to the extremities of arc of segment;
c = chord of segment;
and h = height of segment.

EXAMPLE.—What is the area of a segment whose chord is 10 inches long, angle subtended by chord is 83.46°, radius is 7.5 inches, and height of segment is 1.91 inches?

SOLUTION.—Applying the formula just given,

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r-h) = \frac{3.1416 \times 7.5^2 \times 83.46}{360} - \frac{10}{2}(7.5-1.91)$$
$$= 40.968 - 27.95 = 13.018 \text{ square inches, nearly.} \quad \text{Ans.}$$

18. The area of any triangle may be found by means of the following formula, in which A = the area, and a, b, and c represent the lengths of the sides:

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2+b^2-c^2}{2b}\right)^2}.$$

EXAMPLE.—What is the area of a triangle whose sides are 21 feet, 46 feet, and 50 feet long?

SOLUTION.—In order to apply the formula, suppose we let a represent the side that is 21 feet long; b, the side that is 50 feet long; and c, the side that is 46 feet long. Then, substituting in the formula,

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2+b^2-c^2}{2b}\right)^2} = \frac{50}{2}\sqrt{21^2 - \left(\frac{21^2+50^2-46^2}{2\times 50}\right)^2}$$

$$= \frac{50}{2}\sqrt{441 - \left(\frac{441+2{,}500-2{,}116}{100}\right)^2} = 25\sqrt{441 - \left(\frac{825}{100}\right)^2}$$

$$= 25\sqrt{441 - 8.25^2} = 25\sqrt{441 - 68.0625} = 25\sqrt{372.9375}$$

$$= 25 \times 19.312 = 482.8 \text{ square feet, nearly.} \quad \text{Ans.}$$

19. The operations in the above examples have been extended much farther than was necessary; it was done in order to show the student every step of the process. The last formula is perfectly general, and the same answer would have been obtained had the 50-foot side been represented by a, the 46-foot side by b, and the 21-foot side by c.

20. The Rankine-Gordon formula for determining the least load in pounds that will cause a long column to break is

$$P = \frac{SA}{1 + q\frac{l^2}{G^2}},$$

in which P = load (pressure) in pounds;

S = ultimate strength (in pounds per square inch) of the material composing the column;

A = area of cross-section of column in square inches;

q = a factor (multiplier) whose value depends upon the shape of the ends of the column and on the material composing the column;

l = length of column in inches;

and G = least radius of gyration of cross-section of column.

The values of S, q, and G^2 are given in printed tables in books in which this formula occurs.

EXAMPLE.—What is the least load that will break a hollow wrought-iron column whose outside diameter is 14 inches; inside diameter, 11 inches; length, 20 feet; and whose ends are flat?

SOLUTION.—For steel, $S = 150{,}000$, and for flat-ended steel columns, $q = \frac{1}{25{,}000}$; A, the area of the cross-section, $= .7854(d_1^2 - d_2^2)$ $= .7854(14^2 - 11^2)$, d_1 and d_2 being the outside and inside diameters, respectively; $l = 20 \times 12 = 240$ inches; and $G^2 = \frac{d_1^2 + d_2^2}{16} = \frac{14^2 + 11^2}{16}$. Substituting these values in the formula,

$$P = \frac{SA}{1 + q\frac{l^2}{G^2}} = \frac{150{,}000 \times .7854(14^2 - 11^2)}{1 + \frac{1}{25{,}000} \times \frac{240^2}{\frac{14^2 + 11^2}{16}}}$$

$$= \frac{150{,}000 \times 58.905}{1 + .1163} = \frac{8{,}835{,}750}{1.1163} = 7{,}915{,}211 \text{ pounds.} \quad \text{Ans.}$$

21. EXAMPLE.—When $A = 10$, $B = 8$, $C = 5$, and $D = 4$, what is the value of E in the following:

(*a*) $E = \sqrt[3]{\frac{BCD}{A\left(2 + \frac{D^2}{C^2}\right)}}$? (*b*) $E = \frac{A - \frac{3}{4}D + \frac{4B^2}{A + C}}{A - \sqrt{\frac{2B^2}{A + 22}}}$?

SOLUTION.—(*a*) Substituting,

$$E = \sqrt[3]{\frac{8 \times 5 \times 4}{10\left(2 + \frac{4^2}{5^2}\right)}}.$$

To simplify the denominator, square the 4 and 5, add the resulting fraction to 2, and multiply by 10. Simplifying, we have,

$$E = \sqrt[3]{\frac{160}{10\left(2 + \frac{16}{25}\right)}} = \sqrt[3]{\frac{160}{10 \times \frac{66}{25}}} = \sqrt[3]{\frac{160}{\frac{660}{25}}} = \sqrt[3]{\frac{200}{33}}$$

Reducing the fraction to a decimal, so that it will be easier to extract the cube root,

$$E = \sqrt[3]{6.0606} = 1.823. \quad \text{Ans.}$$

(*b*) Substituting,

$$E = \frac{10 - \frac{3}{4} \times 4 + \frac{4 \times 8^2}{10 + 5}}{10 - \sqrt{\frac{2 \times 8^2}{10 + 22}}} = \frac{10 - 3 + \frac{4 \times 64}{15}}{10 - \sqrt{\frac{2 \times 64}{32}}}$$

$$= \frac{7 + 17.066 +}{10 - \sqrt{4}} = \frac{24.066 +}{8} = 3.008 +. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

Find the numerical values of x in the following formulas, when $A = 9$, $B = 8$, $d = 10$, $e = 3$, and $c = 2$:

1. $x = \dfrac{d + c^2}{d^2 - 40}$. Ans. $x = \frac{7}{30}$.

2. $x = \dfrac{\frac{2}{3}(A + e)}{c\,e}$. Ans. $x = 1\frac{1}{3}$.

3. $x = \sqrt{\dfrac{d^2}{2c}} + \sqrt{A\,B^2}$. Ans. $x = 29$.

4. $x = \dfrac{A\,e}{\sqrt{16\,B\,c}} + \dfrac{5}{16}$. Ans. $x = 2$.

5. $x = (c + 2\,e)\left(\sqrt[3]{B} - \dfrac{1}{c}\right) + \dfrac{e^2 - c^2}{e^2 + c^2}$. Ans. $x = 12\frac{5}{13}$.

6. $x = \sqrt{\dfrac{B\,c\,d}{A\left(2 + \frac{d^3}{e^3}\right)}}$. Ans. $x = .396+$.

GEOMETRY AND MENSURATION.

GEOMETRY.

LINES AND ANGLES.

1. **Geometry** is that branch of mathematics which treats of the properties of lines, angles, surfaces, and volumes.

2. A **point** indicates position only. It has neither length, breadth, nor thickness.

3. A **line** has only one dimension: length.

4. A **straight line** is one that does not change its direction throughout its whole length. See Fig. 1. A straight line is also frequently called a **right line.**

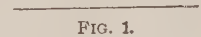


FIG. 1.

5. A **curved line** changes its direction at every point. See Fig. 2.

FIG. 2.

6. A **broken line** is one made up wholly of straight lines lying in different directions. See Fig. 3.

FIG. 3.

7. **Parallel lines** are equally distant from each other at all points. The lines shown in Fig. 4 are parallel.

FIG. 4.

8. A line is **perpendicular** to another when it meets that line so as not to incline towards it on either side. Thus, in Fig. 5, the line denoted by the letters *A B* is perpendicular to that denoted by *C D*.

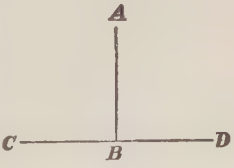


FIG. 5.

For notice of the copyright, see page immediately following the title page.

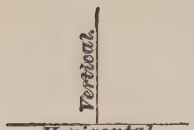


FIG. 6.

9. A **horizontal line** is a line parallel to the horizon, or water level. See Fig. 6.

10. A **vertical line** is a line perpendicular to a horizontal line; consequently, it has the direction of a plumb-line. See Fig. 6.

11. When two lines cross or cut each other, they are said to **intersect,** and the point at which they intersect, as *A*, Fig. 7, **is called the point of intersection.**

FIG. 7.

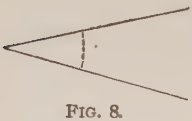


FIG. 8.

12. An **angle** is the opening between two lines which intersect, or meet; the point of meeting is called the **vertex** of the angle. See Fig. 8.

13. In order to distinguish one line from another, two of its points are given if it is a straight line, and as many more as are considered necessary if it is a broken or curved line. Thus, in Fig. 9, the line *A B* would mean the straight line included between the points *A* and *B*. Similarly, the straight line between *C* and *B*, or between *B* and *D*, would be called the line *C B*, or the line *B D*. The broken line made up of the lines *A B* and *C B*, or *B D*, would be called the broken line *C B A* or *A B C*, and *A B D* or *D B A*, according to the point started from.

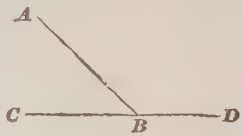


FIG. 9.

To distinguish angles, a point on each line and the point of their intersection, or vertex of the angle, are named; thus, in Fig. 9, the angle formed by the lines *A B* and *C B* is called the angle *A B C* or the angle *C B A*; the letter at the vertex is always placed in the middle. The angle formed by the lines *A B* and *B D* is called the angle *A B D* or the angle *D B A*.

When an angle stands alone so that it cannot be mistaken

for any other angle, only the vertex letter need be given; thus, the angle O, or the angle P, etc.

14. If one straight line meets another straight line at a point between its ends, two angles, $A\,B\,C$ and $A\,B\,D$, Fig. 9, are formed, which are called **adjacent angles.**

15. When these adjacent angles, $A\,B\,C$ and $A\,B\,D$, are equal, they are called **right angles.** See Fig. 10.

FIG. 10.

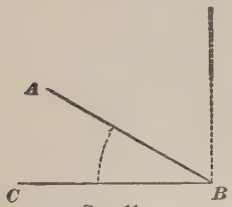


FIG. 11.

16. **An acute angle** is less than a right angle. Thus, $A\,B\,C$, Fig. 11, is an acute angle.

17. An **obtuse angle** is greater than a right angle. The angle $A\,B\,D$, Fig. 12, is an obtuse angle.

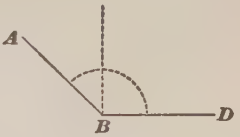


FIG. 12.

18. When two straight lines intersect, they form four angles about the point of intersection. Thus, in Fig. 13, the lines $A\,B$ and $C\,D$, intersecting at the point O, form four angles, $B\,O\,D$, $D\,O\,A$, $A\,O\,C$, and $C\,O\,B$, about the point O. The angles which lie on the *same* side of one straight line, as $D\,O\,B$ and $D\,O\,A$ are **adjacent angles.** The angles which lie *opposite* each other are called **opposite angles.** Thus, $A\,O\,C$ and $D\,O\,B$, also $D\,O\,A$ and $B\,O\,C$, are opposite angles.

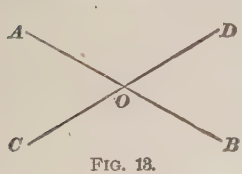


FIG. 13.

When one straight line intersects another straight line, as in Fig. 13, the opposite angles are equal. Thus, $D\,O\,B = A\,O\,C$, and $D\,O\,A = B\,O\,C$.

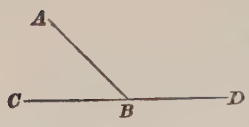


FIG. 14.

19. When one straight line meets another straight line at a point between its ends, the sum of the two adjacent angles, as ABD and ABC, Fig. 14, is equal to two right angles.

20. If a number of straight lines on the same side of a given straight line meet at the same point, the sum of all the angles formed is equal to two right angles. Thus, in Fig. 15, $COB + DOC + EOD + FOE + AOF$ = two right angles.

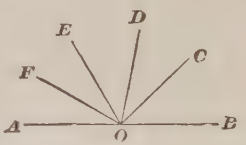


FIG. 15.

21. If a straight line intersects another straight line, so that the adjacent angles are equal, the lines are said to be **perpendicular** to each other. In such a case, four right angles are formed about the point of intersection. Thus, in Fig. 16, $BOC = COA$; hence, BOC, COA, AOD, and DOB are right angles. From this, it is seen that *four right angles* are all that can be formed about a given point.

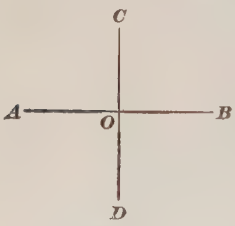


FIG. 16.

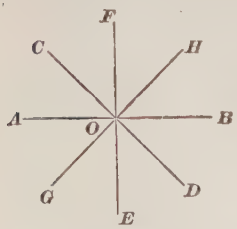


FIG. 17.

It follows that, if through a given point, any number of straight lines are drawn, the sum of all the angles formed about the point of intersection is equal to four right angles. Thus, in Fig. 17, $HOF + FOC + COA + AOG + GOE + EOD + DOB + BOH$ = four right angles.

EXAMPLE.—A circular window has 12 ribs equally spaced. What part of a right angle is included between the center lines of any two ribs?

SOLUTION.—Since there are 12 ribs, there are 12 angles. The sum of all the angles equals four right angles. Hence, one angle equals $\frac{1}{12}$ of four right angles, or $\frac{4}{12} = \frac{1}{3}$ of one right angle. Ans.

22. A perpendicular drawn from a point over or under a given straight line is the shortest distance from the point to the line, or to the line extended. Thus, if A, Fig. 18, is the given point, and CD, the given line, then the perpendicular AB is the shortest distance from A to CD.

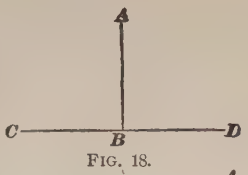


FIG. 18.

23. If two angles have their sides parallel, and lie in the same or in opposite directions, they are equal. Thus, if the side AB, Fig. 19 or Fig. 20, is parallel to the side DE, and if the side BC is parallel to the side EF, then the angle E = the angle B.

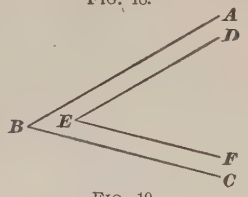


FIG. 19.

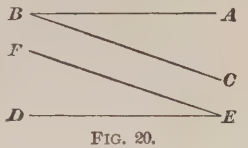


FIG. 20.

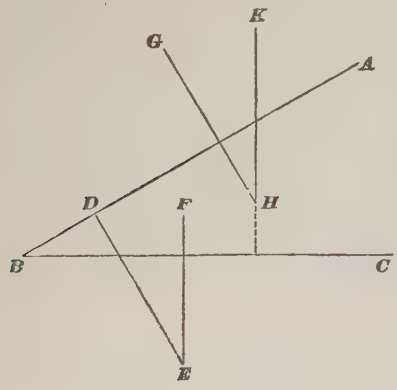


FIG. 21.

24. If two sides of an angle are perpendicular to two sides of another angle, the two angles are equal. Thus, if DE and GH, Fig. 21, are perpendicular to BA, and EF and HK are perpendicular to BC, then will angle E = angle B = angle H.

EXAMPLES FOR PRACTICE.

25. Solve the following examples:

1. In a pulley with five arms, what part of a right angle is included between the center lines of any two arms? Ans. $\frac{4}{5}$ of a right angle.

2. If one straight line meets another straight line so as to form an angle equal to $1\frac{2}{5}$ right angles, what part of a right angle does its adjacent angle equal? Ans. $\frac{3}{5}$ of a right angle.

3. If a number of straight lines meet a given straight line at a given point, all being on the same side of the given line, so as to form six equal angles, what part of a right angle is contained in each angle?

Ans. $\frac{1}{3}$ of a right angle.

PLANE FIGURES.

26. A **surface** has only two dimensions: *length* and *breadth*.

27. A **plane surface** is a flat surface. If a straightedge be laid on a plane surface, every point along the edge of the straightedge will touch the surface, no matter in what direction it is laid.

28. A **plane figure** is any part of a plane surface bounded by straight or curved lines.

29. When a plane figure is bounded by **straight** lines, it is called a **polygon**. The bounding lines are called the **sides**, and the length of the broken line that bounds it (or the whole distance around it) is called the **perimeter** of the polygon.

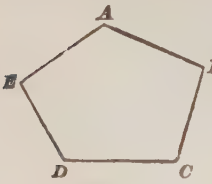


FIG. 22.

30. The angles formed by the sides are called the **angles** of the polygon. Thus, $ABCDE$, Fig. 22, is a polygon. AB, BC, etc. are the **sides**; EAB, ABC, etc. are the **angles**; and the length of the broken line $ABCDE$ is the **perimeter**.

31. Polygons are classified according to the number of their sides: One of three sides is called a **triangle**; one of four sides, a **quadrilateral**; one of five sides, a **pentagon**; one of six sides, a **hexagon**; one of seven sides, a **heptagon**; one of eight sides, an **octagon**; one of ten sides, a **decagon**; one of twelve sides, a **dodecagon**; etc.

32. **Equilateral polygons** are those in which the sides are all equal. Thus, in Fig. 23, $AB = BC = CD = DA$; hence, $ABCD$ is an equilateral polygon.

FIG. 23.

33. An **equiangular polygon** is one in which all the angles are equal. Thus, in Fig. 24, angle A = angle B = angle D = angle C; hence, $A\,B\,D\,C$ is an equiangular polygon.

FIG. 24.

FIG. 25.

34. A **regular polygon** is one in which all the sides and all the angles are equal. Thus, in Fig. 25, $A\,B = B\,D = D\,C = C\,A$, and angle A = angle B = angle D = angle C; hence, $A\,B\,D\,C$ is a regular polygon.

Other regular polygons are shown in Fig. 26.

Pentagon. Hexagon. Heptagon. Octagon. Decagon. Dodecagon.

FIG. 26.

35. The sum of all the interior angles of any polygon is equal to two right angles multiplied by a number which is two less than the number of sides in the polygon. Thus, $A\,B\,C\,D\,E\,F$, Fig. 27, is a polygon of six sides (hexagon), and the sum of all the interior angles $A+B+C+D+E+F$ = 2 right angles $\times 4$ $(= 6-2)$, or 8 right angles.

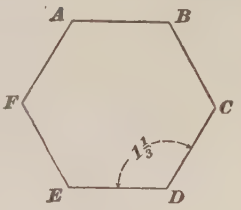


FIG. 27.

EXAMPLE.—Fig. 27 represents a regular hexagon (has equal sides and equal angles). How many right angles are there in each interior angle?

SOLUTION.—The sum of the interior angles is $2 \times (6-2) = 8$ right angles; and, as there are six equal angles, we have $8 \div 6 = 1\frac{1}{3}$ right angles, the number of right angles in each interior angle. Ans.

THE TRIANGLE.

36. Triangles may be divided, with respect to their sides, into *isosceles*, *equilateral*, and *scalene* triangles; and with respect to their angles, into *right-angled* and *oblique-angled* triangles.

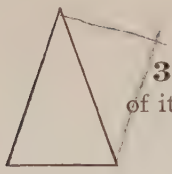


FIG. 28.

37. An **isosceles** triangle is one having two of its sides equal. See Fig. 28.

38. An **equilateral** triangle is one that has the three sides equal. See Fig. 29.

FIG. 29.

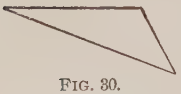


FIG. 30.

39. A **scalene** triangle is one having no two of its sides equal. See Fig. 30.

40. A **right-angled** triangle is any triangle having one right angle. See Fig. 31. The side opposite the right angle is called the **hypotenuse.** For brevity, a right-angled triangle is usually termed a **right triangle.**

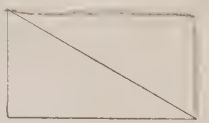


FIG. 31.

FIG. 32.

41. An **oblique-angled** or **oblique** triangle is one which has no right angles. See Fig. 32.

42. The **base** of any triangle is the side upon which the triangle is supposed to stand.

The **altitude** of any triangle is a line drawn from the vertex of the angle opposite the base perpendicular to the base or to the base extended. Thus, in Figs. 33 and 34, the side $A\,C$ is the base of the triangle and the line $B\,D$ is the altitude.

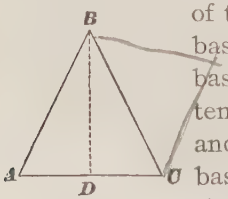


FIG. 33.

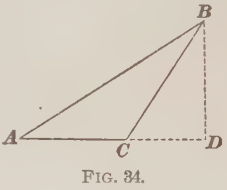


FIG. 34.

In a right triangle, if one of the short sides is taken as

the base, the other short side will be the altitude of the triangle.

43. In an isosceles triangle, the angles opposite the equal sides are equal. Thus, in Fig. 35, $AB = BC$; hence, angle C = angle A.

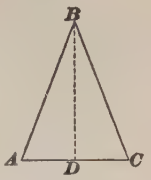


FIG. 35.

In any isosceles triangle, if a perpendicular be drawn from the vertex opposite the unequal side to that side, it bisects (cuts in halves) the side. Thus, AC is the unequal side in the isosceles triangle ABC; hence, the perpendicular BD bisects AC, or $AD = DC$.

If two angles of any triangle are equal, the triangle is isosceles.

44. In any triangle, the sum of the three angles is equal to two right angles. Thus, in Fig. 36, the sum of the angles at A, B, and C = two right angles; that is, $A + B + C$ = two right angles. Hence, if any two angles of a triangle are given, the third may be found by subtracting the sum of the two from two right angles. Suppose that $A + B = 1\frac{7}{10}$ right angles; then, C must equal $2 - 1\frac{7}{10} = \frac{3}{10}$ of a right angle.

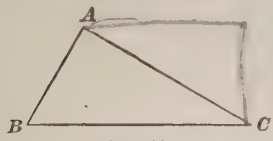


FIG. 36.

45. In any right-angled triangle there can be but one right angle, and since the sum of all the angles equals two right angles, it is evident that the sum of the two acute angles must be equal to a right angle. Therefore, if in any right-angled triangle one acute angle is known, the other can be found by subtracting the known angle from a right angle. Thus ABC, Fig. 37, is a right-angled triangle, right-angled at C. Then, the angles $A + B$ = one right angle. If $A = \frac{3}{7}$ of a right angle, $B = 1 - \frac{3}{7} = \frac{4}{7}$ of a right angle.

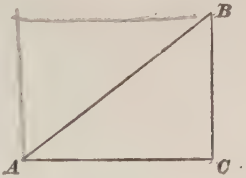


FIG. 37.

46. In any right-angled triangle, the square described on the hypotenuse is equal to the sum of the squares

described upon the other two sides. If $A\,B\,C$, Fig. 38, is a right-angled triangle, right-angled at B, then the square described upon the hypotenuse $A\,C$ is equal to the sum of the squares described upon the sides $A\,B$ and $B\,C$; consequently, if the lengths of the sides $A\,B$ and $B\,C$ are known, we can find the length of the hypotenuse by adding the squares of the lengths of the sides $A\,B$ and $B\,C$, and then extracting the square root of the sum.

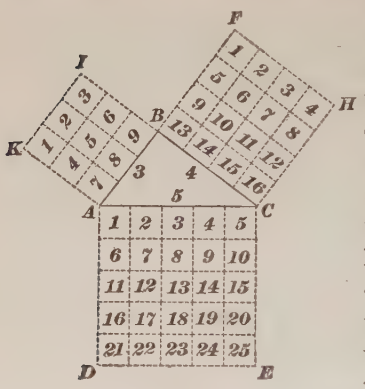


FIG. 38.

If the length of one of the short sides be denoted by a, that of the other short side by b, and that of the hypotenuse by c, then, since $c^2 = a^2 + b^2$, we have, by extracting the square root of both quantities,

$$c = \sqrt{a^2 + b^2}.$$

EXAMPLE.—The width of a house is 20 feet, and the roof is *half pitch*—that is, *the height of the gable is equal to one-half the width.* What is the length of a rafter, if it projects 15 inches over the side?

SOLUTION.—Let one-half the width be called $a = 10$ feet; the height of the gable, $b = \frac{1}{2}$ of 20, or 10 feet; and the hypotenuse, c. Substituting these values in the formula,

$$c = \sqrt{a^2 + b^2} = \sqrt{10^2 + 10^2} = \sqrt{200} = 14.14 \text{ feet.}$$

Adding 15 inches = 1.25 feet, the length of a rafter is 15.39 feet; or, reducing .39 foot to inches, the length is 15 feet $4\frac{3}{4}$ inches, nearly. Ans.

47. If the hypotenuse and one side are given, the other side can be found by subtracting the square of the given side from the square of the hypotenuse, and then extracting the square root of the remainder. That is,

$$a = \sqrt{c^2 - b^2} \text{ or } b = \sqrt{c^2 - a^2},$$

the letters having the same meaning as in Art. **46.**

EXAMPLE.—It is desired to ascertain the height of the ceiling in a room. One end of a 10-foot pole is set in the corner of the ceiling and

wall, and the other end is 6 feet from the base of the wall. What is the height of the ceiling?

SOLUTION.—In this problem, the 10-foot pole is the hypotenuse c, and the short side b is 6 feet. Hence, applying the formula,

$$a = \sqrt{c^2 - b^2} = \sqrt{100 - 36} = 8 \text{ feet},$$

which is the height of the ceiling. Ans.

48. If the sides are equal, or if, in Fig. 39, $a = b$, then the hypotenuse is equal to the square root of twice the square of either side; that is, $c = \sqrt{2\,a^2} = \sqrt{2\,b^2}$. Also, $a = b = \sqrt{\frac{c^2}{2}}$; that is, either side is equal to the square root of one-half the square of the hypotenuse.

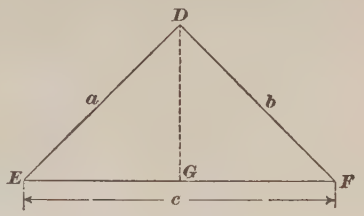


FIG. 39.

In such a triangle, the perpendicular distance from the hypotenuse to the right angle is one-half the hypotenuse. Thus, in roofing, if $G\,D$, Fig. 39, is equal to $E\,G$, or $\frac{1}{2}\,E\,F$, then the roof is called *half pitch*.

EXAMPLE.—What is the length of a slope of a roof having half pitch, the width of the house being 30 feet?

SOLUTION.—In this example, $c = 30$ feet, and $a = b$. Then, by the formula,

$$a = b = \sqrt{\frac{c^2}{2}} = \sqrt{450} = 21.21 \text{ feet}. \quad \text{Ans.}$$

EXAMPLE.—It is desired to lay out a line $A\,C$ at right angles to a line $A\,B$, which is 15 feet long. How can it be done by means of a tape?

SOLUTION.—If we can find any two numbers, the sum of whose squares is equal to the square of another number, these three numbers will be the sides of a right triangle; as, for example, 3, 4, and 5, or 6, 8, and 10. Thus, hold the 6-foot mark on the tape at A, Fig. 40, the 14-foot mark at D, and bring the end of the tape and the 24-foot mark together at C, and mark the point C, then $A\,C$ will be perpendicular to $A\,D$; for $\sqrt{A\,C^2 + A\,D^2} = C\,D$, that is, $\sqrt{6^2 + 8^2} = \sqrt{36 + 64} = 10$ feet $= 24$ feet $- 14$ feet.

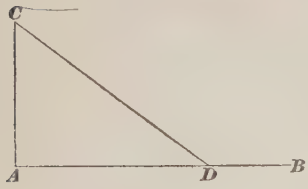


FIG. 40.

49. The principle of the right triangle is of very great value in practical work, and the student should become thoroughly familiar with it in all its variations.

The following is an example showing a double application of the right-triangle principle:

EXAMPLE.—In Fig. 41, $A\,B\,C\,D$ represents a skylight 7 ft. 6 in. $\times$ 9 ft. The point O is 2 feet above the plane of $A\,B\,C\,D$. What is the length of the hip rafter $O\,D$ at the angle of the skylight?

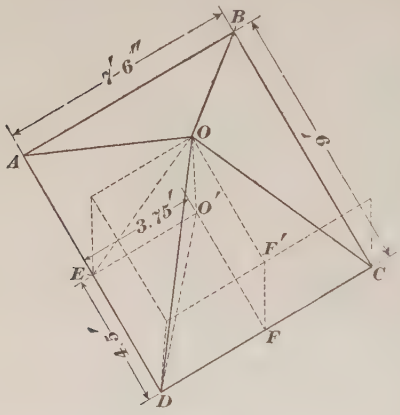


FIG. 41.

SOLUTION.—It will be seen that the point O' is directly under O and 2 feet below it, so that O' is on the same level as $A\,B\,C\,D$. It will also be seen that $D\,O$ is simply the hypotenuse of a right triangle, whose base is $D\,O'$ and whose altitude is $O\,O'$. But $D\,O'$ is also the hypotenuse of a right triangle, whose sides are $E\,O'$ and $E\,D$. $E\,O' = \frac{1}{2}$ of 7 feet 6 inches $= 3.75$ feet, and $E\,D = \frac{1}{2}$ of 9 feet $= 4.5$ feet. Then, $D\,O' = \sqrt{(3.75)^2 + (4.5)^2} = 5.86$ feet. In the triangle $D\,O'\,O$, $D\,O' = 5.86$ feet, and $O\,O' = 2$ feet; whence,

$$D\,O = \sqrt{(5.86)^2 + 2^2} = 6.19 \text{ feet} = 6 \text{ feet } 2\tfrac{1}{4} \text{ inches, nearly.}$$

One extraction of square root may be dispensed with, thus:

$$D\,O = \sqrt{(D\,O')^2 + (O\,O')^2} = \sqrt{[(E\,O')^2 + (E\,D)^2] + (O\,O')^2}$$

$$= \sqrt{(3.75)^2 + (4.5)^2 + 2^2} = \sqrt{38.31} = 6.19 \text{ feet, as before.} \quad \text{Ans.}$$

Again, if we can find the length of a line along the skylight perpendicular to an edge, as $O\,E$ to $A\,D$, then the hip $O\,D$ is the hypotenuse of a right triangle of which $O\,E$ and $E\,D$ are sides; and $O\,E = \sqrt{(O'\,E)^2 + (O\,O')^2}$.

The result will be the same either way.

NOTE.—When decimals occur in solving the examples in this section on *Geometry and Mensuration*, two decimal places are to be retained (except in constants, as .7854, .5236, etc.), and if the third decimal figure is *greater* than 5, the second decimal figure is to be increased by 1; thus, 13.537 should be written 13.54, not 13.53. To easily convert feet and inches to feet and decimals of a foot, feet and decimals of a foot to feet and inches, or to change a fraction of an inch to a decimal, or vice versa, the conversion tables, Art. **82**, may be used.

SIMILAR TRIANGLES.

50. Two triangles are **equal** when the *sides* of one are equal to the sides of the other.

51. Two triangles are **similar** when the *angles* of one are equal to the angles of the other. *The corresponding sides of similar triangles are proportional.*

For example, suppose we have two triangles $A\,B\,C$ and $a\,b\,c$, Fig. 42, in which the side $a\,c$ is perpendicular to $A\,C$, the side $a\,b$, to $A\,B$, and side $c\,b$, to $B\,C$, then, angle A = angle a, since the sides of one are perpendicular to the sides of the other. (See Art. **24.**) In like manner, angle B = angle b, and angle C = angle c. The two triangles are therefore similar, and their corresponding sides are proportional. That is, any two sides of one triangle are to each other as the two corresponding sides of the other triangle; or, one side of one triangle is to the corresponding side of the other as another side of the first triangle is to the corresponding side of the second. The following are examples of the many proportions that may be written. In this case, the corresponding sides of the two triangles are the ones perpendicular to each other.

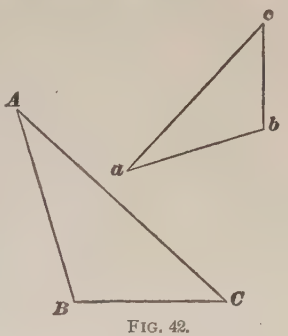


FIG. 42.

$$A\,B : B\,C = a\,b : b\,c, \qquad B\,C : b\,c = A\,B : a\,b,$$
$$A\,B : A\,C = a\,b : a\,c, \qquad A\,C : a\,c = B\,C : b\,c, \text{ etc.}$$

EXAMPLE.—It is required to find the distance $A\,B$, Fig. 43, across a stream.

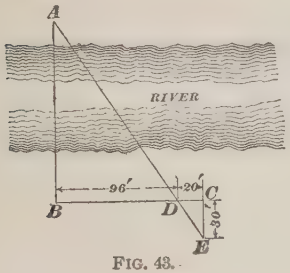


FIG. 43.

SOLUTION.—The line $B\,C$ making any angle with $A\,B$ is measured, and $C\,E$ is made parallel with $A\,B$, and of any convenient length. The point D is marked where $A\,E$ intersects $B\,C$, and $B\,D$ and $D\,C$ are measured. Then since the triangle $A\,B\,D$ and $C\,D\,E$ have their corresponding sides parallel, they are similar, and $A\,B : C\,E = B\,D : D\,C$; or $A\,B = \frac{30 \times 96}{20} = 144$ feet. Ans.

52. If a straight line be drawn through two sides of a triangle parallel to the third side, it divides those sides proportionally. Thus, let the line DE be drawn parallel to the side BC in the triangle ABC, Fig. 44. Then, $AD:AB = AE:AC$, or $AD:DB = AE:EC$. It is to be noticed that the triangles ADE and ABC are similar, and their sides are proportional. Thus $AB:AD = BC:DE$ and $AC:AE = BC:DE$.

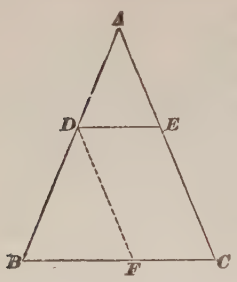


FIG. 44.

If a straight line, as DF, be drawn from D or E, Fig. 44, parallel to AC or AB, then the triangles ADE, ABC, and DBF are all similar, and a number of other proportions may be formed.

EXAMPLE.—Referring to Fig. 45, it is desired to find the length of the line CA, extending across a river. E is on the line AC, D is on the line BA, and DE is parallel to CB; the lengths of CE, ED, and CB are as shown in the figure.

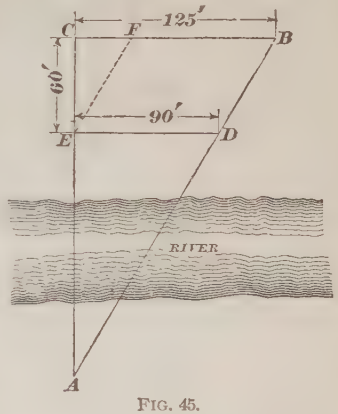


FIG. 45.

SOLUTION.—Draw EF parallel to AB, so that $CF = 125 - 90 = 35$ feet; then $CA:CE = CB:CF$, or $CA:60 = 125:35$, whence, $CA = \frac{60 \times 125}{35}$ = 214.3 feet, nearly.

EXAMPLE.—On a drawing it is required to divide a line 8 inches long into 12 parts.

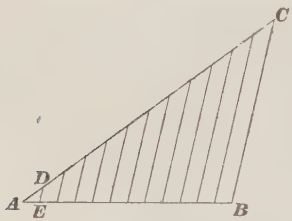


FIG. 46.

SOLUTION.—Let AB, Fig. 46, be the 8-inch line. Through A draw any line AC 12 inches long, and mark the inch points on it. Connect C and B, and through each inch mark on AC, draw a parallel to CB, as DE, etc., cutting AB at E, etc., which will be the points required; for by similar triangles, $AE:AD = AB:AC$; or AE:1 in. = 8 in.:12 in.; hence, $AE = \frac{1 \times 8}{12} = \frac{2}{3}$ inch. Ans.

This principle is very useful when an exact measurement—as for example, $\frac{2}{3}$ inch—cannot be obtained by a scale or rule.

EXAMPLES FOR PRACTICE.

53. Solve the following:

1. The distance from the first to the second floor of a house is 9 feet. It is desired to mark the line of the top of the 14 steps, or *treads*, on a drawing. What is the height of each *riser?* Make a sketch showing how the spacing may be found. Ans. 7.7 in. $= 7\frac{11}{16}$ in., nearly.

2. What length of stone coping is required for each side of a gable, the width of which is 24 feet, and whose height is $\frac{3}{4}$ the span? Ans. 21.63 ft.

3. What is the angle at the top of a gable of a house whose roof slopes are each one-half of a right angle? Ans. A right angle.

4. In running a line FA, Fig. 47, a house stands in the way. AC is laid off square with AF, 14 feet long, and 12 feet behind AC, a line GE 23 feet long is laid off at right angles to AF, so as to give a sight past the house to B. What are the lengths of AB and CB? $CE = \sqrt{9^2 + 12^2}$. Ans. $\begin{cases} AB, 18\frac{2}{3} \text{ ft.} \\ CB, 23\frac{1}{3} \text{ ft.} \end{cases}$

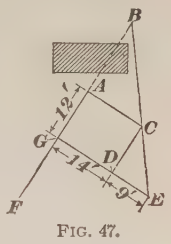


Fig. 47.

5. What is the distance between the 16-inch mark on one leg of a carpenter's square and the 12-inch mark on the other? Ans. 20 in.

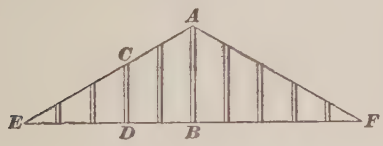


Fig. 48.

6. The distance EB, Fig. 48, is $8\frac{1}{3}$ feet. If the spacing of the roof rafters is 20 inches, and the true length of AB is 12 feet, what is the length of the "jack rafter" CD? Ans. $7\frac{1}{6}$ ft. $= 7$ ft. $2\frac{1}{2}$ in., nearly.

7. In Fig. 49, $CBAEC$ represents a gable at right angles to the main roof, the line AB, where they meet, being called a *valley*. What is the length of the valley rafter AB, CA being 16 feet, ED, the *rise* of the rafter, being 8 feet, and $EB = DF$, the distance of B back of ACE, 4 feet? The principle is the same as explained for *hips*. Ans. 12 ft.

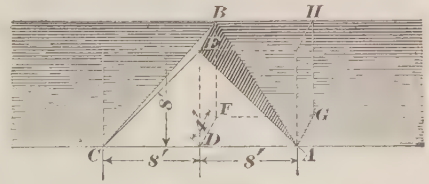


Fig. 49.

THE CIRCLE.

FIG. 50.

54. A **circle** is a plane figure bounded by a curved line, called the **circumference,** every point of which is equally distant from a point within, called the **center.** See Fig. 50.

55. The **diameter** of a circle is a straight line passing through the center and terminated at both ends by the circumference, as $A\,B$, Fig. 51.

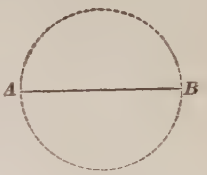


FIG. 51.

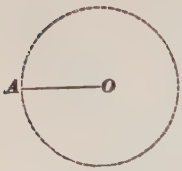


FIG. 52.

56. The **radius** of a circle, $O\,A$, Fig. 52, is a straight line drawn from the center to the circumference. It is equal in length to one-half the diameter. The plural of the word radius is **radii.** All radii of any circle are equal in length.

57. An **arc** of a circle is any part of its circumference, as $A\,E\,B$, Fig. 53.

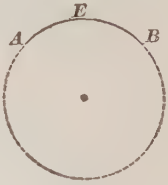


FIG. 53.

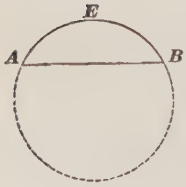


FIG. 54.

58. A **chord** is a straight line joining any two points in a circumference; or, it is a straight line joining the extremities of an arc. Thus, in Fig. 54, $A\,B$ is the chord of the arc $A\,E\,B$.

59. A **segment** of a circle is the space included between the arc and its chord. Thus, in Fig. 54, the space between the arc $A\,E\,B$ and the chord $A\,B$ is a segment.

60. A **sector** of a circle is the space included between an arc and two radii drawn to the extremities of the arc, as *A O B*, Fig. 55.

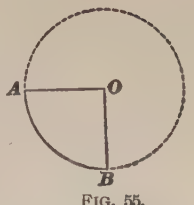


FIG. 55.

61. Two circles are equal when the *radius or diameter* of one is equal to the *radius or diameter* of the other.

Two arcs are equal when the *radius and chord* of one is equal to the *radius and chord* of the other.

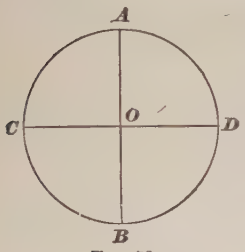


FIG. 56.

62. If *A D B C*, Fig. 56, is a circle in which two diameters *A B* and *C D* are drawn at right angles to each other, then *A O D*, *D O B*, *B O C*, and *C O A* are right angles. The circumference is thus divided into four equal parts; each of these parts is called a **quadrant.**

63. To measure angles, the circumference of circles are divided into 360 equal parts called *degrees*, which are subdivided into 60 equal parts called *minutes;* and the latter are further subdivided into 60 equal parts called *seconds.* Degrees, minutes, and seconds are indicated by the marks °, ′, ″; thus, 65 degrees, 15 minutes, and 40 seconds is written 65° 15′ 40″. Since a quadrant is one-fourth of a circumference, it includes $\frac{1}{4}$ of 360°, or 90°, whence a right angle contains 90°. So, also, if a circle be divided into equal sectors, the angle included between two adjacent radii is equal to 360° divided by the number of sectors. Thus, the angle between radii drawn to the angles of a regular octagon includes $\frac{1}{8}$ of 360°, or 45°.

The intersection of any two straight lines may be considered the center of a circle, and the number of 360ths, or degrees—measured on any circumference described from

this center—included between the lines, measures the angle.

64. An **inscribed** angle is one whose vertex lies on the circumference of a circle, and whose sides are chords. It is measured by *one-half* the intercepted arc. Thus, in Fig. 57, $A\,B\,C$ is an inscribed angle, and it is measured by one-half the arc $A\,D\,C$.

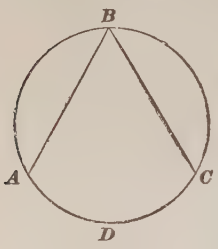


FIG. 57.

EXAMPLE.—If, in Fig. 57, the arc $A\,D\,C = \frac{2}{5}$ of the circumference, how many degrees are there in the inscribed angle $A\,B\,C$?

SOLUTION.—Since the angle is an inscribed angle, it is measured by one-half the intercepted arc, or $\frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$ of the circumference. The whole circumference contains 360°; and $360° \times \frac{1}{5} = 72°$. Ans.

65. If a circle is divided into halves, each half is called a **semicircle,** and each half circumference is called a **semi-circumference.** The measure of a semicircle is one-half of 360°, or 180°.

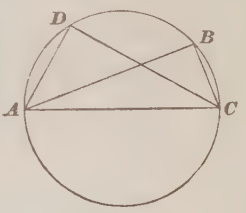


FIG. 58.

Any angle that is inscribed in a semicircle and intercepts a semi-circumference, as $A\,B\,C$ or $A\,D\,C$, Fig. 58, is a right angle, since it is measured by one-half a semi-circumference, or by 90°.

66. If, in any circle, a radius be drawn perpendicular to any chord, it bisects (cuts in halves) the chord. Thus, if the radius $O\,C$, Fig. 59, is perpendicular to the chord $A\,B$, $A\,D = D\,B$.

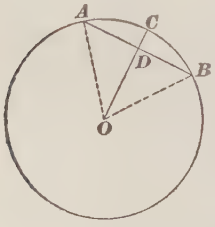


FIG. 59.

A radius which bisects a chord, bisects also the angle included between radii drawn to the ends of the chord. Thus, in Fig. 59, the radius $O\,C$ bisects the angle $A\,O\,B$.

If a straight line be drawn perpendicular to any chord at its middle point, it must pass through the center of the circle.

67. Through any three points not in the same straight line, a circumference can be drawn. Let A, B, and C, Fig. 60, be any three points. Join A and B, and B and C by straight lines. At the middle point of AB, draw HK perpendicular to AB; at the middle point of BC draw EF perpendicular to BC. These two perpendiculars intersect at O. With O as a center, and OB, OA, or OC as a radius, describe a circle; it will pass through A, B, and C.

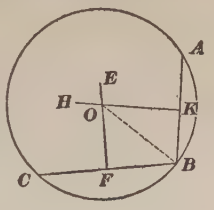


FIG. 60.

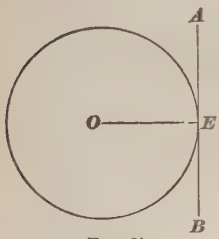


FIG. 61.

68. A **tangent** to a circle is a straight line which touches the circle at one point only; it is always perpendicular to a radius drawn to that point. Thus, AB, Fig. 61, is a tangent to the circle; it touches the circle at E and is perpendicular to the radius OE.

69. If two circles intersect each other, the line joining their centers bisects at right angles the line joining the two points of intersection. Thus, if the two circles, whose centers are O and P, Fig. 62, intersect at A and B, the line OP bisects at right angles the line AB; or $AC = BC$.

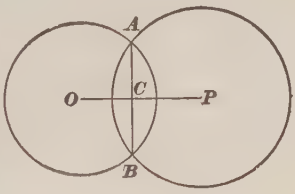


FIG. 62.

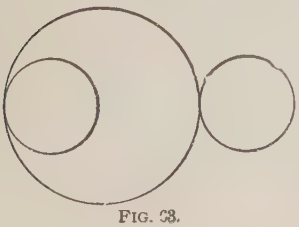


FIG. 63.

70. One circle is said to be **tangent** to another circle when they touch each other at one point only. See Fig. 63. This point is called the **point of contact**, or the **point of tangency**.

71. When two or more circles are described from the same center, they are called **concentric circles.** See Fig. 64.

FIG. 64.

72. If, from any point on the circumference of a circle, a perpendicular be let fall upon a given diameter, this perpendicular will be a mean proportional between the two parts into which it divides the diameter.

If AB, Fig. 65, is the diameter, and C any point on the circumference, then the perpendicular CD is a mean proportional between AD and DB, or $AD : CD = CD : DB$. Therefore, $\overline{CD}^2 = AD \times DB$, and $CD = \sqrt{AD \times DB}$. This principle furnishes a method of drawing a mean proportional between two lines. Let AD and DB, Fig. 65, be any two lines. Join them together in one line as AB, and on this as a diameter, draw a circle. Then CD, perpendicular to this diameter at the common end D, is the mean proportional.

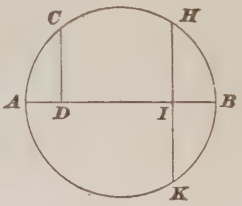


FIG. 65.

EXAMPLE.—In arches the *span* is the distance across the opening, as AC, Fig. 66, measured from the ends of the arch as A and C. The *rise* DB is the perpendicular distance from AC to the highest point B measured on the center line OB; DB = the radius $- OD$.

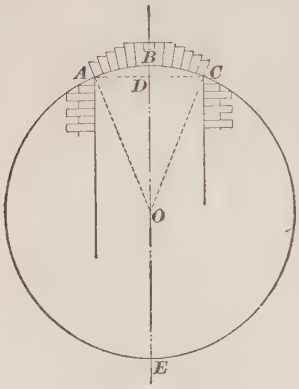


FIG. 66.

The span AC, Fig. 66, of an arch is 6 feet, and the rise DB is 8 inches. What is the length of the radius OB?

SOLUTION. — Here the span is the chord of a circle, and since a radius perpendicular to a chord bisects it, AD is $\frac{1}{2}$ of $AC = 36$ inches. If we draw the complete circle, it will be seen that ED and DB are the parts of the diameter and AD the perpendicular from the point A. Then $\overline{AD}^2 = ED \times DB$, or $36^2 = ED \times 8$, whence, $ED = 1{,}296 \div 8 = 162$ inches, and $EB = 162 + 8 = 170$ inches. $OB = \frac{1}{2} EB = 85$ inches, or 7 feet 1 inch. Ans.

INSCRIBED AND CIRCUMSCRIBED POLYGONS.

73. An **inscribed** polygon is one whose vertices lie on the circumference of a circle and whose sides are chords, as $KLMNPQ$, Fig. 67.

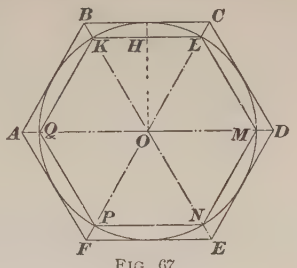


FIG. 67.

74. A **circumscribed** polygon is one whose sides are tangent to a circle, as $ABCDEF$, Fig. 67.

Circles may be inscribed in and circumscribed about any regular polygon. The radius of a circle is the distance from the center to an angle of the inscribed polygon, and the perpendicular distance from the center to the side of the circumscribed polygon, as shown in Fig. 67.

75. If lines be drawn from the center to the angles of a regular polygon, they will make equal angles with the sides of the polygon and will form isosceles triangles. As the sum of all the angles formed by lines drawn from a point is 4 right angles, or 360°, the angle at the center of a regular polygon between two radii equals 360° divided by the number of sides. Thus the angle between two radii drawn to the end of a side of a regular hexagon is $360° \div 6 = 60°$. The sum of the angles of a triangle is 2 right angles, or 180°; hence, since the triangles formed by drawing radii to the angles of a regular polygon are isosceles, the angles OAB and OBA, Fig. 67, are each equal to $\frac{1}{2}$ the difference between 180° and the central angle. Thus, each angle $= \frac{1}{2} \times (180° - 60°) = 60°$, and the triangle OBA is equilateral. In a regular hexagon, therefore, the sides are equal to the radius of the circumscribed circle.

A line drawn from the center of a regular polygon to an angle bisects the angle; thus OA, Fig. 67, bisects the angle BAF, which, therefore, is equal to $2 \times OAB$, or $2 \times OAF$.

EXAMPLE.—As the principles of Art. **75** are used in forming bevel angles, etc., let us find at what angle the bevel must be set to mark the cuts for the moldings around an octagonal room of equal sides, as in Fig. 68.

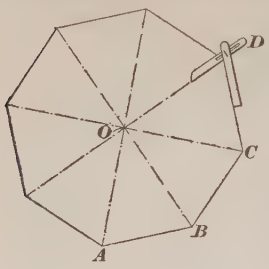


FIG. 68.

SOLUTION.—Draw $A\,O$, $B\,O$, etc. to the center O, forming triangles $A\,O\,B$, $B\,O\,C$, etc. Then each of the angles at O is $\frac{1}{8}$ of $360° = 45°$. The angles $O\,A\,B$ and $O\,B\,A$ are each $\frac{1}{2} \times (180° - 45°) = 67\frac{1}{2}°$. If the blade of the bevel is set at this angle, as at D, and the ends of the pieces of molding are cut to it, they will fit together properly.

EXAMPLE.—In Fig. 69, $G\,H\,E\,F$ and $H\,K\,D\,E$ represent two boards forming a center for supporting a brick arch during building; the shaded parts being cut off, leaving a curve, as $A\,L\,B$. What is the length of each board, and the angle of bevel $G\,H\,O$ where they meet? The radius $A\,O$ of the arch is 2 feet.

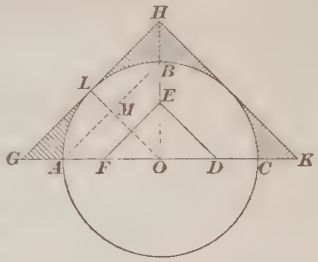


FIG. 69.

SOLUTION.—Draw $A\,B$ parallel to $G\,H$ and meeting $O\,G$ and $O\,H$, and also $O\,L$ perpendicular to $A\,B$; then $A\,M = M\,B$ (Art. **66**), and by similar triangles, $L\,G = L\,H$. $O\,L$ bisects the right angle $A\,O\,B$, hence, $A\,O\,M = \frac{1}{2}$ of 90°, or 45°. Then the other acute angle in the right triangle $O\,M\,A = 90° - 45° = 45°$ also. The angle $L\,G\,O$ is equal to angle $M\,A\,O$, or 45°, and the right triangle $O\,L\,G$, having two angles equal, the opposite sides are equal also, and since $O\,L$ = the radius, or 2 feet, $L\,G$ is also 2 feet and $G\,H = 2 \times 2 = 4$ feet, the required length. Angle $O\,H\,G$ = angle $O\,G\,H = 45°$, the angle at which to set the bevel.

EXAMPLES FOR PRACTICE.

76. Solve the following:

1. What angle does the minute hand of a clock travel over in 5 minutes? Ans. 30°.

2. The span of an arch is 7 feet, and the rise is 1 inch for each foot of span. What is the radius of the arch? Ans. 10 ft. $9\frac{1}{2}$ in.

3. The centering for an arch, Fig. 70, 8 feet in diameter is made of 3 pieces of equal length. Knowing that the side of a regular

inscribed hexagon is equal to the radius, what must be the length, as $A\,B$, of each piece?

Ans. 4.62 ft., or 4 ft. $7\frac{1}{2}$ in., nearly.

SUGGESTION.—Find the length of $O\,C$ in the right triangle $O\,C\,E$; then by similar triangles, find the length of $A\,B$.

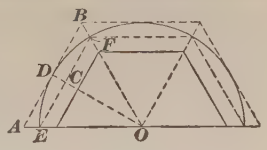


FIG. 70.

4. In the preceding example, what is (*a*) the bevel angle, and (*b*) the angle between any two pieces?

Ans. { (*a*) 60°. (*b*) 120°.

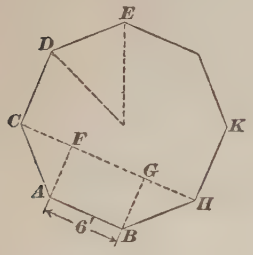


FIG. 71.

5. The flooring in a regular octagonal room, Fig. 71, is laid parallel with the side $A\,B$, which is 6 feet long. What are the angles of bevel for cutting the ends along $A\,C$ and $C\,D$?

Ans. 135° and 90°.

6. What is the length of the flooring between $C\,D$ and $H\,K$, Fig. 71, knowing that $C\,F$ and $A\,F$ are equal, and that angle $C\,F\,A$ is a right angle? (See Art. 48.)

Ans. 14.48 ft., or 14 ft. $5\frac{3}{4}$ in., nearly.

MENSURATION.

INTRODUCTION.

77. **Mensuration** is that part of geometry which treats of the measurement of lines, surfaces, and solids.

78. The practical application of mensuration is the computation of lengths, areas of surfaces, or volumes of solids. The dimensions which furnish the data required are usually obtained from working drawings, or plans; therefore, before formally taking up the subject of mensuration, it is necessary to explain how drawings and plans are made, and how the dimensions are taken from them.

79. Working drawings are generally made to *scale;* that is, the lines on the drawing have a certain ratio to the corresponding dimensions of the full-size object which the drawing represents; this ratio is called the scale. For example, if the scale of a drawing is $\frac{1}{2}$ inch to 1 foot, it

means that each half inch on the drawing represents 1 foot on the object, and the drawing is $\frac{1}{2} \div 12 = \frac{1}{24}$ of full size.

80. To make drawings and to obtain measurements from them, divided rules called *scales* are used. These consist of strips or pieces of wood or metal 1 or 2 feet long, having the edges beveled, and upon which are engraved the graduations for the different scales, as 1 inch to 1 foot $\frac{1}{4}$ inch to 1 foot, etc. The marks are numbered in order from the end; the space next the end being subdivided into twelfths, representing inches, and in the larger scales, these spaces are again subdivided for fractions of an inch. There are usually two scales marked on each edge, and in one the divisions are twice as long as in the other, and are numbered from the other end.

To measure a line on a drawing, place the scale with the proper edge along the line, and move it until one of the foot marks is opposite one end of the line, as at *b*, Fig. 72, while

FIG. 72.

the other end of the line is at the *0* mark on the scale, or opposite one of the small graduations, as at *a*. Then read the number of large divisions from the *0* mark to the far end of the line, and also the number of small divisions from *0* to the near end of the line. Thus in Fig. 72, the line *a b* extends over 9 large spaces and 5 small ones, or $\frac{5}{12}$ of a large one, so that the length of the line represented by *a b* is 9 feet 5 inches.

81. Working drawings usually have the principal dimensions marked on them, as shown in Fig. 73, which represents a side and an end view of a cast-iron lintel. The length is 7 feet 6 inches, as indicated by the broken line *c d* extending between *e c* and *f d*, and drawn perpendicular to *e f* at the ends *e* and *f*. The arrowheads indicate the ends of the

7′ 6″ space. At *h* is shown that the height from *g* to *K* is 6 inches, being the distance between the parallel lines *m K* and *n g*. At *l* are shown two arrowheads pointed towards each other; these are used where the space is too small to

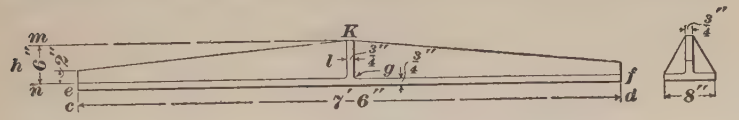


FIG. 73.

insert the arrowheads and figures in the ordinary manner. In this case it means that the thickness of the rib is $\frac{3}{4}$ inch, as shown by the nearby figures.

Having read or scaled all the necessary dimensions from the drawing, the proper formulas or rules may be applied to obtain the required length, area, or contents.

82. The following tables are useful in changing inches or fractions of an inch, to decimals of a foot or inch, and vice versa:

CONVERSION TABLES.

INCHES TO DECIMALS OF A FOOT.

Inches or Fractions.	Decimal of Foot.	Approximate Decimal.	Inches.	Decimal of Foot.	Approximate Decimal.
$\frac{1}{16}$	.0052	.005	3	.2500	.25
$\frac{1}{8}$	.0104	.010	4	.3333	.33
$\frac{1}{4}$	.0208	.020	5	.4167	.42
$\frac{3}{8}$	.0313	.030	6	.5000	.50
$\frac{1}{2}$	.0417	.040	7	.5833	.58
$\frac{5}{8}$	.0521	.050	8	.6667	.67
$\frac{3}{4}$	.0625	.060	9	.7500	.75
$\frac{7}{8}$	.0729	.070	10	.8333	.83
1	.0833	.080	11	.9167	.92
2	.1667	.170	12	1.0000	1.00

FRACTIONS OF AN INCH TO DECIMALS OF AN INCH.

Fraction.	Exact Decimal.	Approximate Decimal.	Fraction.	Exact Decimal.	Approximate Decimal.
$\frac{1}{32}$	.03125	.03	$\frac{9}{16}$	.5625	.56
$\frac{1}{16}$	.06250	.06	$\frac{5}{8}$	.6250	.63
$\frac{1}{8}$	.12500	.13	$\frac{11}{16}$	.6875	.69
$\frac{3}{16}$	.18750	.19	$\frac{3}{4}$	.7500	.75
$\frac{1}{4}$	.25000	.25	$\frac{13}{16}$	.8125	.81
$\frac{5}{16}$	.31250	.31	$\frac{7}{8}$	.8750	.88
$\frac{3}{8}$	.37500	.38	$\frac{15}{16}$	.9375	.94
$\frac{7}{16}$	.43750	.44	1	1.0000	1.00
$\frac{1}{2}$	.50000	.50			

EXAMPLE.— 7 ft. $8\frac{7}{8}$ in. = 7.00 ft. + .67 ft. + .07 ft. = 7.74 ft.
18.19 ft. = 18 ft. + .17 ft. + .02 ft. = 18 ft. $2\frac{1}{4}$ in.
$19\frac{9}{16}$ in. = 19.56 in., approximately.
13.26 in. = $13\frac{1}{4}$ in., approximately.

MENSURATION OF PLANE SURFACES.

83. The **area** of a surface is expressed by the number of unit squares it will contain.

84. A **unit square** is the square having the unit for its side. For example, if the unit is 1 inch, the unit square is the square whose sides measure 1 inch in length, and the area would be expressed by the number of square inches that the surface contains. If the unit were 1 foot, the unit square would measure 1 foot on each side, and the area would be the number of square feet that the surface contains, etc.

The square that measures 1 inch on a side is called a **square inch,** and the one that measures 1 foot on a side is called a **square foot.** Square inch and square foot are abbreviated to sq. in. and sq. ft., or to $\square''$ and $\square'$.

THE TRIANGLE.

85. Rule.—*To find the area of a triangle, multiply the base by the altitude and divide the product by 2.*

Let b = base;
h = altitude;
A = area.

Then, $$A = \frac{b\,h}{2}.$$

Example.—How many square feet of 1-inch boards will be needed for the 2 gables of a house having a half-pitch roof and width of 24 feet?

Solution.—In half-pitch roofs the rise equals one-half the span; hence, $h = \frac{1}{2}$ of 24 = 12 feet, and b = 24 feet. Applying the formula, $A = \frac{1}{2}\,b\,h = \frac{1}{2} \times 24 \times 12 = 144$ square feet for each gable, or $144 \times 2 = 288$ square feet of boards for the 2 gables. Ans.

86. If the triangle is a right-angled triangle, one of the short sides may be taken as the base, and the other short side as the altitude; hence, *the area of a right-angled triangle is equal to one-half the product of the two short sides.*

87. The area of any triangle may be found, when the length of each side is known, by means of the following formula, in which a, b, and c represent the lengths of the sides, s the half sum of the lengths, and A the area of the triangle:

$$A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}, \text{ where } s = \frac{a+b+c}{2}.$$

Example.—How many feet of 6-inch clapboards, laid $4\frac{1}{2}$ inches to the weather, will be required for the gable shown in Fig. 74?

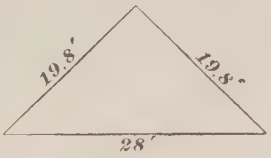


Fig. 74.

Solution.—It is immaterial which side is called a, b, or c. Applying the formula, $s = \frac{a+b+c}{2} = \frac{28+19.8+19.8}{2} = 33.8$, the half sum; taking b and c as the short sides, $s-a = 33.8-28 = 5.8$ and $s-b$ and $s-c$ are each $33.8-19.8 = 14$. Now applying the formula,

$$A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)} = \sqrt{33.8 \times 5.8 \times 14 \times 14}$$
$$= 196+ \text{ square feet.}$$

A clapboard with $4\frac{1}{2}$-inch exposure and 1 foot long, has 54 square inches exposed; 1 square foot, then, will require $144 \div 54 = 2\frac{2}{3}$ linear feet of clapboards, and 196 square feet will require $196 \times 2\frac{2}{3} = 522\frac{2}{3}$ feet. Ans.

THE QUADRILATERAL.

88. A **parallelogram** is a quadrilateral whose opposite sides are parallel.

There are four kinds of parallelograms: the **square,** the **rectangle,** the **rhombus,** and the **rhomboid.**

FIG. 75.

89. A **rectangle** is a parallelogram whose angles are all right angles. See Fig. 75.

FIG. 76.

90. A **square** is a rectangle, all of whose sides are equal. See Fig. 76.

91. A **rhomboid** is a parallelogram whose opposite sides only are equal, and whose angles are not right angles. See Fig. 77.

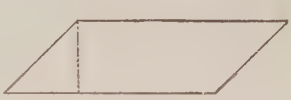


FIG. 77.

FIG. 78.

92. A **rhombus** is a parallelogram having equal sides, and whose angles are not right angles. See Fig. 78.

93. A **trapezoid** is a quadrilateral which has only two of its sides parallel. Fig. 79 is a trapezoid.

FIG. 79.

94. A **trapezium** is a quadrilateral having no two sides parallel. Fig. 80 is a trapezium.

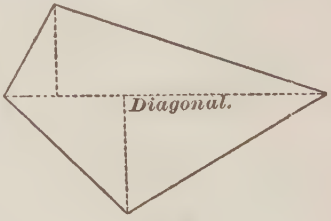


FIG. 80.

95. The **altitude** of a parallelogram, or of a trapezoid, is the perpendicular distance between the parallel sides.

96. A **diagonal** is a straight line drawn from the vertex of any angle of a quadrilateral to the vertex of the angle opposite; a diagonal divides the quadrilateral into two triangles. A diagonal divides a parallelogram into two *equal* and *similar* triangles.

97. **Rule.**—*To find the area of any parallelogram, multiply the base by the altitude.*

Let b = length of base;
h = altitude;
A = area.

Then, $A = b\,h$.

Example.—A lot is 4 rods wide and 8 rods long. (*a*) How many square feet does it contain? (*b*) What part of an acre?

Solution.—(*a*) Reducing rods to feet, 4 rods = 66 feet, and 8 rods = 132 feet. Either side may be taken as the base. Applying the formula,

$$A = b\,h = 66 \times 132 = 8{,}712 \text{ square feet.} \quad \text{Ans.}$$

(*b*) As there are 43,560 square feet in an acre, 8,712 square feet $= 8{,}712 \div 43{,}560 = .2 = \frac{1}{5}$ acre. Ans.

98. **Rule.**—*To find the area of a trapezoid, multiply one-half the sum of the parallel sides by the altitude of the trapezoid.*

Let a and b represent the lengths of the parallel sides, and h the altitude;

then, $$A = h\left(\frac{a+b}{2}\right).$$

Example.—How many square feet of floor space are there in a 5-story warehouse having the dimensions given in Fig. 81?

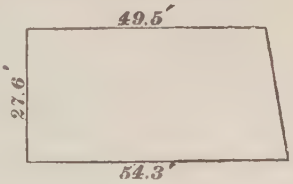


Fig. 81.

Solution.—As either side may be called a, let it be the shorter; then, applying the formula,

$$A = h\left(\frac{a+b}{2}\right) = 27.6\left(\frac{49.5+54.3}{2}\right)$$

$= 27.6 \times 51.9 = 1{,}432.44$ square feet.

Therefore, for 5 stories, there are 1,432.44 square feet $\times 5 = 7{,}162.2$ square feet of floor space. Ans.

POLYGONS.

99. Rule.—*To find the area of a regular polygon, divide the figure into isosceles triangles, compute the area of one triangle, and multiply by the number of triangles. The result will be the area of the polygon.*

Let n = number of sides of polygon;
l = length of one side;
h = perpendicular distance from the center to a side;
A = area of polygon.

Then, $$A = \frac{n\,l\,h}{2}.$$

EXAMPLE.—Find the floor surface of an octagonal room, the length of whose sides is 5 feet. The distance between parallel sides is 12 feet 1 inch nearly.

SOLUTION.—As the room is a regular octagon, $n = 8$, $l = 5$ feet, and $h = \frac{1}{2}$ of 12 feet 1 inch $= \frac{1}{2}$ of 12.08 feet $= 6.04$ feet. Applying the formula,

$$A = \frac{n\,l\,h}{2} = \frac{8 \times 5 \times 6.04}{2} = 120.8 \text{ square feet.} \quad \text{Ans.}$$

100. Rule.—*To find the area of an irregular polygon, or any figure bounded by straight lines, divide the figure into triangles, parallelograms, and trapezoids, and find the area of each. The sum of these partial areas will be the area of the figure.*

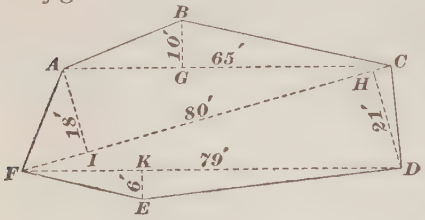


FIG. 82.

EXAMPLE.—A farmer fenced in a piece of land having the dimensions given in Fig. 82. How many square feet does it contain?

SOLUTION.—The distances marked on Fig. 82 being measured, the partial areas are found to be:

$$ABC = \frac{65 \times 10}{2} = 325 \text{ square feet;}$$

$$FAC = \frac{80 \times 18}{2} = 720 \text{ square feet;}$$

$$FCD = \frac{80 \times 21}{2} = 840 \text{ square feet;}$$

$$FDE = \frac{79 \times 6}{2} = 237 \text{ square feet.}$$

The area of the field is, therefore, the sum of these, or 2,122 square feet. Ans.

THE CIRCLE.

101. The ratio of the circumference of a circle to its diameter is an incommensurable number, and is usually denoted in technical books by the Greek letter π (pronounced *pi*). The approximate value of the ratio, correct to four decimal places, is 3.1416; hence, approximately $\pi = 3.1416$. For offhand calculations, the ratio is frequently taken as $3\frac{1}{7}$; that is, the circumference is $3\frac{1}{7}$ times the diameter.

102. Rule.—*To find the circumference of a circle, multiply the diameter by 3.1416.*

To find the diameter of a circle, divide the circumference by 3.1416.

103. Denoting the circumference by c, the diameter by d, and the radius by r,

$$c = \pi d = 2\pi r;$$

$$d = \frac{c}{\pi};$$

and

$$r = \frac{c}{2\pi}.$$

EXAMPLE.—A wheelwright wishes to cut a length of tire iron long enough to go around a $4\frac{1}{2}$-foot wagon wheel. How long must he cut it, allowing 4 inches for welding?

SOLUTION.—Here $d = 4\frac{1}{2}$ feet. Applying the formula,

$$c = \pi d = 3.1416 \times 4\tfrac{1}{2} = 14.14 \text{ feet.}$$

Adding the 4 inches = .33 foot, the length required is 14.47 feet. As this is only .03 foot = $\frac{3}{8}$ inch less than $14\frac{1}{2}$ feet, he would cut off $14\frac{1}{2}$ feet, and make the weld $4\frac{3}{8}$ inches.

104. As the circle is divided into 360 degrees, and the length of the circumference is $2\pi r$, the length of 1 degree is $\frac{2\pi r}{360}$; or, dividing both terms by 2π (= 6.2832), this reduces to $\frac{r}{57.3}$, very closely. If the angle is 57.3°, the length of the arc is equal to the radius; this angle is called a *radian*.

105. Rule.—*To find the length of an arc of a circle, multiply the number of degrees in the arc by the radius, and divide by 57.296 (or, sufficiently close, 57.3).*

Let l = length of arc;
n = number of degrees in arc;
r = radius of arc.

Then, $$l = \frac{r \times n}{57.3}.$$

If the angle contains degrees, minutes, and seconds, reduce them to degrees and decimals of a degree, thus:

$37° 30' 15'' = 37° + \frac{30}{60}° + \frac{15}{3600}° = 37° + .5° + .004° = 37.504°.$

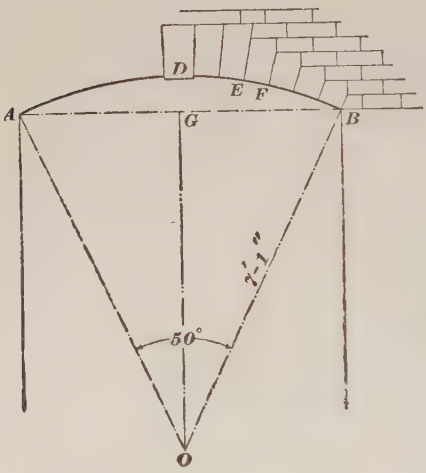


FIG. 83.

EXAMPLE.—In Fig. 83 is shown a segmental stone arch having a radius of 7 feet 1 inch = 85 inches. The angle $A\,O\,B$ is 50°. If there are 13 ring stones in the arch, what will be the width, as $E\,F$, of each stone?

SOLUTION.—Here $A\,D\,B = l$; $r = 85$ inches, and $n = 50°$. Applying the formula, $l = \frac{r\,n}{57.3} = \frac{85 \times 50}{57.3} = 74.17$ inches. Dividing by the number of stones, $\frac{74.17}{13} = 5.70$ inches $= 5\frac{11}{16}$ inches, nearly. Ans.

106. When the chord of the arc and the height of the segment (that is, $A\,B$ and $C\,D$, Fig. 84) are known, the length of the arc may be found by the following formula, in which

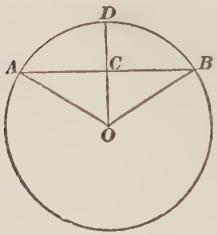


FIG. 84.

r = radius of arc;
l = length of the arc = length of $A\,D\,B$, Fig. 84;
c = length of the chord = length of $A\,B$;
h = height of segment = length of $C\,D$;

$$l = \frac{4\sqrt{c^2 + 4\,h^2} - c}{3}.$$

If the chord $A\,B$ and radius $O\,A$ are given, the height $C\,D$ of the segment may be found by the formula

$$h = r - \tfrac{1}{2}\sqrt{4r^2 - c^2}.$$

EXAMPLE.—If in Fig. 83 the span $A\,B$ is 6 feet, or 72 inches, and the rise $G\,D$ is 8 inches, what is the length of the intrados (arc $A\,D\,B$) of the arch?

SOLUTION.—Here the span $A\,B$ is the chord, and the rise is the height of the segment. Applying the formula,

$$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3} = \frac{4\sqrt{72^2 + 4 \times 8^2} - 72}{3} = \frac{4 \times 73.76 - 72}{3} = 74.35$$

inches = $74\frac{3}{8}$ inches, nearly. Ans.

107. When the quotient obtained by dividing the chord by the height is less than 4.8, that is, when $\frac{c}{h}$ is less than 4.8, the formula does not work well, the results not being sufficiently exact. In such a case, bisect the arc, and then apply the formula. Thus, in Fig. 85, suppose that $\frac{A\,B}{C\,D}$ is less than 4.8; then we should bisect $A\,C\,B$ by drawing $O\,C$ perpendicular to $A\,B$. We then find the length of the arc $A\,C$ and multiply the result by 2. But, in order to find the length of the arc $A\,C$, we must know the length of the radius $O\,A$ and the height of the segment included between the chord $A\,C$ and the arc $A\,C$. These may be found by means of the following formulas, in which

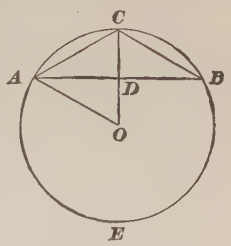


FIG. 85.

r = radius $O\,A$ of the arc;
C = chord $A\,B$ of the arc;
c = chord $A\,C$ of half the arc;
H = height $C\,D$ of the segment;
h = height of segment included between chord $A\,C$ and arc $A\,C$.

(*a*) $r = \dfrac{C^2 + 4H^2}{8H}$;

(*b*) $c = \tfrac{1}{2}\sqrt{C^2 + 4H^2}$;

(*c*) $h = r - \tfrac{1}{4}\sqrt{16r^2 - C^2 - 4H^2}$.

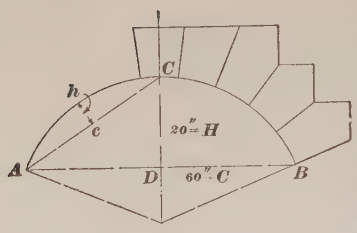


FIG. 86.

EXAMPLE.—The segmental arch in Fig. 86 has a span of 60 inches and a rise of 20 inches. What is the length of its intrados $A\,C\,B$?

SOLUTION.—Since $60 \div 20 = 3$, a number smaller than 4.8, we must find the length $A\,C$ of half the arc. Applying formula (a) to find the radius,

$$r = \frac{C^2 + 4H^2}{8H} = \frac{60^2 + 4 \times 20^2}{8 \times 20} = 32.5 \text{ inches.}$$

Now applying formula (b),

$$c = \tfrac{1}{2}\sqrt{C^2 + 4H^2} = \tfrac{1}{2}\sqrt{60^2 + 4 \times 20^2} = 36.06 \text{ inches.}$$

Applying formula (c),

$$h = r - \tfrac{1}{4}\sqrt{16r^2 - C^2 - 4H^2} = 32.5 - \tfrac{1}{4}\sqrt{16 \times 32.5^2 - 60^2 - 4 \times 20^2}$$
$$= 5.46 \text{ inches.}$$

Applying the formula of Art. **106**,

$$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3} = \frac{4\sqrt{36.06^2 + 4 \times 5.46^2} - 36.06}{3}$$
$$= 38.22 \text{ inches, length of arc } A\,C.$$

Therefore, $38.22 \times 2 = 76.44$ inches, nearly = length of arc $A\,C\,B$. Ans.

108. **Rule.**—*To find the area of a circle, square the diameter and multiply by .7854; or, square the radius and multiply by 3.1416.*

Let d = diameter of circle;
r = radius of circle;
A = area of circle.

Then, $A = \frac{1}{4}\pi d^2 = .7854\,d^2$;
$A = \pi r^2 = 3.1416\,r^2$.

EXAMPLE.—What is the area of a circle whose radius is 14 inches?

SOLUTION.—Applying the formula,

$A = 3.1416 \times 14^2 = 615.75$ square inches. Ans.

EXAMPLE.—The contract for a brick warehouse provides that all openings over 4 feet wide shall be deducted. What must be deducted for a semicircular arch 4 ft. 8 in. in diameter?

SOLUTION.—Here $d = 4$ feet 8 inches $= 4\frac{2}{3}$ feet. Applying the formula,

$A = .7854\,d^2 = .7854 \times (4\frac{2}{3})^2 = .7854 \times 21.78 = 17.11$ square feet.

For a half circle the area is $\frac{17.11}{2} = 8.55$ square feet, practically $8\frac{1}{2}$ square feet. Ans.

109. **Rule.**—*To find the diameter of a circle, the area being given, divide the area by .7854 and extract the square root of the quotient.*

$$d = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{4A}{\pi}}.$$

EXAMPLE.—To supply a certain quantity of water, it is necessary to have a pipe with an area of 28 square inches. What will be the diameter of the pipe?

SOLUTION.—Applying the formula,

$$d = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{28}{.7854}} = \sqrt{35.65} = 5.97 \text{ inches.}$$

As this is almost 6 inches, a pipe of that diameter would be chosen. Ans.

110. **Rule.**—*To find the area of a flat circular ring, subtract the area of the smaller circle from that of the larger.*

Let d = the longer diameter;
d_1 = the shorter diameter;
A = area of ring.

Then, $A = .7854\,d^2 - 7854\,d_1^2 = .7854(d^2 - d_1^2)$.

EXAMPLE.—What is the sectional area of a brick stack whose external and internal diameters are 6 feet 6 inches, and 4 feet respectively?

SOLUTION.—Here $d = 6.5$ feet, and $d_1 = 4$ feet. Applying the formula,

$A = .7854(6.5^2 - 4^2) = .7854 \times 26.25 = 20.62$ square feet. Ans.

If one diameter and the area of the ring are known, the other diameter may be found by adding to or subtracting from the area of the given circle that of the ring, and finding the diameter corresponding to the resulting area.

111. Rule.—*To find the area of a sector of a circle, divide the number of degrees in the arc of a sector by 360. Multiply the result by the area of the circle of which the sector is a part.*

Let n = number of degrees in arc;
A = area of circle;
r = radius of circle;
a = area of sector.

Then, $$a = \frac{n A}{360} = .0087267\, n r^2.$$

Example.—A circular window 4 feet in diameter is divided by radial ribs into 12 equal sections. What is the area of each space?

Solution.—Here $n = \frac{360^\circ}{12} = 30^\circ$. Applying the formula,

$$a = \frac{n A}{360} = \frac{30 \times .7854 \times 4^2}{360} = 1.05 \text{ square feet. Ans.}$$

Or, $a = .0087267\, n r^2 = .0087267 \times 30 \times 4 = 1.05$ square feet. Ans.

112. The area may also be found as follows:

Rule.—*To find the area of a sector, multiply one-half of the length of the arc by the radius.*

Let l = length of the arc;
r = radius of the arc;
a = area of sector.

Then, $$a = \tfrac{1}{2} l r.$$

Example.—In the sector $O A D B$, Fig. 84, the radius ($O A = O D$) of the arc is 6 inches, and the length of the chord $A B$ is 7 inches; what is the area of the sector?

Solution.—In order to find the area, we must first find the length of the arc, by applying the formula given in Art. **106**; but before applying this formula, we must find the height $C D$ of the segment. Since $AC = \frac{1}{2} AB = \frac{1}{2} \times 7 = 3.5$, $OC = \sqrt{OA^2 - AC^2} = \sqrt{6^2 - 3.5^2} = 4.87$ inches. Then, $CD = OD - OC = 6 - 4.87 = 1.13$ inches.

Applying the formula of Art. **106**,

$$l = \frac{4\sqrt{7^2 + 4 \times 1.13^2} - 7}{3} = 7.48 \text{ inches.}$$

Now applying the formula given above,

$a = \frac{1}{2} l r = \frac{1}{2} \times 7.48 \times 6 = 22.44$ square inches. Ans.

113. **Rule.**—*To find the area of a segment of a circle, find the area of the sector of which the segment is a part, and from this area subtract the area of the triangle formed by drawing radii to the extremities of the chord of the segment. The result is the area of the segment.*

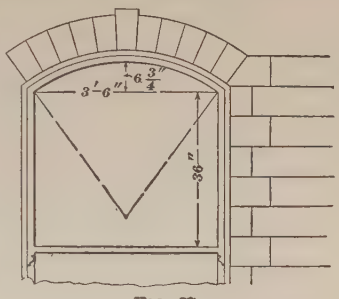


FIG. 87.

EXAMPLE.—What is the area of the upper pane of glass in the window shown in Fig. 87?

SOLUTION.—The radius is found by applying the formula

$$r = \frac{c^2 + 4h^2}{8h};$$

$c = 3$ feet 6 inches $= 42$ inches, and $h = 6\frac{3}{4}$ inches $= 6.75$ inches. Substituting,

$$r = \frac{42^2 + 4 \times 6.75^2}{8 \times 6.75} = 36 \text{ inches.}$$

The length of the arc is

$$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3} = \frac{4\sqrt{42^2 + 4 \times 6.75^2} - 42}{3} = 44.83 \text{ inches.}$$

The area of the sector is

$$a = \frac{lr}{2} = \frac{44.83 \times 36}{2} = 806.94 \text{ square inches.}$$

The altitude of the triangle is 36 inches $- 6\frac{3}{4}$ inches $= 29\frac{1}{4}$ inches, and its area is $\frac{42 \times 29\frac{1}{4}}{2} = 614.25$ square inches. The area of the segment is, therefore, $806.94 - 614.25 = 192.69$ square inches. The area of the rectangular portion of the glass is $42 \times 36 = 1{,}512$ square inches; adding to this 192.69, there results 1,704.69 square inches. Ans.

THE ELLIPSE.

114. An ellipse is a plane figure bounded by a curved line, to any point of which the sum of the distances from two fixed points within, called the foci, is equal to the sum of the distances from the foci to any other point on the curve.

In Fig. 88 let A and B be the foci, and let C and D be any two points on the perimeter. Then, according to the above definition, $AC + CB = AD + DB$, and both these sums are also equal to the major axis EF. The long diameter EF is called the major axis; the short diameter GD, the minor axis. The foci may be located from D or G as a center, and radius $DA = DB = \frac{1}{2}EF$.

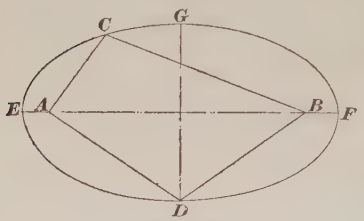


FIG. 88.

115. There is no exact method of finding the perimeter of an ellipse; but the following is close enough for most cases:

Rule.—*Multiply the major axis by 1.82, and the minor axis by 1.315. The sum of the results will be the perimeter.*

Let D = major diameter;
d = minor diameter;
C = perimeter, or circumference.

Then, $C = 1.82\,D + 1.315\,d$.

116. **Rule.**—*To find the exact area of an ellipse, multiply the product of its two diameters by .7854.*

$$A = \tfrac{1}{4}\pi\,d\,D = .7854\,d\,D,$$

where A represents the area, and D and d the two diameters.

EXAMPLE.—The soffit (under surface) of the semi-elliptic stone arch shown in Fig. 89 is to be carved at a cost of $3.50 per square foot. If the arch is 15 inches wide, what will the carving cost?

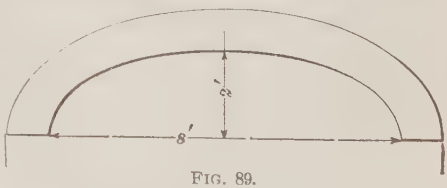


FIG. 89.

SOLUTION.—Here $D = 8$ feet, and $d = 2 \times 2 = 4$ feet; hence, applying the formula,

$$C = 1.82\,D + 1.315\,d = 1.82 \times 8 + 1.315 \times 4 = 19.82 \text{ feet.}$$

For one-half the ellipse, the length is $\frac{19.82}{2} = 9.91$ feet. The area is 9.91×1.25 (= 15 in.) = 12.39 square feet. Therefore, the cost of carving is $12.39 \times \$3.50 = \$43.36$. Ans.

EXAMPLE.—What is the area of the elliptical top of a table whose long and short diameters are 4 and 3 feet, respectively?

SOLUTION.—Here $D = 4$ feet and $d = 3$ feet. Applying the formula,

$$A = .7854\, D\, d = .7854 \times 4 \times 3 = 9.42 \text{ square feet.} \quad \text{Ans.}$$

AREA OF ANY PLANE FIGURE.

117. Rule.—*To find the area of any plane figure, divide it into triangles, quadrilaterals, circles, or parts of circles, and ellipses, find the area of each part separately and add the partial areas.*

EXAMPLE.—Find the sectional area of the half-arch and abutment shown in Fig. 90.

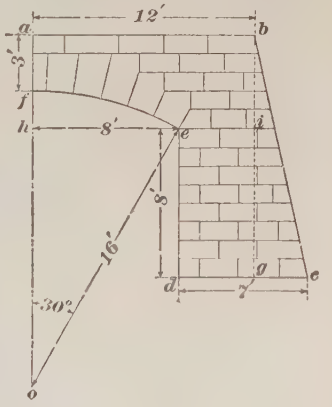


FIG. 90.

SOLUTION.—Divide the section into parts, as $a\,b\,i\,h$, $e\,i\,g\,d$, and $b\,g\,c$. Find $f\,h = f\,o - h\,o = 16 - \sqrt{16^2 - 8^2} = 2.14$ feet; then,

area $a\,b\,i\,h = 12 \times 5.14 = 61.68$ sq. ft.;

area $e\,i\,g\,d = 8 \times 4 = 32$ sq. ft.;

area $b\,g\,c = \frac{3 \times 13.14}{2} = 19.71$ sq. ft.

Adding these partial areas, the sum is 113.39 square feet. The area of sector $f\,o\,e = .0087267\, n r^2 = .0087267 \times 30 \times 16^2 = 67.02$ square feet. The area of triangle $h\,e\,o = \frac{1}{2}(h\,o \times e\,h)$; $h\,o = 16 - 2.14 = 13.86$ feet; area $= \frac{13.86 \times 8}{2} = 55.44$ square feet. The area of segment $f\,e\,h = 67.02 - 55.44 = 11.58$ square feet. Deducting 11.58 square feet from 113.39 square feet, the net area = 101.81 square feet. Ans.

AREAS OF IRREGULAR FIGURES.

118. If the figure is so irregular in shape that the ordinary rules cannot be easily applied, the area may be obtained as follows: Let Fig. 91 represent a drawing of such an area. Suppose it to be divided by equidistant parallel lines into strips, as ab, bc, etc.; then each of these small areas may be considered a trapezoid, and the whole area is the sum of the partial areas.

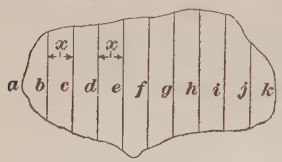


FIG. 91.

From these considerations we obtain the following rule:

119. **Rule.**—*To find the area of any irregular figure, divide the figure into any number of strips by equidistant parallel lines. Measure the length of the lines; add together one-half the lengths of the end lines and the lengths of the remaining lines, and multiply this sum by the distance between the lines.*

Let a = length of one end line;
k = length of other end line;
$b, c,$ etc. = lengths of intermediate lines;
x = distance between parallels;
A = area.

Then, $$A = \left(\frac{a+k}{2} + b + c + \text{etc.} + j\right)x.$$

The shorter the distance x is, the more accurate will be the result.

EXAMPLE.—Fig. 92 is a map of a small island whose area is to be determined. The parallel lines are 10 feet apart, and are of the lengths marked thereon. Required, the area of the island.

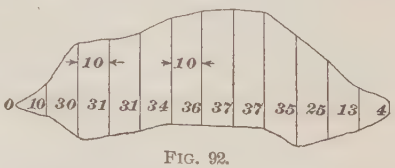


FIG. 92.

SOLUTION.—The extreme left end of the island is a point; hence, its length, or a, is 0; k is 4 feet.

Applying the formula, $A = \left(\frac{a+k}{2} + b + \text{etc.}\right)x$

$= \left(\frac{0+4}{2} + 10 + 30 + 31 + 31 + 34 + 36 + 37 + 37 + 35 + 25 + 18\right) \times 10$

$= 321 \times 10 = 3{,}210$ sq. ft. Ans.

EXAMPLES FOR PRACTICE.

(*See note at bottom of page 12.*)

120. Solve the following examples:

1. (*a*) How many squares (100 sq. ft.) of shingling, and (*b*) how many bundles of shingles, 250 to the bundle, each shingle being 6 inches wide and laid 5 inches to the weather, will be required for the gable shown in Fig. 93, adding 5 per cent. for waste?

Ans. { (*a*) $2\frac{1}{4}$ squares. (*b*) $4\frac{1}{2}+$ bundles.

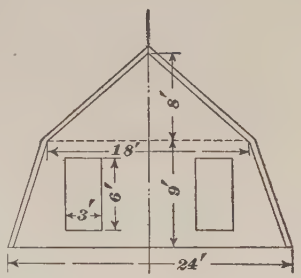


FIG. 93.

SUGGESTION.—Gable consists of trapezoid and triangle. Find exposed area of each shingle and number required per square of 14,400 square inches.

2. What is the cost of plastering the wall and ceiling of a semicircular alcove, 6 feet in diameter and 8 feet high, at 35 cents per square yard? Ans. $3.48.

SUGGESTION.—Total area = area of semicircle + (length of semicircumference × height of wall).

3. What is the length of the intrados *A B E C D* of the 3-centered arch shown in Fig. 94? Ans. 16.76 ft.

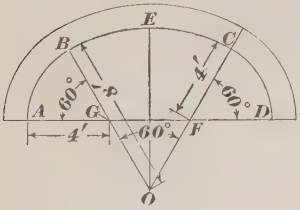


FIG. 94.

SUGGESTION.—Find lengths of arcs *B E C*, *A B*, and *C D* separately. *A B* = *C D*.

4. The space above the line *A D*, in Fig. 94, is to be covered with grille-work, costing $1.50 per square foot. What will be the total cost? Ans. $65.02.

SUGGESTION.—Area required = area of three sectors less the area of triangle *G O F*. *G F* = *G O* = *F O*, because angles are all equal; then lengths of three sides are known, to find the area of triangle.

5. How many laths 4 feet long and $1\frac{1}{2}$ inches wide, spaced $\frac{1}{4}$ inch apart transversely, will be required for the walls of a room 14 ft. 6 in.

$\times$ 10 ft. 9 in. $\times$ 8 ft. high, deducting 2 doors 36 in. $\times$ 7 ft. and 2 windows 2 ft. 6 in. $\times$ 5 ft. 6 in. ? Add 5% for waste. Ans. 601 laths.

SUGGESTION.—Number of lath per square foot = 144 $\div$ [(width of 1 lath + 1 space) $\times$ length of lath in inches].

6. A hollow circular cast-iron column, having an external diameter of 8 inches, is to carry a load of 47,120 pounds, and must not be loaded more than 4,000 pounds per square inch. What will be the thickness of the metal ring? Ans. $\frac{1}{2}$ in.

SUGGESTION.—Area of ring = total load $\div$ load per square inch. Area of smaller circle = area of 8-inch circle $-$ area of ring; from this find smaller diameter.

7. If brick masonry will safely carry 10 tons per square foot, what must be the size of the square base of the column given in the previous example? (Tons of 2,000 lb.) Ans. 1.54 ft., or $18\frac{1}{2}$ in., square.

SUGGESTION.—Side of square = square root of area required.

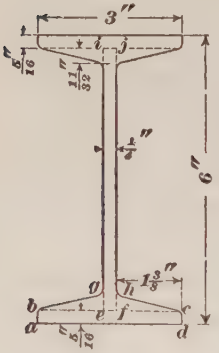


FIG. 95.

8. A bar of iron 1 yard long and 1 inch square weighs 10 pounds. What will be the weight per yard of the I beam shown in Fig. 95? Disregard curved edges and corners. All work in fractions.

Ans. $41\frac{2}{3}$ lb. per yd.

SUGGESTION.—Area required = areas of 3 rectangles + 4 triangles. Length $i\ e = 6$ inches $-(\frac{5}{16} \times 2)$. Express results in 256ths of a square inch, and reduce the sum.

9. What is the area of the cross-section of the dam shown in Fig. 96? $K\ C = D\ K$, and $O\ C$ and $O\ D$ are perpendicular, respectively, to $K\ C$ and $D\ K$.

Ans. 138.93 sq. ft.

SUGGESTION.—Divide area into trapezoids, rectangles, etc. Area $D\ M\ C\ K$ = areas of equal right triangles $D\ O\ K$ and $K\ O\ C$ less area of sector. The angle $D\ O\ C$ being 63° 26′ = 63.43°, find length of arc $D\ C$, and area of sector. Area $D\ M\ C\ K$ = 7.53 sq. ft.

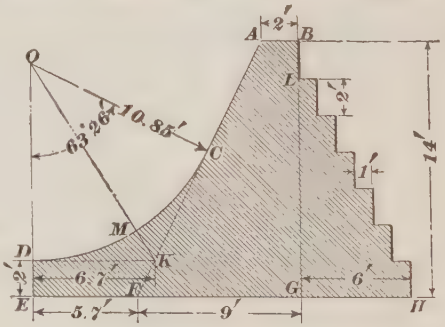


FIG. 96.

10. The segment $A\ B\ C$ of the window shown in Fig. 97 is to be filled with ornamental glass. What is the cost at \$6 per square foot? Ans. \$5.64.

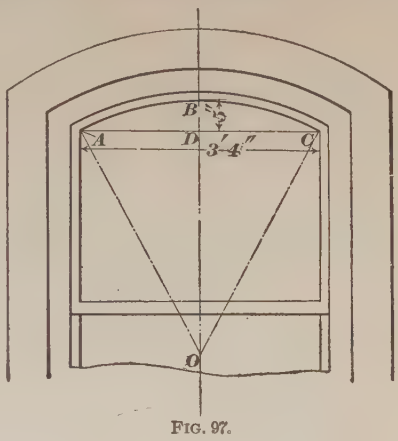


FIG. 97.

SUGGESTION.—Area = area of sector − area of triangle. Find radius $A\ O$ and arc $A\ B\ C$ by proper formulas; $O\ D$ = radius − 5 inches.

11. (*a*) What is the length of sash for an elliptical window whose axes are 8 feet and 4 feet 8 inches? (*b*) If the sash is 4 inches wide, what is the exposed area of the glass?

Ans. { (*a*) 20.70 ft. (*b*) 23.03 sq. ft.

SUGGESTION.—The axes of the glass are 4 inches $\times 2$ = 8 inches less than those of the sash.

12. In an octagonal room having equal sides the distance between the parallel sides is 12 feet and the distance between opposite corners is 13 feet. How many feet B. M. of 1-inch white-pine flooring will be required? Add 5% for waste and poor stuff. Ans. 126 ft. B. M.

SUGGESTION.—Length of side = 2 times base of triangle. Area = 8 triangles.

13. Pressed brick are estimated at 7 per square foot of external area. How many will be required for a building 20 ft. $\times$ 40 ft. and 20 feet high; with 2 chimneys 20 in. $\times$ 28 in. and 8 feet high; and 1 chimney 20 in. $\times$ 28 in. and 12 feet high? Deduct 6 window openings, 3 ft. $\times$ 6 ft. 6 in.; 1 double window, 2 openings, each 2 ft. 8 in. $\times$ 9 ft.; 1 front door, 4 ft. 8 in. $\times$ 9 ft.; 1 rear door, 3 ft. $\times$ 9 ft.; 6 window openings, 3 ft. $\times$ 6 ft.; and 1 double window, 2 openings, 2 ft. 8 in. $\times$ 6 ft.

Ans. 15,750 brick.

14. (*a*) How many squares (100 square feet) of slating are there on a house 25 ft. $\times$ 40 ft. with half-pitch roof, supposing the roof to extend beyond the walls 1 foot (along the roof) at eaves and ends? (*b*) How many slates, 14 inches wide and exposed 8½ inches, will be required?

Ans. { (*a*) 15.69 squares. (*b*) 1,899 slates.

SUGGESTION.—Rise of roof = one-half width. Find exposed area of each slate and number required per square of 14,400 square inches.

THE MENSURATION OF SOLIDS.

DEFINITIONS.

121. A **solid,** or body, has three dimensions: length, breadth, and thickness. The sides which enclose it are called the **faces,** and their intersections are called **edges.**

122. The **entire surface** of a solid is the area of the whole outside of the solid, including the ends.

The **convex surface** of a solid is the same as the entire surface, except that the areas of the ends are not included.

123. The **volume** of a solid is expressed by the number of times it will contain another volume, called the unit of volume. Instead of the word *volume*, the expression **cubical contents** is frequently used.

THE PRISM AND CYLINDER.

124. A **prism** is a solid whose ends are equal parallel polygons and whose sides are parallelograms.

Prisms take their names from their bases. Thus, a triangular prism is one whose bases are triangles; a pentagonal prism is one whose bases are pentagons, etc.

FIG. 98.

125. A **parallelopiped** is a prism whose bases (ends) are parallelograms. Fig. 98.

126. A **cube** is a parallelopiped whose faces and ends are squares. Fig. 99.

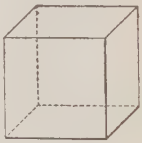


FIG. 99.

127. The cube whose edges are equal to the unit of length is taken as the unit of volume when finding the volume of a solid.

Thus, if the unit of length is 1 inch, the unit of volume will be the cube whose edges measure 1 inch, or 1 *cubic inch;* and the number of cubic inches the solid contains will be its volume. If the unit of length is 1 foot, the unit of volume will be 1 *cubic foot*, etc. Cubic inch, cubic foot, and cubic yard are abbreviated to cu. in., cu. ft., and cu. yd., respectively.

128. A **cylinder** is a solid whose ends are equal and similar *curved* figures. A *circular cylinder* is one any section of which, perpendicular to the axis, is a circle. See Fig. 100. Unless otherwise expressed, the word *cylinder* always means a *circular* cylinder.

FIG. 100.

129. A *right* prism or *right* cylinder is one whose center line (axis) is perpendicular to its base.

130. The *altitude* of a prism or cylinder is the perpendicular distance between its two ends.

131. **Rule.**—*To find the area of the convex surface of any right prism, or right cylinder, multiply the perimeter of the base by the altitude.*

Let p = perimeter of base;
h = altitude;
S = convex surface.

Then, $$S = p\,h.$$

To find the entire area, add the areas of the two ends to the convex areas.

EXAMPLE.—How many feet, board measure, of 1-inch sheathing are needed for a house 20 ft. $\times$ 28 ft. $\times$ 18 ft. high, including gables under a half-pitch roof, making no allowance for openings?

SOLUTION.—The perimeter p of the house $= 20 \times 2 + 28 \times 2 = 96$ feet; $h = 18$ feet; hence, $S = 96 \times 18 = 1{,}728$ square feet. Adding 2 gables, 20 feet base, and (since the roof is half pitch) 10 feet high $= 200$ square feet, the total is 1,928 square feet. As the sheathing is 1 inch thick the feet B. M. is also 1,928. Ans.

EXAMPLE.—A circular iron chimney, 7 feet in diameter and 85 feet high, is to be covered outside with a preservative paint. What will it cost to paint at 20 cents per square yard?

SOLUTION.—The perimeter p of the base is $\pi d = 21.99$ feet, and $h = 85$ feet. The surface of the chimney is $21.99 \times 85 = 1{,}869.2$ square feet, or 207.7 square yards. The cost of painting at 20 cents per square yard is, therefore, $207.7 \times \$.20 = \$41.54$. Ans.

EXAMPLE.—How many square feet of zinc will be required to line a refrigerator having interior dimensions of 4 ft. $\times$ 6 ft. $\times$ 8 ft. high?

SOLUTION.—The area of the sides $= (4 \times 2 + 6 \times 2) \times 8 = 160$ square feet. Adding for floor and ceiling $(4 \times 6) \times 2 = 48$ square feet, the total is 208 square feet. Ans.

EXAMPLE.—What will be the cost, at 35 cents per square yard, of plastering the walls and ceiling of an octagonal room, having sides 5 feet long and 11 feet high, the distance between the parallel sides being 12 feet?

SOLUTION.—The area of the sides is $8(5 \times 11) = 440$ square feet, the area of the ceiling is $\frac{8 \times 5 \times 6}{2} = 120$ square feet; the total is 560 square feet, or 62.22 square yards. At 35 cents per square yard, the cost is \$21.78. Ans.

132. Rule.—*To find the volume of a right prism, or cylinder, multiply the area of the base by the altitude.*

Let a = area of base;
h = altitude;
V = volume.

Then, $V = a\,h$.

If the given prism is a cube, the three dimensions are all equal, and the volume is equal to the cube of one of the edges.

If the volume and area are given, the altitude $= \frac{V}{a}$. If the cylinder or prism is hollow, the volume is equal to the area of the ring multiplied by the altitude.

EXAMPLE.—If brickwork averages 21 brick per cubic foot, how many brick are there in a pier 18 inches square and 6 feet high?

SOLUTION.—The sectional area or a is $1.5 \times 1.5 = 2.25$ square feet; $h = 6$ feet; hence, $V = a\,h = 2.25 \times 6 = 13.5$ cubic feet; at 21 brick per cubic foot, the number of bricks in the pier is $13.5 \times 21 = 283.5$. Ans.

EXAMPLE.—If cast iron weighs .26 pound per cubic inch, what length of sash weight will be necessary to balance a window weighing 8 pounds, the diameter of the cylindrical weight being $1\frac{1}{2}$ inches?

SOLUTION.—As there are 2 weights, the weight of each is 4 pounds. The number of cubic inches required, or V, is $4 \div .26 = 15.38$. The area a of a $1\frac{1}{2}$-inch circle is 1.77 square inches; hence, h, the length, is $\frac{V}{a} = \frac{15.38}{1.77} = 8.69$ in., practically $8\frac{3}{4}$ in. **Ans.**

THE PYRAMID AND CONE.

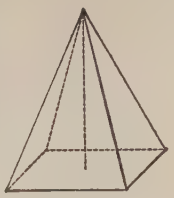


FIG. 101.

133. A **pyramid** is a solid whose base is a polygon and whose sides are triangles uniting at a common point, called the **vertex.** See Fig. 101.

FIG. 102.

134. A **cone** is a solid whose base is a circle and whose convex surface tapers uniformly to a point called the **vertex.** See Fig. 102.

135. A **right pyramid** or **cone** is one whose axis is perpendicular to the base.

136. The **altitude** of a pyramid or cone is the perpendicular distance from the vertex to the base.

137. The **slant height** of a *pyramid* is a line drawn from the vertex perpendicular to one of the sides of the base. The slant height of a *cone* is any straight line drawn from the vertex to the circumference of the base.

138. **Rule.**—*To find the convex area of a right pyramid or cone, multiply the perimeter of the base by one-half of the slant height.*

Let p = perimeter of base;
s = slant height;
A = convex area.

Then, $$A = \frac{ps}{2}.$$

EXAMPLE.—An octagonal church steeple is 60 feet high. The outer radius of the base (see Fig. 103) is 6 feet, and the length of one of the sides of the base is 4.6 feet. Compute the convex surface.

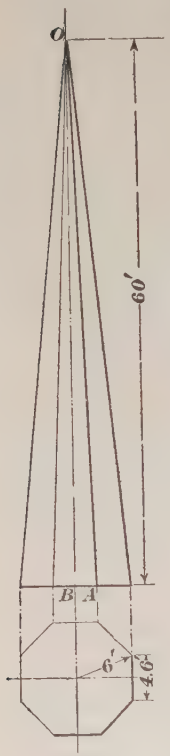


FIG. 103.

SOLUTION.—To find the slant height, the length OA along the hips must first be found. This is the hypotenuse of a right-angled triangle of which the altitude is 60 feet, the height of the tower, and the base is the radius, 6 feet. Hence $OA = \sqrt{60^2 + 6^2} = 60.3$. Now, in the triangle OAB, $OA = 60.3$ feet and $AB = \frac{4.6}{2} = 2.3$ feet. The slant height is, therefore, $OB = \sqrt{60.3^2 - 2.3^2} = 60.3$ feet, nearly. The perimeter of the base is $4.6 \times 8 = 36.8$ feet. Applying the formula, $A = \frac{ps}{2} = \frac{36.8 \times 60.3}{2} = 1{,}109.5$ sq. ft. Ans.

139. Rule.—*To find the volume of a pyramid or cone, multiply the area of the base by one-third of the altitude.*

Let a = area of base;
h = altitude;
V = volume.

Then, $$V = \frac{ah}{3}.$$

EXAMPLE.—The granite capstone of a monument is a square pyramid having a base 4 feet square and an altitude of 6 feet. If granite weighs 170 pounds per cubic foot, what will be the freight charges on the piece at 20 cents per 100 pounds?

SOLUTION.—The area a of the base is $4 \times 4 = 16$ square feet, and the altitude h is 6 feet. Applying the formula, $V = \frac{ah}{3} = \frac{16 \times 6}{3} = 32$ cubic feet; at 170 pounds per cubic foot, the weight is $32 \times 170 = 5{,}440$ pounds $= 54.4$ hundredweight. The freight is therefore $54.4 \times \$.20 = \$10.88$. Ans.

EXAMPLE.—If in the preceding example the capstone were conical, having a base 4 feet in diameter and a height of 6 feet, how much less would it weigh than the pyramidal stone?

SOLUTION.—The area of the base is $.7854 d^2 = 12.57$ square feet. Using the formula, the volume is $\frac{12.57 \times 6}{3} = 25.14$ cubic feet, and the weight at 170 pounds per cubic foot is 4,274 pounds, nearly. As the pyramidal capstone weighs 5,440 pounds, the difference in weight is $5{,}440 - 4{,}274 = 1{,}166$ lb. Ans.

THE FRUSTUM OF A PYRAMID OR CONE.

140. If a pyramid or cone is cut by a plane parallel to the base, so as to form two parts, the lower part is called the frustum of the pyramid or cone. See Figs. 104 and 105.

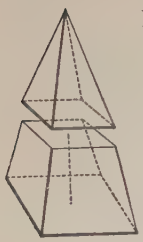


FIG. 104.

The upper end of the frustum of a pyramid or cone is called the upper base, and the lower end the lower base. The altitude of a frustum is the perpendicular distance between the bases.

FIG. 105.

141. **Rule.**—*To find the convex area of a frustum of a right pyramid or right cone, multiply one-half the sum of the perimeters of the bases by the slant height.*

Let P = perimeter of lower base;
p = perimeter of upper base;
s = slant height;
A = convex area.

Then, $$A = \frac{(P+p)s}{2}.$$

To find the entire area, add to the convex area the areas of the two bases.

EXAMPLE.—A square mansard-tower roof has the dimensions shown in Fig. 106. To compute the slate required to cover the tower the convex area is required. What is this convex area?

SOLUTION.—The tower has the form of a frustum of a pyramid. 6 ft. 6 in. $= 6\frac{1}{2}$ feet and 3 ft. 6 in. $= 3\frac{1}{2}$ feet. The perimeter of the lower base is $4 \times 6\frac{1}{2} = 26$ feet, and that of the upper base is $4 \times 3\frac{1}{2} = 14$ feet. In the triangle $A\,B\,C$, $A\,C = 6$ feet, and $B\,C = \frac{1}{2}(6\frac{1}{2} - 3\frac{1}{2}) = 1\frac{1}{2}$ feet. Hence, the hypotenuse $A\,B$, which is the slant height, is $\sqrt{\overline{B\,C}^2 + \overline{A\,C}^2} = \sqrt{1\frac{1}{2}^2 + 6^2} = 6.18$ feet.

Applying the formula, $A = \frac{(P+p)s}{2}$

$= \frac{(26+14) \times 6.18}{2} = 123.6$ sq. ft. Ans.

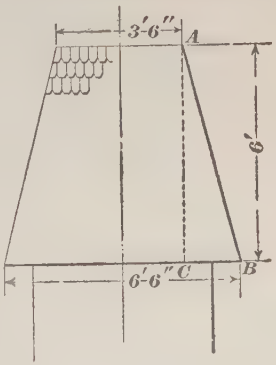


FIG. 106.

142. Rule.—*To find the volume of the frustum of a pyramid or cone, add the areas of the upper base, the lower base, and the square root of the product of the areas of the two bases; multiply this sum by one-third of the altitude.*

Let A = area of lower base;
a = area of upper base;
h = altitude;
V = volume.

Then, $$V = \frac{h}{3}(A + a + \sqrt{A \times a}).$$

EXAMPLE.—A marble monument is $2\frac{1}{2}$ feet square at the base, 1 foot square at the top, and is 16 feet high. If marble weighs 160 pounds per cubic foot, what is the weight of the stone?

SOLUTION.—The area of the lower base is $2\frac{1}{2} \times 2\frac{1}{2} = 6.25$ square feet; that of the upper base is 1 square foot. Applying the formula,

$V = \frac{h}{3}(A + a + \sqrt{A \times a}) = \frac{16}{3}(6.25 + 1 + \sqrt{6.25 \times 1}) = 51.97$ cu. ft.;

at 160 pounds per cubic foot, the weight is $51.97 \times 160 = 8{,}315$ lb.

THE WEDGE.

143. A **wedge** is a solid having plane surfaces, of which the base is a parallelogram, the ends are triangles, and the sides are quadrilaterals meeting in a line parallel to the sides of the base. See Fig. 107.

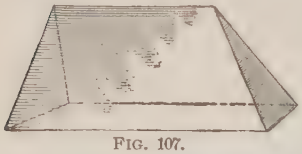


FIG. 107.

144. Rule.—*To find the volume of a wedge, multiply together the width of the base, the perpendicular distance from the base to the edge, and the sum of the lengths of the three parallel edges; divide the product by 6.*

Let w = width of base;
h = perpendicular distance from base to edge;
s = sum of lengths of the three parallel edges;
V = volume.

Then, $$V = \frac{w\,h\,s}{6}.$$

If the base is a rectangle, and the triangular ends are parallel, the wedge becomes a triangular prism, and the rule for prisms may be used.

EXAMPLE.—Steel weighs .28 pound per cubic inch. How heavy is a steel wedge 8 inches long, and $1\frac{1}{2}$ in. $\times$ 3 in. at the head?

SOLUTION.—Here w is $1\frac{1}{2}$ inches, h is 8 inches, s is $3+3+3=9$ inches. Then $V = \frac{1.5 \times 8 \times 9}{6} = 18$ cubic inches. Or, as this wedge is a prism, the volume = area of end $\times$ length of edge; area of triangle $= \frac{1.5 \times 8}{2} = 6$ square inches; multiplying by the length of the edge, 3 inches, the volume is 18 cubic inches. The weight at .28 pound per cubic inch is $.28 \times 18 = 5.04$ lb. Ans.

THE PRISMOIDAL FORMULA.

145. If any solid be sliced in pieces whose adjacent surfaces are flat, any piece is called a **plane section** of the solid.

Plane sections are divided into three classes: Longitudinal sections, cross-sections, and right sections. A **longitudinal section** is any plane section taken lengthwise through the solid. Any other plane section is called a **cross-section.** If the surface exposed by taking a plane section of a solid is perpendicular to the center line of the solid, the section is called a **right section.** The surface exposed by any longitudinal section of a cylinder is a rectangle. The surface exposed by a right section of a cube is a square; of a cylinder or cone, a circle; an oblique cross-section of a cylinder is an ellipse.

146. A **prismoid** is a solid having for its ends two *parallel* plane surfaces which are connected by plane triangular or quadrilateral faces.

Thus, the solid shown in Fig. 108 is a prismoid. The parallel ends are the pentagon $A\,B\,C\,D\,E$ and the quadrilateral $F\,G\,H\,I$, and these ends are connected by the triangular face $B\,C\,H$ and the four quadrilateral faces $A\,B\,H\,I$, $A\,E\,F\,I$, $D\,E\,F\,G$, and $C\,D\,G\,H$.

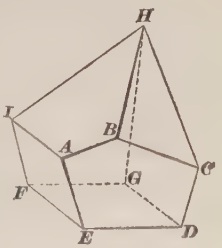


FIG. 108.

147. **Rule.**—*To find the volume of a prismoid, add together the areas of the two parallel ends, and four times the*

area of a parallel section midway between them; multiply the sum by one-sixth of the perpendicular distance between the parallel ends.

Let A = area of one end;
a = area of other end;
M = area of middle section;
h = distance between ends;
V = volume.

Then, $$V = \frac{h}{6}(A + a + 4M).$$

The area of the middle section is not in general a mean between the end areas, but the lengths of its sides are means between the corresponding lengths on the ends. This formula, called the *prismoidal formula*, is of very extended application and may be used to calculate the volume of a prism, cylinder, pyramid, cone, frustum of pyramid and cone, wedge, sphere, and segments of spheres, in addition to the irregularly shaped bodies to which it is usually applied. For a pyramid, cone, and wedge, the upper area is 0; for a sphere, the end areas are both 0; for a cylinder and prism, the areas are all equal.

EXAMPLE.—What is the volume in cubic yards of a masonry pier 30 feet high, the upper and lower rectangular bases being, respectively, 7 ft. $\times$ 24 ft. and 11 ft. $\times$ 30 ft.?

SOLUTION.—The area of the upper base is $7 \times 24 = 168$ square feet. The area of the lower base is $11 \times 30 = 330$ square feet. The dimensions of the middle section are the means of the corresponding dimensions of the bases: therefore, the width is $\frac{24+30}{2} = 27$ feet, and the thickness is $\frac{7+11}{2} = 9$ feet. The area of the middle section is $27 \times 9 = 243$ square feet. Applying the prismoidal formula, the volume is $V = \frac{30}{6}(330 + 168 + 4 \times 243) = 7{,}350$ cubic feet $= \frac{7{,}350}{27}$ cubic yards $= 272\frac{2}{3}$ cu. yd. Ans.

148. The prismoidal formula, when used to obtain the volumes of frustums of pyramids and cones, saves the labor of extracting a square root, as required by the ordinary rule.

EXAMPLE.—The upper and lower bases of the frustum of a right cone are respectively 30 inches and 14 inches in diameter, and the perpendicular distance between them is 36 inches. Required, the volume of the frustum.

SOLUTION.—The diameter of the middle section is the mean of the diameters of the ends, or $\frac{30+14}{2} = 22$ inches.

Area of lower base $= A = .7854 \times 30^2 = 706.86$ sq. in.
Area of upper base $= a = .7854 \times 14^2 = 153.94$ sq. in.
Area of middle section $= M = .7854 \times 22^2 = 380.13$ sq. in.

Using the prismoidal formula,

$$V = \frac{36}{6}(706.86 + 153.94 + 4 \times 380.13) = 14{,}287.92 \text{ cu. in.} = 8.27 \text{ cu. ft.}$$ Ans.

THE SPHERE.

149. A **sphere** is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within called the center. Fig. 109. The word *ball* is commonly used instead of sphere.

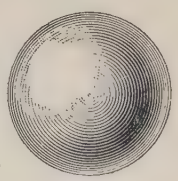


FIG. 109.

150. **Rule.**—*To find the area of the surface of a sphere, multiply the square of the diameter by 3.1416.*

Let d = diameter of sphere;
S = surface.

Then, $S = \pi d^2 = 3.1416\, d^2$.

EXAMPLE.—A ball on a flagstaff is 10 inches in diameter and is to be gilded. Deducting 20 square inches for space covered by attachment to pole, how many books of gold leaf, each containing 25 leaves $3\frac{3}{8}$ inches square, will be required for gilding it?

SOLUTION.—Applying the formula, $S = \pi d^2 = 3.1416 \times 10 \times 10 = 314.16$ square inches. Deducting 20 square inches, the net surface is 294.16 square inches. Each gold leaf has an area of $3\frac{3}{8}$ in. $\times 3\frac{3}{8}$ in., or 11.4 square inches, nearly; hence for 294.16 square inches, there will be needed $294.16 \div 11.4 = 25.8$ leaves, or 1 book and 1 leaf. Ans.

151. **Rule.**—*To find the volume of a sphere, multiply the cube of the diameter by .5236.*

Let d = diameter;
V = volume.

Then, $$V = \frac{\pi}{6} d^3 = .5236\, d^3.$$

Since $\frac{\pi d^3}{6} = \frac{1}{6} \times \pi d^2 \times d$, the volume of a sphere is equal to one-sixth of the surface multiplied by the diameter.

EXAMPLE.—What is the weight of a cast-iron ball 6 inches in diameter, if the metal weighs .26 pound per cubic inch?

SOLUTION.—Applying the formula,

$V = .5236\, d^3 = .5236 \times 216 = 113.1$ cubic inches.

The weight is, therefore, $113.1 \times .26 = 29.41$ lb. Ans.

152. The volume of a spherical shell is equal to the difference in volume between two spheres having the outer and inner diameters of the shell.

153. **Rule.**—*To find the diameter of a sphere of known volume, divide the volume by .5236 and extract the cube root of the quotient.*

$$d = \sqrt[3]{\frac{V}{.5236}}.$$

EXAMPLE.—A cast-iron ball weighing 25 pounds is required for a certain purpose. If cast iron weighs .26 pound per cubic inch, what will be the diameter of the ball?

SOLUTION.—The volume of the ball will be $25 \div .26 = 96.1$ cubic inches. Using the formula,

$$d = \sqrt[3]{\frac{V}{.5236}} = \sqrt[3]{\frac{96.1}{.5236}} = \sqrt[3]{183.5} = 5.68 \text{ inches} = 5\tfrac{11}{16} \text{ in., nearly.}$$

Ans.

SYMMETRICAL AND SIMILAR FIGURES.

154. An **axis of symmetry** is any line so drawn that, if the part of the figure on one side of the line be folded on this line, it will coincide exactly with the other part, point for point and line for line. Thus, in Fig. 110, if the upper semicircle be folded over on the diameter $C\,D$, it will coincide exactly with the lower semicircle; also, if the part on the right of the diameter $A\,B$ be folded over on $A\,B$, it will coincide exactly with the part of the left of this line.

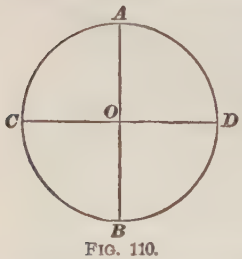


FIG. 110.

It is evident from the above that a circle may have any number of axes of symmetry. In certain cases, however, a figure may be symmetrical with regard to only one axis. Thus, the isosceles triangle $A\,B\,C$, Fig. 111, is symmetrical with regard to the axis $B\,D$, because the part $B\,C\,D$ would coincide with the part $B\,A\,D$ if folded over on the line $B\,D$; but no other axis of symmetry could be drawn. A rectangle has two axes of symmetry at right angles to each other. A hexagon has six axes of symmetry.

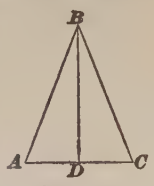


FIG. 111.

155. Similar figures are those which are alike in form. As in the case of triangles, which have been considered, two figures, to be similar, must have their corresponding sides in proportion, and the angles of one equal to the corresponding angles of the other. Circles are, of course, similar figures.

156. *The areas of two similar figures are to each other as the squares of any one dimension.* Thus, a regular octagon whose sides are 1 inch long contains 4.828 square inches; another with sides 4 inches long contains 4^2 or 16 times 4.828 square inches = 77.25 square inches, for let A = required area; then A : 4.828 sq. in. $= 4^2 : 1^2$, or $A = 4.828 \times 16$ square inches.

This principle often saves considerable labor in determining the areas of similar figures, as, for instance, the end areas of frustums of pyramids, cones, etc.

EXAMPLE.—Required, the volume of the frustum of a pyramid, the bases of which are equilateral triangles, one with sides 6 feet in length, the other with sides 2 feet in length. The altitude of the frustum is 8 feet.

SOLUTION.—The area of the lower triangular base may be found from the formula $A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$, where $s = \dfrac{6+6+6}{2} = 9$. and the factors $(s-a)$, etc. are each $9-6 = 3$.

Hence, $A = \sqrt{9 \times 3 \times 3 \times 3} = 15.59$ square feet.

Since the ends are similar triangles, their areas are proportional to the squares of the sides; or, denoting the area of the upper base by a, $a : 15.59 = 2^2 : 6^2$.

Hence, $a = 15.59 \times \frac{2^2}{6^2} = 15.59 \times \frac{1}{9} = 1.73$ square feet.

A section midway between the bases is evidently, also, an equilateral triangle with sides $\frac{6+2}{2} = 4$ feet in length; hence, its area may be obtained from the proportion

$$M : 15.59 = 4^2 : 6^2,$$

or $$M = 15.59 \times \frac{4^2}{6^2} = 6.93 \text{ square feet.}$$

Now using the prismoidal formula,

$$V = \tfrac{2}{6}(15.59 + 1.73 + 4 \times 6.93) = 60.05 \text{ cu. ft.} \quad \text{Ans.}$$

157. The cubical contents (and weights) of similar solids are to each other as the cubes of any one dimension.

EXAMPLE.—If a cast-iron ball 9 inches in diameter weighs 100 pounds, what would a ball 15 inches in diameter weigh?

SOLUTION.— $100 : x = 9^3 : 15^3$, or $x = \frac{100 \times 3{,}375}{729} = 462.96$ pounds, the weight of the larger ball. Ans.

EXAMPLES FOR PRACTICE.

(*See note at bottom of page 12.*)

158. Solve the following examples:

1. What is the weight of the cast-iron lintel shown in Fig. 112, estimating iron at .26 pound per cubic inch? Ans. 201.08 lb.

SUGGESTION.—The lintel consists of a rectangular base, a longitudinal rib whose faces are trapezoids, and two triangular cross-ribs.

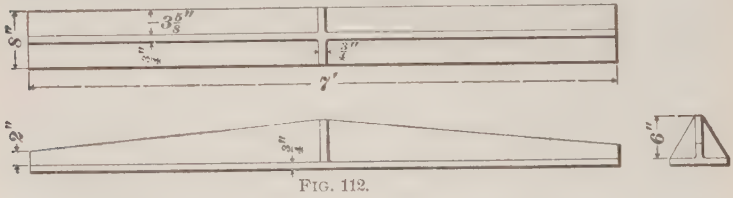


FIG. 112.

2. (*a*) Figure the number of feet B. M. (board measure) in the following bill of material, 1 foot B. M. being 1 foot square and 1 inch thick; (*b*) also the cost at $16.50 per M. (thousand) feet.

23 pieces 3″ × 10″; 18′ 6″ long.	2 pieces 3″ × 6″; 10′ 6″ long.
3 pieces 3″ × 10″; 15′ 0″ long.	5 pieces 3″ × 8″; 18′ 6″ long.
3 pieces 3″ × 10″; 12′ 0″ long.	56 pieces 2″ × 4″; 9′ 9″ long.
34 pieces 3″ × 6″; 14′ 0″ long.	81 pieces 2″ × 4″; 9′ 3″ long.
4 pieces 3″ × 6″; 6′ 0″ long.	42 pieces 2″ × 4″; 1′ 2″ long.
38 pieces 3″ × 4″; 5′ 6″ long.	

SUGGESTION.—An easy method to figure B. M. is as follows: Note that in the first item, a board 10 inches wide and 1 foot long contains

$\frac{12}{12}$ or $\frac{5}{6}$ of a board foot for each inch of thickness. Hence, a 3″ × 10″ plank contains $\frac{5}{6} \times 3 = 2.5$ feet, B. M. per foot of length.

Ans. { (*a*) 3,338 ft. B. M., nearly. (*b*) $55.08.

3. What is the weight of the boiler front shown in Fig. 113 if the metal is $1\frac{1}{4}$ inches thick, and cast iron weighs .26 pound per cubic inch? Ans. 1,638 lb.

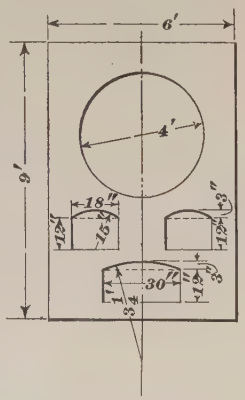


FIG. 113.

SUGGESTION.—First find net area of surface; door openings consist of rectangular portion + circular segment whose chord and height are known.

4. Figure the cost of digging and lining a well 24 feet deep and $3\frac{1}{2}$ feet clear diameter, the lining to be 1 brick thick, laid without mortar; allow 18 brick per square foot of surface of interior of lining. The excavation is to be $5\frac{1}{4}$ feet in diameter. The digging is to cost $1.25 per cubic yard, and the brickwork $12 per thousand, furnished and laid. Ans. $81.04.

5. Find the diameter of the balls of a 10-pound dumbbell, the cylindrical handle of which is $4\frac{1}{2}$ inches long and $1\frac{1}{4}$ inches in diameter. Cast iron weighs .26 pound per cubic inch. Disregard the small volumes at ends of handle, which are figured twice. Ans. 3.16 in. = $3\frac{1}{8}$+ in.

SUGGESTION.—Find contents of handle; one-half of remainder is volume of each ball.

6. A window weighing 10 pounds is to be balanced by two $1\frac{1}{2}$-inch (inside diameter) lead-pipe weights. If the metal is $\frac{1}{4}$ inch thick, and lead weighs .41 pound per cubic inch, how long pieces are required? Ans. 8.9 in., say 9 in.

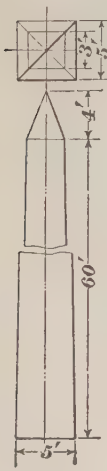


FIG. 114.

SUGGESTION.—Find number of cubic inches in each weight. Length required = volume ÷ area of ring.

7. A shop 28 ft. × 40 ft. and 18 ft. high, with gables 8 ft. 8 in. high, is to be built of brick. The lower walls are 12 inches, and the gables 8 inches in thickness. Deduct 12 windows 3 ft. × 6 ft. 4 in., and 4 doors 4 ft. × 7 ft., all in 12-inch wall. Estimating 7 brick to each square foot of wall *4 inches thick*, and counting corners twice, how many thousand brick will be required, with 5 per cent. allowance for breakage, etc.? Ans. 50.05 thousand.

SUGGESTION.—Find areas and not volumes.

8. (*a*) What will be the weight of the square granite shaft shown in Fig. 114, estimating the stone at 170 pounds per cubic foot? (*b*) Allowing 4,000 pounds per square foot on the soil, what will be the size of the foundation required?

Ans. { (*a*) 168,640 lb. = 84.3 T. (*b*) 6.5 ft. square, nearly.

Query.—What two solids make up the monument?

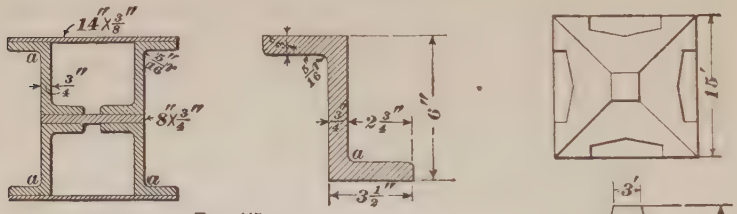


FIG. 115.

FIG. 116.

9. What is the weight per foot of the **Z** bar column shown in Fig. 115, figuring steel at .283 pound per cubic inch? Neglect rounding of edges of **Z** bars, but figure the fillets, as at *a*. Check result by multiplying area of section by 3.4, which is the weight per foot of steel for each square inch of section.

Ans. { 173.67 lb.
{ 173.88 lb.

SUGGESTION.—Area of a fillet = ¼ difference between square of twice the radius, and circle.

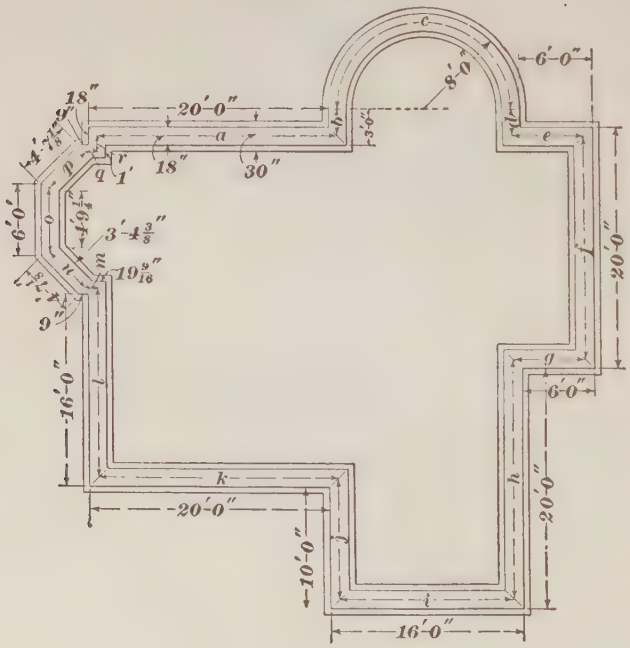


FIG. 117.

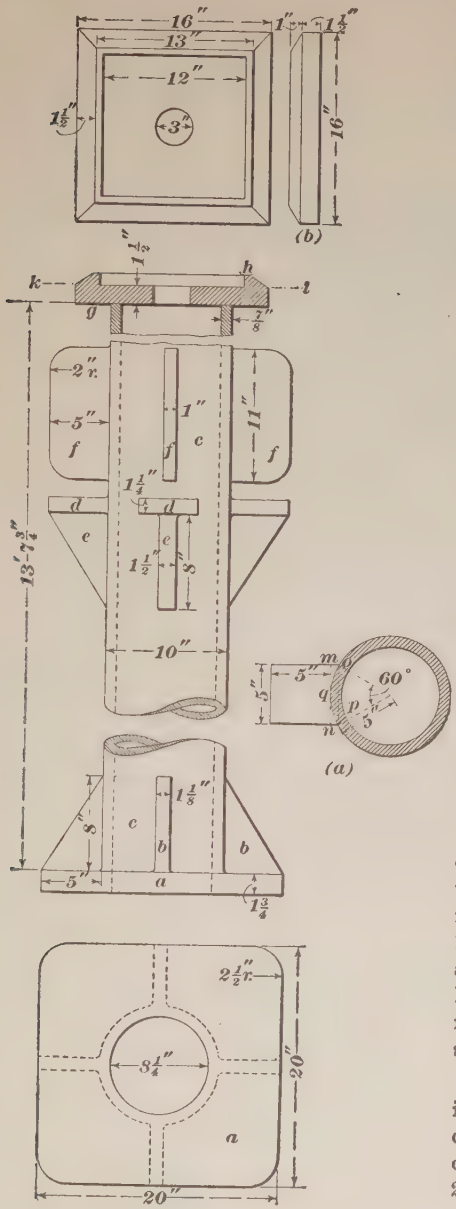


FIG. 118.

10. What is the net area of the 4-sided pyramidal-tower roof shown in Fig. 116, the openings being measured along the slope?

Ans. 710.68 sq. ft.

11. Find the number of cubic yards of masonry in the foundation walls shown in Fig. 117, the wall being 18 inches wide and 9 feet high, and the footing course 2 ft. 6 in. wide and 6 inches thick, projecting 6 inches each side of the foundation wall.

Ans. 95.73 cu. yd., nearly.

SUGGESTION.— $19\frac{9}{16}$ in. = 1.63 ft.; reduce feet and inches to feet and decimals by conversion table. Find the length of center line around the walls; for example, b = 18 in. + ½ the thickness of wall; f = 20 ft. − thickness of wall; o = mean of inside and outside measurements, etc. The same center line applies to both footing and wall.

12. Figure the volume of each portion and also the total weight of the cast-iron column shown in Fig. 118, the metal to be taken at .26 pound per cubic inch. Deduct for rounded corners, but figure d as 5 inches square on top.

ANS.—Base, a, 597.06 cu. in.; 4 triangular ribs, b, 90 cu. in.; cylinder, c, 4,106.85 cu. in.; 4 brackets, $d + e$, 245 cu. in.; 4 lugs, f, 213.16 cu. in.; portion of cap below

k l, 373.39 cu. in.; portion above *k l*, 67 cu. in.; total, 5,692.46 cu. in. Weight, 1,480 lb.

SUGGESTION.—Deduction of each rounded corner $= \frac{1}{4}$ of difference between a square of twice the radius and a circle of the given radius. Top of caps above line *k l* = (16 in. square − 12 in. square) × 1 in. thick − 4 wedges, 16 in. × 1 in. at back and 13 in. at edge and $1\frac{1}{2}$ in. long. See sketch (*b*). Area of top of *d* is strictly more than 5 in. × 5 in. by areas *m o q* and *q n p* [see sketch (*a*)] = difference between rectangle *m o p n* and segment of circle *o q p*. Although not figured in the example, the student should calculate it, as a test of his knowledge.

ARCHITECTURAL ENGINEERING.

INTRODUCTION.

1. Architectural engineering is the study of the anatomy of a building. It differs from architecture in that it does not deal with utility and appearance, but with *strength* and *stability*. The architectural engineer designs and assembles the skeleton or bony framework of the structure, while the architect plans the building to best accomplish its purpose. and beautifies it, inside and out, by covering with becoming vesture the frame-like structure erected by the engineer. This structure must comply with the conditions demanded by the plan and design of the building, and adhere closely to the general lines laid down by the architect.

A knowledge of engineering is essential to architects and those engaged in building operations. Lack of such knowledge on the part of architects or builders has resulted in lamentable catastrophes. The dangers attendant upon reckless building have, in fact, so thoroughly impressed themselves upon civilized communities that their governments, state and municipal, have prepared the most stringent rules and ordinances to enforce safe construction.

2. The purpose of correct structural design is not merely to secure sufficient strength, for, if the supply of material is unlimited, this result may be accomplished by a person with a limited knowledge of the principles of engineering. The purpose is to erect the strongest possible

For notice of copyright, see page immediately following the title page.

structure required by the existing conditions, with a minimum amount of material. The architect or builder who is able to accomplish this saves a considerable expenditure of money, and, at the same time, secures the required strength and stability.

3. To make a successful design for a structure, the engineer must keep in mind the following principles: 1. Every part of the structure must be strong enough to carry the greatest load to which it may ever be subjected. 2. The material must be used in the most economical manner consistent with the conditions demanded and prescribed.

The first principle implies a knowledge of the forces that may act on a building, such as the weight of the materials composing it, the effect of persons or machinery in motion in the building, and the effect of winds and storms on the structure, and also a knowledge of the properties of the available building materials and their ability to resist these forces.

The second principle implies a knowledge of the relative cost of different building materials and their adaptability to different purposes; the available commercial sizes, and the details of the processes by means of which the commercial forms of these materials are prepared for their respective places in the building, should also be known.

The rolling mills make, for example, a set of standard sizes of steel beams which they are always prepared to furnish. Now, if the calculations show that a steel beam, weighing $17\frac{5}{8}$ pounds per foot, is just strong enough to carry the load on a given floor, it would be very poor engineering to specify a beam of that weight, unless it was found that such a beam is a standard commercial size. The extra cost per pound of the unusual size would add much more to the cost of the building than the added weight of material required by the use of the larger commercial size nearest that called for by the calculation. The successful

design does not, then, necessarily imply a structure in which each part is just strong enough for the work required of it, but that, through a careful selection of the available materials, the most economical use of each, consistent with the strength and durability of the structure, is made.

THE ELEMENTS OF MECHANICS.

DEFINITIONS.

4. **Mechanics** is that science which treats of the action of forces upon bodies, and the effects which they produce; it treats of the laws which govern the movement and state of rest of bodies, and shows how these laws may be utilized.

5. A **force** is that which produces or tends to produce or destroy motion.

6. **Motion** is the opposite of rest, and indicates a change of position in relation to some object.

7. **Equilibrium** is the state of a body when at rest; that is, a body is said to be in equilibrium when, through the action and effect of opposing systems of forces upon it, there is no tendency to movement within it.

8. **Statics** is that division of the science of mechanics which treats of the relation between the forces acting on a body at rest or in equilibrium. Statics includes among other things the action of forces on the parts of a building or other similar structure.

9. **Dynamics** is that division of the science of mechanics which treats of the relation between the forces acting on a body in motion.

EFFECTS OF A FORCE.

10. The **effect** of a **force** upon a body may be compared with another force when the three following conditions are fulfilled in regard to both forces:

1. *The point of application, or point at which the force acts upon the body, must be known.*

2. *The direction of the force, or, what is the same thing, the straight line along which the force tends to move the point of application, must be known.*

3. *The magnitude of the force, when compared with a given standard, must be known.*

In this section and the succeeding sections, the unit of force will always be taken as *one pound*, and the magnitudes of all forces will be expressed in pounds.

11. The fundamental principles of the **relations** between **force** and **motion** were first stated by Sir Isaac Newton. They are called "Newton's Three Laws of Motion," and are as follows:

I. *All bodies continue in a state of rest or of uniform motion in a straight line, unless acted upon by some external force that compels a change.*

II. *Every motion or change of motion is proportional to the acting force, and takes place in the direction of the straight line along which the force acts.*

III. *To every action there is always opposed an equal and contrary reaction.*

From the *first law* of *motion*, it is inferred that a body once set in motion by any force, no matter how small, will move forever in a straight line, and always with the same velocity, unless acted upon by some other force which compels a change.

The deduction from the *second law* is that, if two or more forces act upon a body, their final effect upon that body will be in proportion to their magnitude and to the directions in which they act. Thus, if the wind is blowing due west, with a velocity of 50 miles per hour, and a ball is thrown due north, with the same velocity, or 50 miles per hour, the wind will carry the ball just as far west as the force of the throw carried it north, and the combined effect will be to cause it to move northwest. The amount of departure from

due north will be proportional to the force of the wind, and independent of the velocity due to the force of the throw.

In Fig. 1, a ball *e* is supported in a cup, the bottom of which is attached to the lever *o* in such a manner that a movement of *o* will swing the bottom horizontally and allow the ball to drop. Another ball *b* rests in a horizontal groove that is provided with a slit in the bottom. A swinging arm is actuated by the spring *d* in such a manner that, when drawn back as shown and then released, it will strike the lever *o* and the ball *b* at the same time. This gives *b* an impulse in a horizontal direction and swings *o* so as to allow *e* to fall.

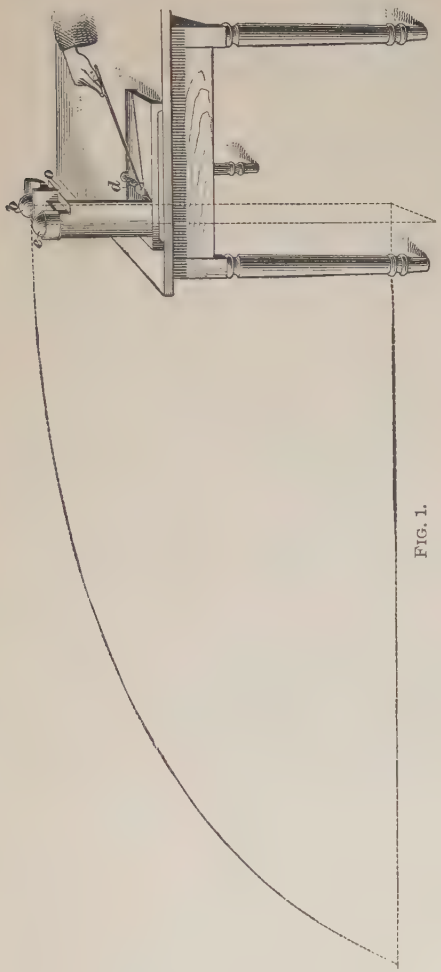


FIG. 1.

On trying the experiment, it is found that *b* follows a path shown by the curved dotted line, and reaches the floor at the same instant as *e*, which drops vertically. This shows that the force which gave the first ball its horizontal movement had no effect on the vertical force which compelled both balls to fall to the floor, the vertical force producing the same effect as if the horizontal force had not acted. The second law may also be stated as follows: *A force has the same effect in*

producing motion, whether it acts upon a body at rest or in motion, and whether it acts alone or with other forces.

The *third law* states that action and reaction are equal and opposite. A man cannot lift himself by his boot straps, for the reason that he presses downwards with the same force that he pulls upwards; the downward reaction equals the upward action, and is opposite to it.

12. A **force** may be represented by a line; thus, in Fig. 2, let *A* be the *point of application* of the force; let the length of the line *A B* represent its *magnitude*, and let the arrowhead indicate the *direction* in which the force acts, then the line *A B* fulfils the three required conditions in regard to point of application, direction, and intensity, and the force is fully represented.

FIG. 2.

THE COMPOSITION OF FORCES.

13. Parallelogram of Forces.—When two forces act upon a body at the same time, but at different angles, their final effect may be obtained as follows:

In Fig. 3, let *A* be the common *point of application* of the two forces, and let *A B* and *A C* represent the *magnitude* and *direction* of the forces. The final effect of the movement due to these two forces will be the same whether they act singly or together. Let, for instance, the line *A B* represent the distance that the force *A B* would cause the body to move; similarly, let *A C* represent the distance which the force *A C* would cause the body to move, when both forces were acting separately. The force *A B*, acting alone, would carry the body to *B*; if the force *A C* were now to act upon the body, it would carry it along the line *B D*, parallel to *A C*, to a point *D*, at a distance from *B* equal to *A C*. Join *C* and *D*, then *C D* is parallel to *A B*

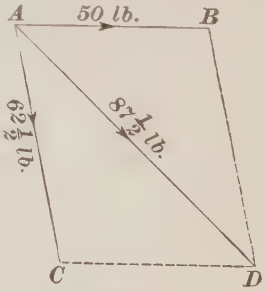


FIG. 3.

and $A\,B\,D\,C$ is a parallelogram. Draw the diagonal $A\,D$. According to the second law of motion, the body will stop at D whether the forces act separately or together, but if they act together, the path of the body will be along $A\,D$, the diagonal of the parallelogram. Moreover, the length of the line $A\,D$ represents the *magnitude* of a force, which, acting at A in the *direction* $A\,D$, would cause the body to move from A to D; in other words, $A\,D$, measured to the same scale as $A\,B$ and $A\,C$, represents the *magnitude* and *direction* of the combined effect of the two forces $A\,B$ and $A\,C$.

The force represented by the line $A\,D$ is called the **resultant** of the forces $A\,B$ and $A\,C$. Suppose that the scale used was 50 pounds to the inch, then, if $A\,B = 50$ pounds, and $A\,C = 62\frac{1}{2}$ pounds, the length of $A\,B$ would be $\frac{50}{50} = 1$ inch, and the length of $A\,C$ would be $\frac{62.5}{50} = 1\frac{1}{4}$ inches. If the line $A\,D$ measures $1\frac{3}{4}$ inches, the *magnitude* of the resultant, which it represents, would be $1\frac{3}{4} \times 50 = 87\frac{1}{2}$ pounds.

Therefore, a force of $87\frac{1}{2}$ pounds, acting upon a body at A, in the direction $A\,D$, will produce the same result as the combined effects of a force of 50 pounds acting in the direction $A\,B$ and a force of $62\frac{1}{2}$ pounds acting in the direction $A\,C$.

14. The above method of finding the resulting action of two forces acting upon a body at a common point, is correct, whatever may be their direction and magnitudes. Hence, to find the **resultant** of two forces when their common point of application, their direction, and magnitudes are known:

Rule I.—*Through an assumed point draw two lines parallel with the direction of the two forces. With any scale, measure from the point of intersection, in the direction of the forces, distances corresponding to the magnitudes of the respective forces, and from the points thus obtained complete the parallelogram. Draw the diagonal of the parallelogram from the point of intersection of the two forces; this diagonal will represent the resultant, and its direction will be away from the point of intersection of the two forces. To find the*

magnitude of the resultant, the diagonal must be measured with the same scale that is used to lay off the two forces.

This method is called the **graphical method** of the **parallelogram of forces.**

Experimental Proof.—The principle of the parallelogram of forces is clearly shown in Fig. 4. *A B D C* is a wooden frame, jointed to allow motion at its four corners. The length *A B* is equal to the length *C D*; also, *A C* is equal to *B D*, and the corresponding adjacent sides are in the ratio

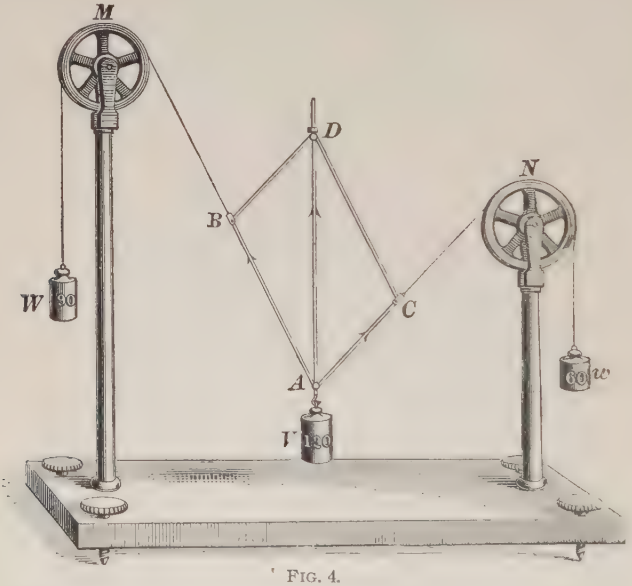


Fig. 4.

of 2 to 3. Cords pass over the pulleys *M* and *N*, carrying weights *W* and *w*, of 90 and 60 pounds, respectively. The ratio between the weights is equal to the ratio of the corresponding adjacent sides. A weight *V*, of 120 pounds, is hung from the corner *A*.

When the frame comes to rest, the sides *A B* and *A C* lie in the direction of the cords. These sides *A B* and *A C* are accurate graphic representations of the two forces acting upon the point *A*. It will be found that the diagonal *A D* is vertical, and twice as long as *A C*; hence, since *A C*

represents a force of 60 pounds, $A\,D$ will represent a force of 2×60, or 120 pounds.

Thus, we see that the line $A\,D$ represents the *resultant* of the two forces $A\,B$ and $A\,C$; in other words, it represents the resultant of the two weights W and w. This resultant is equal and opposite to the vertical force, which is due to the weight of V.

Satisfactory results of this kind will be secured when we have the proportion,

$$A\,B : A\,C = W : w.$$

EXAMPLE.—If two forces act upon a body at a common point, both acting away from the body, and the angle between them is 80°, what is the value of the resultant, the magnitude of the two forces being 60 pounds and 90 pounds, respectively.

SOLUTION.—Draw two indefinite lines having an angle of 80° between them (Fig. 5). With any convenient scale, say 10 pounds to the inch, measure off $A\,B = 60 \div 10 = 6$ inches, and $A\,C = 90 \div 10 = 9$ inches.

Through B draw $B\,D$ parallel to $A\,C$, and through C draw $C\,D$ parallel to $A\,B$, intersecting at D. Then draw $A\,D$, and $A\,D$ will be the *resultant;* its *direction* is towards the point D, as shown by the arrow.

Measuring $A\,D$, we find that its length is 11.7 inches. Hence, the magnitude of the resultant is $11.7 \times 10 = 117$ pounds. Ans.

15. Triangle of Forces.—The above example might also have been solved by the method called the **triangle of forces,** which is as follows:

In Fig. 5, suppose that the two forces acted separately, first from A to B, and then from B to D, in the direction of the arrows.

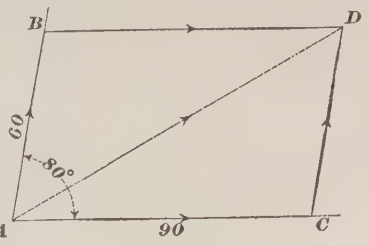


FIG. 5.

Draw $A\,D$; then $A\,D$ is the *resultant* of the forces $A\,B$ and $A\,C$, since $B\,D = A\,C$; but $A\,D$ is a side of the triangle $A\,B\,D$. It will also be noticed that the direction of $A\,D$ is *opposed* to that of $A\,B$ and $B\,D$; hence, to find the **resultant** of two forces acting upon a body at a common point, by the method of triangle of forces:

Rule II.—*Through any point, draw a line to represent one of the forces in magnitude and direction. At the extremity of this line draw a second line parallel to the line of action of the other force and representing this force in magnitude and direction. Join the extremities of the two lines by a straight line; then this joining line will represent the resultant, and its direction will be opposite to that of the two forces.*

When we speak of the resultant being opposed in direction to the other two forces, we mean that, starting from the point where we began to draw the triangle, and tracing each line in succession, the pencil will have the same general direction around the triangle as if passing around a circle, from left to right, or from right to left, but the closing line or resultant must have an *opposite direction;* that is, *the two arrowheads, the one on the resultant and the other on the last side, must point towards the intersection of the resultant and the last side.*

16. Resultant of Several Forces.—When three or more forces act upon a body at a given point, their *resultant* may be found by the following rule:

Rule III.—*Find the resultant of any two forces; treat this resultant as a single force, and combine it with a third force to find a second resultant. Combine this second resultant with a fourth force, to find a third resultant, etc. After all the forces have been thus combined, the last resultant will be the resultant of all the forces, both in magnitude and direction.*

EXAMPLE.—Find the resultant of all the forces acting on the point O in Fig. 6, the length of the lines being proportional to the magnitude of the forces.

SOLUTION.—Draw OE parallel and equal to AO, and EF parallel and equal to BO; then OF is the resultant of these two forces, and its direction is from O to F, opposed to OE and EF. Consider OF as replacing OE and EF, and draw FG parallel and equal to OC; OG will be the resultant of OF and FG; but OF is the resultant of OE and EF; hence, OG is the resultant of OE, EF, and FG, and likewise of AO, BO, and CO. The line FG, parallel to CO, could not

be drawn from the point O to the right of $O\,E$, for in that case it would be opposed in direction to $O\,F$; but $F\,G$ must have the same direction as $O\,F$, in order that the resultant may be opposed to $O\,F$ and $F\,G$.

For the same reason, draw $G\,L$ parallel and equal to $D\,O$. Join O and L, and $O\,L$ will be the *resultant* of all the forces $A\,O$, $B\,O$, $C\,O$, and $D\,O$ (both in magnitude and direction) acting at the point O. If $L'\,O$ be drawn parallel and equal to $O\,L$, and having the same direction, it would represent the effect produced on the body by the combined

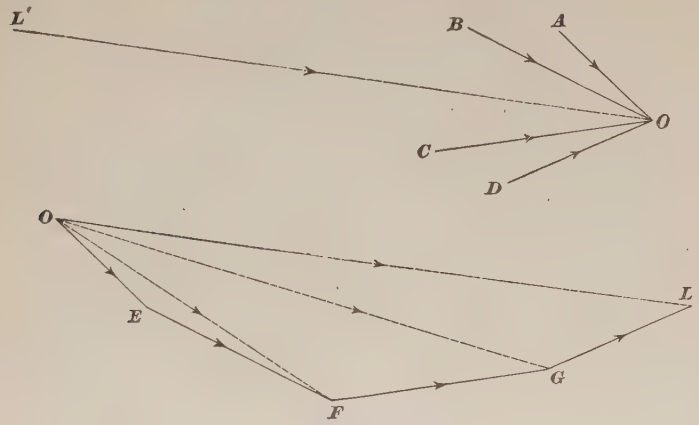


FIG. 6.

action of the forces $A\,O$, $B\,O$, $C\,O$, and $D\,O$. In this solution we have, for brevity, spoken of the forces $A\,O$, $B\,O$, etc., and the resultants $O\,F$, $O\,G$, and $O\,L$. It must be remembered, however, that these are merely lines that *represent* the forces in magnitude and direction.

17. In the last figure, it will be noticed that $O\,E$, $E\,F$, $F\,G$, $G\,L$, and $L\,O$ are sides of a polygon $O\,E\,F\,G\,L$, in which $O\,L$, the resultant, is the closing side, and that its direction is opposed to that of all the other sides. This fact is made use of in what is called the **method** of the **polygon of forces.**

To find the resultant of several forces acting upon a body at the same point:

Rule IV.—*Let the several forces be represented in direction and magnitude by lines, as explained in Art.* **12.** *Through any point draw a line parallel to one of the forces, and having the same direction and magnitude. At the end of this line, draw another line, parallel to a second force, and*

having the same direction and magnitude as this second force; at the end of the second line, draw a line parallel and equal in magnitude and direction to a third force. Thus continue until lines have been drawn equal in magnitude, and having the same directions, respectively, as the lines representing the several forces.

The straight line joining the free ends of the first and last lines will be the closing side of the polygon; mark its direction opposite to that of the other forces around the polygon, and it will represent in magnitude and direction the resultant of all the forces.

EXAMPLE.—If five forces act upon a body at angles of 60°, 120°, 180°, 240°, and 270°, towards the same point, and their respective magnitudes are 60, 40, 30, 25, and 20 pounds, find the magnitude and direction of their resultant by the method of polygon of forces.*

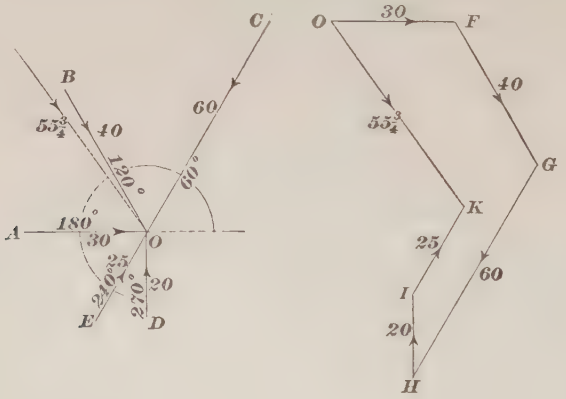


FIG. 7.

SOLUTION.—From a common point O, Fig. 7, draw the lines of action of the forces, making the given angles with a horizontal line through O, and mark them as acting towards O, by means of arrowheads, as shown. Now, choose some convenient scale, such that the whole figure may be drawn in a space of the required size on the drawing. Choose any one of the forces, as $A\,O$, and draw $O\,F$ parallel to it, and equal in length to 30 pounds on the scale. It must also act in the same direction as $A\,O$. From F, draw $F\,G$ parallel to $B\,O$, and make its length equal to 40 pounds on the assumed scale. In a similar manner,

* All the angles in the figure are measured from a horizontal line in a direction opposite to the movement of the hands of a watch, from 1° up to 360°.

draw GH, HI, and IK parallel to CO, DO, and EO, and equal on the scale to 60, 20, and 25 pounds, respectively. Join O and K by OK; then OK will represent in magnitude and direction the resultant of the combined action of the five forces. The direction of the resultant is opposed to that of the other forces around the polygon $OFGHIK$, and its magnitude is $55\frac{3}{4}$ pounds. Ans.

18. If the resultant OK, in Fig. 7, were to act alone upon the body in the direction shown by the arrowhead with a force of $55\frac{3}{4}$ pounds, it would produce exactly the same effect as the combined action of the five forces.

If OF, FG, GH, HI, and IK represent the distances and directions that the forces would move the body, if acting separately, OK is the direction and distance of movement of the body when all the forces act together.

From what has been said before, it is evident that any number of forces acting on a body at the same point, or having their lines of action pass through the same point, can be replaced by a *single force* (resultant), whose line of action shall pass through that point.

Heretofore, it has been assumed that the forces acted upon a single point on the *surface* of the body, but it will make no difference where they act, so long as the lines of action of all the forces intersect at a *single point* either within or without the body, only so that the resultant can be drawn through the *point of intersection.* If two forces act upon a body in the same straight line and in the same direction, their *resultant* is the *sum* of the two forces; but if they act in opposite directions, their *resultant* is the *difference* of the two forces, and its direction is the same as that of the greater force. If they are equal and opposite, the resultant is *zero*, or one force just balances the other.

EXAMPLE.—Find the resultant of the forces whose lines of action pass through a single point, as shown in Fig. 8.

SOLUTION.—Take any convenient point g, and draw a line gf, parallel to one of the forces, say the one marked *40*, making it equal in length to 40 pounds on the scale, and indicate its direction by the arrowhead. Take some other force—the one marked *37* will be convenient; the line fe represents this force. From the point e draw a line parallel to some other force; say, the one marked *29*, and make it

equal in magnitude and direction to that force. So continue with the other forces, taking care that the general direction around the polygon is not changed. The last force drawn in the figure is *b a*, representing the force marked *25*. Join the points *g* and *a*; then, *g a* represents the resultant of all the forces shown in the figure. Its direction is from *g*

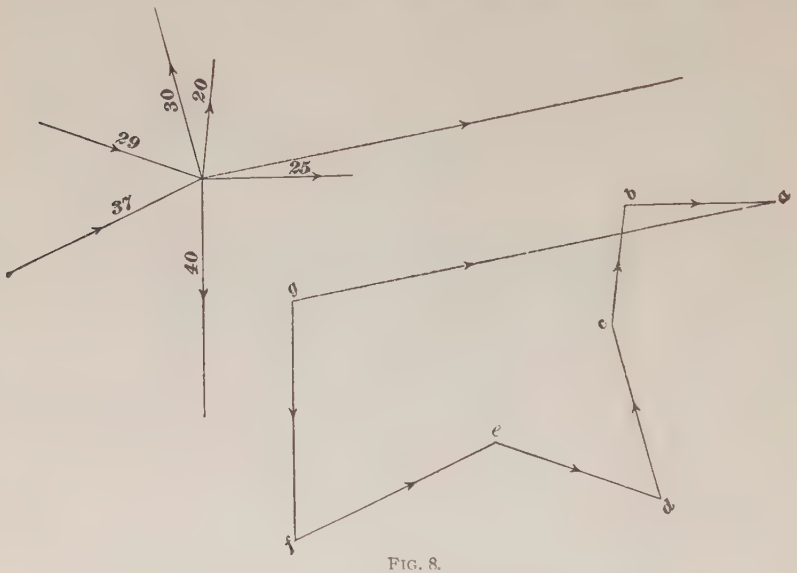


FIG. 8.

to *a*, opposed to the general direction of the others around the polygon. It does not matter in what order the different forces are taken, the resultant will be the same in magnitude and direction, if the work is done correctly.

The various methods of finding the resultant of several forces are all grouped under one head: the **composition of forces.**

THE RESOLUTION OF FORCES.

19. Since two forces can be combined to form a single resultant force, we may also treat a single force as if it were the resultant of two forces whose action upon a body will be the same as that of a single force. Thus, in Fig. 9, the force *O A* may be resolved into two forces, *O B* and *B A*.

If the force *O A* acts upon a body, moving or at rest upon

a horizontal plane, and the resolved force OB is vertical, and BA horizontal, OB, measured to the same scale as OA, is the magnitude of that part of OA which pushes the body *downwards*, while BA is the magnitude of that part of the force OA which is exerted in pushing the body in a *horizontal* direction. OB and BA are called the **components** of the force OA, and when these components are vertical and horizontal, as in the present case, they are called, respectively, the *vertical component* and the *horizontal component* of the force OA.

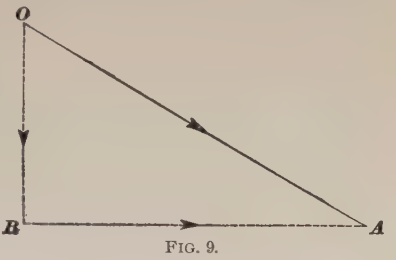


FIG. 9.

20. It frequently happens that the position, magnitude, and direction of a certain force are known, and that it is desired to know the effect of the force in some direction other than that in which it acts. Thus, in Fig. 9, suppose that OA represents, to some scale, the magnitude, direction, and line of action of a force acting upon a body at A, and that it is desired to know what effect OA produces in the direction BA. Now, BA, instead of being horizontal, as in the illustration, may have any direction. To find the value of the component of OA which acts in the direction BA, we employ the following rule:

Rule.—*From one extremity of the line representing the given force draw a line parallel to the direction in which it is desired that the component shall act; from the other extremity of the given force, draw a line perpendicular to the component first drawn, and intersecting it. The length of the component, measured from the point of intersection to the intersection of the component with the given force, will give the magnitude of the effect produced by the given force in the required direction.*

Thus, suppose OA, Fig. 9, represents a force acting upon a body resting upon a horizontal plane, and it is desired to know what *vertical pressure* OA produces on the body.

Here the desired direction is vertical; hence, from one extremity, as O, draw $O B$ parallel to the desired direction (vertical in this case), and from the other extremity draw $A B$ perpendicular to $O B$, and intersecting $O B$ at B. Then $O B$, when measured to the same scale as $O A$, will give the magnitude of the vertical pressure produced by $O A$.

EXAMPLE.—If a body weighing 200 pounds rests upon an inclined plane whose angle of inclination to the horizontal is 18°, what force does it exert perpendicular to the plane, and what force does it exert parallel to the plane, tending to slide downwards?

SOLUTION.—Let $A B C$, Fig. 10, be the plane, the angle A being 18°, and let W be the weight. Draw a vertical line $F D = 200$ pounds, to represent the magnitude of the weight. Through F draw $F E$ parallel to $A B$, and through D draw $D E$ perpendicular to $E F$, the two lines intersecting at E. $F D$ is now resolved into two components, one $F E$ tending to pull the weight down the incline, and the other $E D$ acting as a perpendicular pressure on the plane.

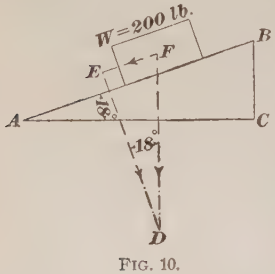


FIG. 10.

Upon measuring $F E$ with the same scale by which the weight $F D$ was laid off, it is found to be about 61.8 pounds, and the perpendicular pressure $E D$ on the plane is found to measure 190.2 pounds. Ans.

EQUILIBRIUM.

21. When a body is at rest, the forces which act upon it must *balance* one another; the forces are then said to be in **equilibrium.** The most important of the forces is gravity.

22. A body is in **stable equilibrium** when, if slightly displaced from its position of rest, the forces acting upon it *tend to return it to that position.* For example, a cube, a cone resting on its base, a pendulum, etc.

23. A body acted on by a system of forces is in **unstable equilibrium** when the application of a small force is sufficient to produce motion. A cone standing upon its apex, an egg balanced on end are examples of bodies in unstable equilibrium.

24. A force acting on a body tends to produce motion in two ways:

1. It tends to move the body in the direction of the line of action of the force.

2. If a point in the body, not in the line of action of the force, is fixed, the force tends to turn the body around that point.

25. Conditions of Equilibrium.—Since a force acting on a body tends to produce motion in two ways, the following conditions must be fulfilled in order that a body be in equilibrium:

1. The resultant of all the forces tending to move the body in any direction must be zero.

2. The resultant of all the forces tending to turn the body about any point must be zero.

MOMENTS OF FORCES.

26. In Fig. 11, W is a weight which acts downwards with a force of 10,000 pounds. If we take some fixed point as a, not in the line along which the weight W acts, and connect the point a with the line of action of W by a rigid arm, so that W pulls on one end of this arm, while the other end is firmly held at a, our daily experience teaches us that the pull of W tends to turn or rotate the arm around the point.

Experience also teaches that the tendency to rotation is directly proportional to the magnitude of the force, provided that the arm remains at the same length, and directly proportional to the length of the arm, if the force remains constant. In general, therefore, the rotative effect is proportional to the product of the magnitude of the force multiplied by the length of the lever arm. This product is called the **moment** of the force with respect

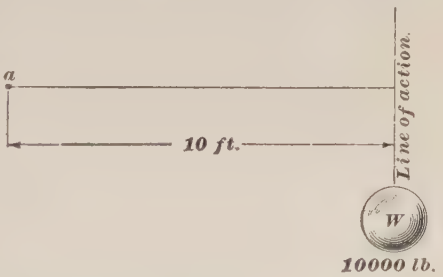


FIG. 11.

to the point in question. Thus, in Fig. 11, the moment of the force W with respect to the point a, is the product obtained by multiplying the magnitude, 10,000 pounds, by the perpendicular distance, 10 feet, from the point a to the line of action of W.

27. The point a which is assumed as the center around which there is a tendency to rotate, is called the **center, or origin, of moments.**

28. The perpendicular distance from the center of moments to the line along which the force acts, is the **lever arm** of the force, also called the **leverage** of the force.

29. Since the unit of force is the *pound*, and the ordinary unit of length is the foot, the unit of *moment* will be a derived unit, the **foot-pound,** and moments will usually be expressed in foot-pounds. In Fig. 11, for example, the moment of the force W with respect to the point a is $10,000 \times 10 = 100,000$ foot-pounds.

EXAMPLE.—What is the moment of the force of 10,000 pounds whose line of action is $a\,b$, Fig. 12, the center of the moments being at d?

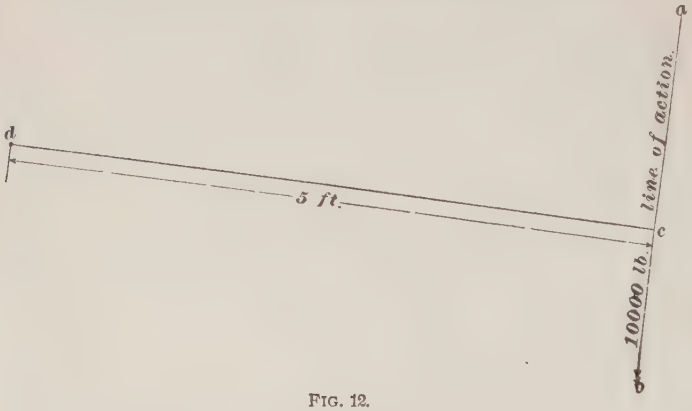


FIG. 12.

SOLUTION.—The perpendicular distance $c\,d$ from the line of action of the force to the center of moments being 5 feet, and the magnitude of the force 10,000 pounds, the moment is $10,000 \times 5 = 50,000$ foot-pounds. Ans.

30. In Fig. 13 the line of action of the force of 10,000 pounds passes directly through the point d; consequently,

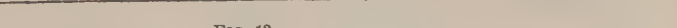


FIG. 13.

the perpendicular distance from the line of action to the point d is *zero*, and there is no tendency to rotate around that point; therefore, there is no motion.

31. The moment of a force may be expressed in *inch-pounds*, *foot-pounds*, or *foot-tons*, depending upon the unit of measurement used to designate the magnitude of the force and the length of its lever arm. For instance, if the magnitude of a force is measured in pounds, and the lever arm through which it acts in inches, the moment will be in *inch-pounds;* again, if a force of 10 tons acts through a lever arm of 20 feet, the moment of the force is $10 \times 20 = 200$ *foot-tons.*

EXAMPLE.—What is the moment in inch-pounds of a force of 8,000 pounds, if the length of the lever arm is 13 feet?

SOLUTION.—Since the moment is to be in inch-pounds, the length of the lever arm must be in inches. 13 feet $= 13 \times 12 = 156$ inches, and the moment is $8{,}000 \times 156 = 1{,}248{,}000$ inch-pounds. Ans.

32. Equilibrium of Moments.—In Art. **21** it was stated that when a body is at rest, all the forces acting on it balance one another; this condition is expressed by saying: *the forces are in equilibrium.* That there may be perfect balance among the forces, it is necessary that there be not only no unbalanced force tending to move the body along some given line, but that there be, also, no unbalanced moment, the effect of which would turn the body about some point.

In Fig. 14 we have a beam, or lever, resting on the support c; a force b of 5 pounds acts downwards at the right-hand end of this lever, and tends to turn it around the point of support c, in the direction traveled by the hands of a clock, that is, to produce **right-hand** rotation. The measure of this tendency is $5 \times 10 = 50$ foot-pounds.

Another force *a* acts downwards on the left-hand end of the lever, its tendency being to produce **left-hand** rotation, or to turn the lever in the direction opposite to that traveled by the hands of a clock. Since the force *a* is 10 pounds, and it acts with a lever arm of 5 feet, its moment is $10 \times 5 = 50$ foot-pounds, the same as the moment of the force *b*. We thus have two equal moments, one tending to turn the lever to the right, and the other to the left; as a result, the effect

FIG. 14.

of one is neutralized by the effect of the other, and the second condition of equilibrium is fulfilled. This condition is expressed by saying: there is *equilibrium of moments.*

33. Positive and Negative Moments.—We may distinguish between the directions in which there is a tendency to produce rotation by the use of the signs + and −. Thus, if a force tends to produce right-hand rotation, its moment may be called positive and have the plus sign, while a force that tends to produce rotation in the opposite direction is called negative, and its moment have the minus sign. That there may be equilibrium of moments, the above considerations show that the difference between the sum of the positive moments and the sum of the negative moments must be zero; this difference is called the **algebraic sum** of the moments. We therefore have the principle: *In order that there may be equilibrium, the algebraic sum of the moments of all the forces acting on a body must be zero.*

34. Resultant Moments.—In Fig. 15 is shown a lever composed of two arms at right angles to each other, and free to turn about the center *c*. A force *a* acts on the horizontal arm in such a manner that it tends to produce left-hand

rotation, its moment being $10 \times 5 = 50$ foot-pounds, which, since it tends to produce left-hand rotation, shall be called negative. Another force *b*, whose moment with respect to the center *c* is $12 \times 3 = 36$ foot-pounds, tends to produce right-hand rotation. Considering the effect of these two forces only, we see that, to secure equilibrium, there must be another force acting in such a manner as to overcome the difference between the moments of the two; that is, it must

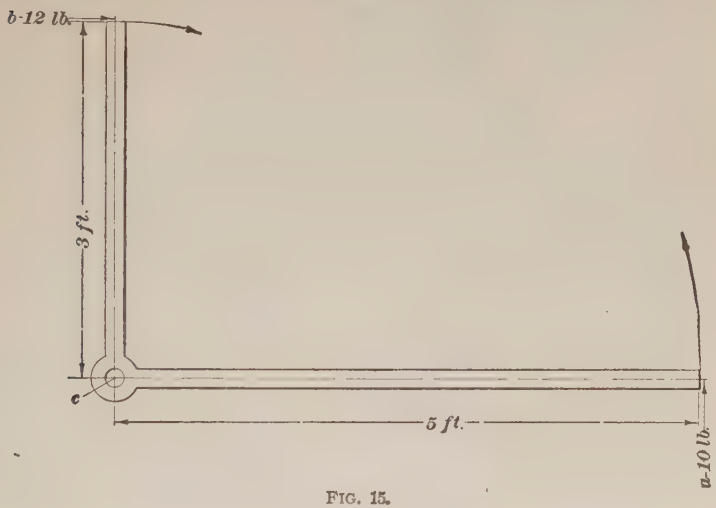


FIG. 15.

tend to produce right-hand rotation with a moment equal to $-50+36 = -14$ foot-pounds. This moment is called the *resultant moment* of the two forces *a* and *b*.

If the length of the lever arm of the force which acts to produce the resultant moment is known, the magnitude of the force may readily be found. Thus, in the present case, the resultant moment is -14 foot-pounds; let it be required to find the force to produce equilibrium, when acting with a lever arm 7 feet long. Since the moment is the product of the force multiplied by its lever arm, it follows that the required force may be found by dividing the given moment by the length of the lever arm; consequently, the required force is $14 \div 7 = 2$ pounds.

If, instead of the two forces just considered, we have a body acted on by any number of forces whose moments about a given center are known, the resultant moment of these forces, that is, the moment of the force required to produce equilibrium, is the algebraic sum of the moments of the given forces; and, further, if the length of the lever arm of the resultant moment is known, the magnitude of the required force can be found by dividing the moment by the length of the lever arm.

35. The above principles may be expressed as follows:

Rule.—*To find the force required to produce equilibrium of moments, when the moments of any number of given forces and the lever arm of the required force are given, divide the algebraic sum of the given moments by the length of the given lever arm. If the algebraic sum is positive, the tendency of the required force is to produce left-hand rotation; if negative, the tendency of the force is to produce right-hand rotation.*

EXAMPLE.—In Fig. 16 we have a system of forces, shown by the arrows, acting in various directions and at various distances from the center O. The force F' is 25 pounds and its lever arm $O\,p'$ is 8 feet, F'' is 16 pounds with a lever arm $O\,p''$ of 12 feet, F''' is 40 pounds with a lever arm $O\,p'''$ of 6 feet, and the force F^{v} is 100 pounds, acting directly through the center O. If the distance $O\,p^{iv}$ is 12 feet, what must be the magnitude of the force F^{iv} in order to produce equilibrium of moments?

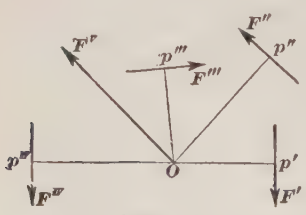


FIG. 16.

SOLUTION.—As shown by the arrows, the forces tending to produce right-hand rotation are F' and F''', and their moments, called positive, are, respectively, $25 \times 8 = +200$ foot-pounds, and $40 \times 6 = +240$ foot-pounds. The lever arm of the force F^{v} is zero; consequently, it has no moment with respect to the center O. The force F'' tends to produce left-hand rotation, and its moment is $16 \times 12 = -192$ foot-pounds. The algebraic sum of the moments of the given forces is $+200 + 240 - 192 = +248$ foot-pounds; therefore, according to the rule, the force F^{iv} must be $248 \div 12 = 20\frac{2}{3}$ pounds,

which, since the algebraic sum of the given moments is positive, must tend to produce left-hand rotation, as shown by the arrow. Ans.

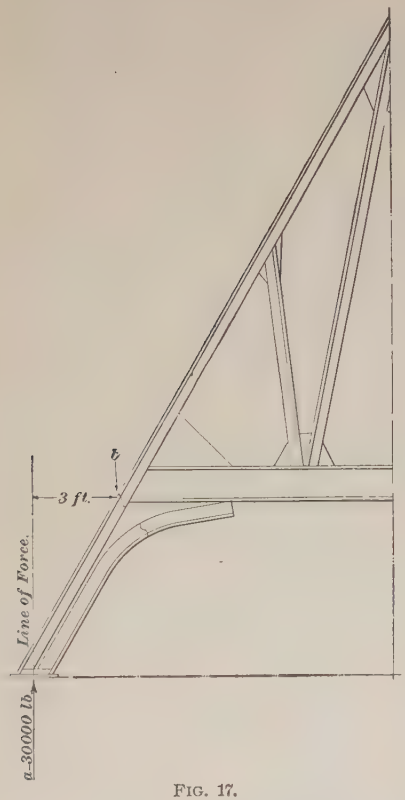


Fig. 17.

36. The principles involved in the theory of moments are among the most simple in mechanics, and at the same time of the greatest practical importance in the solution of problems relating to the strength of beams, girders, and trusses.

Example.—In Fig. 17, the lower tie member in the roof truss has been raised to get a vaulted-ceiling effect in the upper story of the building, which the truss covers. The weight transmitted through this member to the pier wall is 30,000 pounds; there is, consequently, an equal upward force due to the reaction of the wall. This force of 30,000 pounds tends to break the truss by producing rotation about the point b. What is its moment around the point b?

Solution.—Since the perpendicular distance from the line of action of the force is 3 feet, the moment of the force a around the point b is $30{,}000 \times 3 = 90{,}000$ foot-pounds. Ans.

THE LEVER.

37. A **lever** is a bar capable of being turned about a pin, pivot, or point, as in Figs. 18, 19, and 20.

The object W to be lifted is called the **weight**; the force P used is called the **power**; and the point or pivot F is called the **fulcrum.**

That part of the lever between the weight and the fulcrum, or Fb, is called the **weight arm,** and the part

between the power and the fulcrum, or Fc, is called the **power arm.**

Take the fulcrum, or point F, as the center of moments; then, in order that the lever shall be in equilibrium, the moment of P about F, or $P \times Fc$, must be equal to the

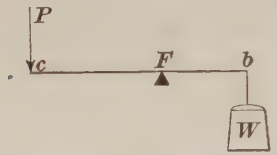


FIG. 18.

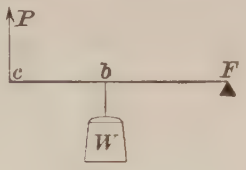


FIG. 19.

moment of W about F, or $W \times Fb$. That is, $P \times Fc = W \times Fb$, or, in other words, *the product of the power and the power arm is equal to the product of the weight and the weight arm.*

If F be taken as a center, and arcs be described through b and c, it will be seen that, if the weight arm is moved through a certain angle, the power arm will move through the same angle; also, that the distance that W moves will be proportional to the distance that P moves. From this it is seen that the power arm is proportional to the distance through which the power moves, and the weight arm is proportional to the distance through which the weight moves.

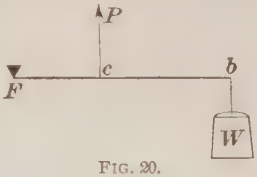


FIG. 20.

Hence, instead of writing $P \times Fc = W \times Fb$, we might have written it $P \times$ distance through which P moves $= W \times$ distance through which W moves. This is the general law of all machines, and can be applied to any mechanism, from the simple lever up to the most complicated arrangement. Stated in the form of a rule it is as follows:

Rule.—*The power multiplied by the distance through which it moves is equal to the weight multiplied by the distance through which it moves.*

EXAMPLE.—If the weight arm of a lever is 6 inches long and the power arm is 4 feet long, how great a weight can be raised by a force of 20 pounds at the end of the power arm?

SOLUTION.— 4 feet = 48 inches. Hence, $20 \times 48 = W \times 6$, or $W = 160$ pounds. Ans.

EXAMPLE.—(*a*) What is the ratio between the power and the weight in the last example? (*b*) In the last example, if P moves 24 inches, how far does W move? (*c*) What is the ratio between the two distances?

SOLUTION.—(*a*) $20 : 160 = 1 : 8$; that is, the weight moved is 8 times the power. Ans.

(*b*) $20 \times 24 = 160 \times x$. $x = \frac{480}{160} = 3$ inches, the distance that W moves. Ans.

(*c*) $3 : 24 = 1 : 8$, or the ratio is $1 : 8$. Ans.

The law which governs the straight lever also governs the bent lever; but care must be taken to determine the

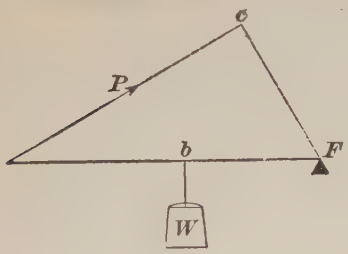


FIG. 21.

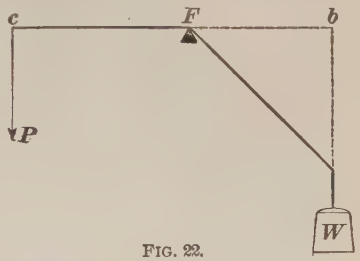


FIG. 22.

true lengths of the lever arms, which are in every case *the perpendicular distances from the fulcrum to the line of direction of the weight or power.*

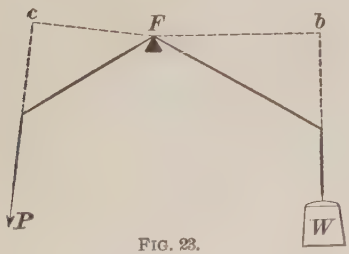


FIG. 23.

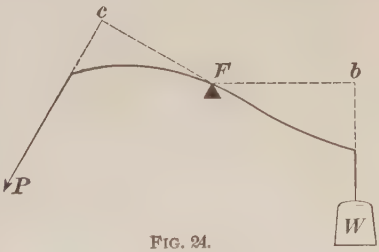


FIG. 24.

Thus, in Figs. 21, 22, 23, and 24, Fc in each case represents the power arm, and Fb the weight arm.

EXAMPLES FOR PRACTICE.

1. A lever arm has a length of 10 feet; the load acting upon the end of the lever is 6,000 pounds. What is the moment of this load in inch-pounds? Ans. 720,000.

2. A piece of timber 20 feet long is balanced at a point 8 feet from one end, the load at this end being 9,000 pounds. What is the load at the other end? Ans. 6,000 lb.

3. The one support of a beam 20 feet long is 8 feet from the left-hand end; at this end is a load of 25 pounds; at the right of the support, 3 feet distant, is a load of 5 pounds; and at 7 feet to the right is a load of 10 pounds. What load, and at which end should it be placed to produce balance, or equilibrium, in the beam?

Ans. 9.58 lb. at right-hand end.

4. A steel I beam, which extends 6 feet outside of the center of a building wall, and 3 feet inside, is required to support a load upon the outside end of 4,000 pounds. What load on the inner end will keep the beam from tilting? Ans. 8,000 lb.

CENTER OF GRAVITY.

38. The **center of gravity** of a body, or of a system of bodies, is that point from which, if the body or system were suspended, it would be in equilibrium. If the body or system were suspended from any other point than the center of gravity, and in such a manner as to be free to turn about the point of suspension, the body would rotate until the center of gravity reached a position directly under the point of suspension.

39. Center of Gravity of Plane Figures.—If a plane figure has an *axis of symmetry*, this axis passes through its center of gravity. If the figure has *two* axes of symmetry, its center of gravity is at their point of intersection.

The center of gravity of a *triangle* lies on a line drawn from a vertex to the middle point of the opposite side, and at a distance from that side equal to $\frac{1}{3}$ of the length of the line. Or, draw a line from another vertex to the middle point of the side opposite, and the intersection of the two lines will be the center of gravity.

The perpendicular distance of the center of gravity of a triangle from the base is equal to $\frac{1}{3}$ of the altitude.

The center of gravity of a *parallelogram* is at the intersection of its two diagonals.

The center of gravity of an *irregular four-sided figure* may be found by first dividing it by a diagonal into two triangles and joining their centers of gravity by a straight line;

then, by means of the other diagonal, divide it into two other triangles, and join their centers of gravity by another straight line; the center of gravity of the figure is at the intersection of the lines joining the centers of gravity of the two sets of triangles.

The distance of the center of gravity of the *surface of a half circle* from the center is equal to the product of the radius multiplied by .424.

LOADS CARRIED BY STRUCTURES.

DEAD LOAD.

WEIGHT OF BUILDING MATERIALS.

40. The materials used in building construction have considerable weight. Every piece of iron, timber, masonry, and brickwork has a tendency to move towards the center of the earth, and this tendency of the parts of the structure to fall to the ground must be resisted by the strength of the various members composing it. The portion of this force to be resisted by any member of the structure is called the **dead load**; it is the sum of the weights of every piece of material in the building which must be supported by that member.

Before the dead load can be computed, the weight of various building materials must be known. The following tables give the weights of building materials in common use. The units in which these weights are expressed are those most often used to make estimates of loads in engineering calculations. Thus, Table 1 gives the weight per cubic foot of the materials usually measured by that unit, together with the weight per cubic inch of a few often measured in inches; while Table 2 gives the weights of such materials as are used in the construction of floors, roofs, ceilings, etc., where the quantities are generally expressed in square feet. While it is not necessary for the student to memorize all of Table 1, it is well for him to keep in mind the weights of the materials printed in Italic.

TABLE 1.

WEIGHT OF BUILDING MATERIALS.

Name of Material.	Average Weight in Pounds.	
	Per Cu. In.	Per Cu. Ft.
Aluminum	.096	166
Asphalt pavement composition		130
Bluestone		160
Brass	.302	523
Brickwork, in lime mortar		120
Brickwork, in cement mortar		130
Bronze	.319	552
Cement, Portland		80 to 100
Cement, Rosendale		56 to 60
Concrete, in cement		140
Copper, cast	.319	550
Earth, dry and loose		72 to 80
Earth, dry and moderately rammed		90 to 100
Gneiss, common		168
Gneiss, in loose piles		96
Gravel		117 to 125
Iron, cast	.26	450
Iron, wrought	.277	480
Lead, commercial cast	.412	712
Limestone		170
Marble		164
Masonry, granite or limestone		165
Masonry, granite or limestone rubble		150
Masonry, granite or limestone dry rubble		138
Masonry, sandstone		145
Mortar, hardened		90 to 100
Quartz, common pure		165
Sand, pure quartz, dry		90 to 106
Sandstone, building, dry		144 to 151
Slate		160 to 180
Snow, fresh fallen		5 to 12
Steel, structural	.283	490
Terra cotta		110
Terra-cotta masonry work		112
Tile		110 to 120

TABLE 2.

WEIGHT OF BUILDING MATERIALS.

Name of Material.	Average Weight per Square Foot in Pounds.
Corrugated galvanized iron No. 20, unboarded	$2\frac{1}{4}$
Copper, 16 oz., standing seam	$1\frac{1}{4}$
Felt and asphalt, without sheathing	2
Glass, $\frac{1}{8}$ inch thick	$1\frac{3}{4}$
Hemlock sheathing, 1 inch thick	$2\frac{1}{2}$
Lead, about $\frac{1}{8}$ of an inch thick	6 to 8
Lath and plaster ceiling (ordinary)	6 to 8
Mackite, 1 inch thick, with plaster	10
Neponset roofing felt, 2 layers	$\frac{1}{2}$
Spruce sheathing, 1 inch thick	2
Slate, $\frac{3}{16}$ inch thick, 3 inches double lap	$6\frac{3}{4}$
Slate, $\frac{1}{8}$ inch thick, 3 inches double lap	$4\frac{1}{2}$
Shingles, $6'' \times 18''$, $\frac{1}{3}$ to weather	2
Skylight of glass, $\frac{3}{16}$ inch to $\frac{1}{4}$ inch, including frame	4 to 10
Slag roof, 4-ply	4
Tin, IX	$\frac{3}{4}$
Tiles, $10\frac{1}{2}'' \times 6\frac{1}{4}'' \times \frac{5}{8}''$; $5\frac{1}{4}''$ to weather (plain)	18
Tiles, $14\frac{1}{2}'' \times 10\frac{1}{2}''$; $7\frac{1}{4}''$ to weather (Spanish)	$8\frac{1}{2}$
White-pine sheathing, 1 inch thick	$2\frac{1}{2}$
Yellow-pine sheathing, 1 inch thick	4

41. In obtaining the dead load upon roof trusses, it is necessary, after having found the weight of the sheathing and roofing, to add a certain weight per square foot, to represent the weight of the truss or members supporting the roof. Not knowing, as yet, the size and weight of the different members in the roof truss, we must assume approximate weights.

Table 3 gives the approximate weights of the trusses, or principals, as they are called, for roofs of different spans.

These weights are, of course, only assumed, and may not be within 25 per cent. of the actual weight of the principals. They are, however, generally on the side of safety.

TABLE 3.

POUNDS TO BE ADDED FOR THE WEIGHT OF THE PRINCIPALS, OR ROOF TRUSSES.

Spans up to 40 feet, 4 pounds per sq. ft. of area covered.
Spans 40 to 60 feet, 5 pounds per sq. ft. of area covered.
Spans 60 to 80 feet, 6 pounds per sq. ft. of area covered.
Spans 80 to 100 feet, 7 pounds per sq. ft. of area covered.

It is required, in the application of Table 3, to obtain the weight in pounds per square foot of *roof surface*. As the weights given in the table are in pounds per square foot of *area covered*, and as the area of the roof is considerably greater than this, owing to the pitch of the roof, it is neces-

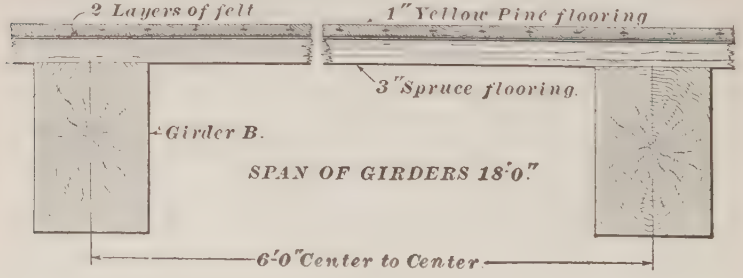


FIG. 25.

sary to divide the area covered by the area of the roof and multiply the result by the quantities given in the table. For example, the area of a building covered by a roof with a span of 50 feet is 10,000 square feet, and the area of the roof is 15,000 square feet; $10{,}000 \div 15{,}000 = \frac{2}{3}$, or .67, and, since the span of the roof is 50 feet, according to Table 3, the weight of the truss is 5 pounds for each square foot of area covered. Therefore, $5 \times .67 = 3.35$ pounds are to be added to the weight of each square foot of roof surface.

EXAMPLE.—In Fig. 25, what is the total dead load on the girder B?

SOLUTION.—

Yellow-pine flooring, 1 inch thick	=	4 lb. per sq. ft.
2 layers of felt	=	$\frac{1}{2}$ lb. per sq. ft.
Rough spruce flooring, 3 inches thick	=	6 lb. per sq. ft.
Assume the weight of the girder	=	8 lb. per sq. ft.
Total dead load of floor surface	=	$18\frac{1}{2}$ lb. per sq. ft.

The area of the floor carried by the girder is $6 \times 18 = 108$ square feet. Then $108 \times 18\frac{1}{2} = 1{,}998$ pounds is the entire dead load upon the girder B. Ans.

EXAMPLES FOR PRACTICE.

1. A $2'' \times 3''$ wrought-iron bar is $36\frac{1}{2}$ inches long. What is its weight? Ans. 60.66 lb.

2. The outside diameter of a cast-iron column is 10 inches, and the thickness of the material composing the column is $\frac{3}{4}$ inch. What is its weight per foot of length? Ans. 68 lb.

3. The wall of a brick building was laid in cement mortar and is 24 inches thick, 36 feet high, and 100 feet long; in it are located 20 window openings, 2 feet 6 inches wide by 6 feet high. What is the weight of this wall? Ans. 858,000 lb.

4. What is the weight of a structural steel angle 6 in. $\times$ 6 in. $\times \frac{1}{2}$ in. $\times$ 20 ft. long? Ans. 390.54 lb.

5. The roof of a building is made of No. 20 corrugated galvanized iron, laid upon 1-inch spruce boarding. What is the weight of the roof covering per square foot? Ans. $4\frac{1}{4}$ lb.

6. What will be the difference in weight between a 4-ply slag roof, laid upon 3-inch tongued-and-grooved yellow-pine planking, and a $\frac{3}{16}$-inch slate roof laid upon 2-inch hemlock sheathing, covered with Neponset roofing felt, two layers thick? Ans. $3\frac{3}{4}$ lb.

7. The span of a roof truss is 40 feet, and its rise 10 feet. What weight per square foot of roof surface should be assumed so as to allow for the weight of the principal or roof truss? Ans. 3.58 lb.

LIVE LOAD.

42. Besides the dead load, which includes the weight of all the material used in the structure itself, there is a load due to the weight of the people and merchandise; this load is called the **live load.** The live load comprises people in

the building, furniture, movable stocks of goods, small safes, and varying weights of any character. Large safes and extremely heavy machinery require some special provision, usually embodied in the construction. Table 4 gives the live loads per square foot recommended as good practice in conservative building construction.

TABLE 4.

Dwellings	70 lb.
Offices	70 lb.
Hotels and apartment houses	70 lb.
Theaters	120 lb.
Churches	120 lb.
Ballrooms and drill halls	120 lb.
Factories	from 150 up.
Warehouses	from 150 to 250 up.

The load of 70 pounds will probably never be realized in dwellings; but inasmuch as a city house may, at some time, be applied to some purpose other than that of a dwelling, it is not generally advisable to use a lighter load. In the case of a country house, a hotel, or a building of light character, where economy demands it, and its actual use for a long time, for some fixed purpose, is almost certain, a live load of 40 pounds per square foot of floor surface is ample for all rooms *not* used for public assembly.

For rooms thus used, a live load of 80 pounds will be sufficient, experience having demonstrated that a floor cannot be crowded to more. If the desks and chairs are fixed, as in a schoolroom or church, a live load of more than 40 to 50 pounds will never be attained. Retail stores should have floors proportioned for a live load of 100 pounds and upwards. Wholesale stores, machine shops, etc. should have the floors proportioned for a live load of not less than 150 pounds per square foot.

The static load in factories seldom exceeds 40 to 50 pounds per square foot of floor surface, and, therefore, in the majority of cases, a live load of 100 pounds, including the effects of vibrations due to moving machinery, is ample. The conservative rule is, in general, to assume loads not less than the above, and to be sure that the beams are proportioned so as to avoid excessive deflection. Stiffness is a factor as important as strength.

EXAMPLE.—What will be the entire live load coming upon a large girder supporting a portion of a church floor, if the floor area to be supported is 600 square feet?

SOLUTION.—From the list given in Table 4, 120 pounds is usually considered safe for a live load in a church. Therefore, $600 \times 120 = 72{,}000$ pounds is the total live load on the girder. Ans.

EXAMPLES FOR PRACTICE.

1. What will be the entire live load on the floor of a church 50 ft. $\times$ 120 ft.? Ans. 720,000 lb.

2. What live load will a joist in a city dwelling be required to bear, the distance between centers being 14 inches, and the span of the joist, 20 feet? Ans. 1,633 lb.

3. A steel beam, supporting a portion of the floor in an office building, sustains an area of 80 square feet. What will be the live load coming upon the beam? Ans. 5,600 lb.

SNOW AND WIND LOADS.

43. In calculating the weight upon roofs, there are two other loads always to be considered when obtaining the stresses on the various members of the truss. These are snow and wind loads. Where the roof is comparatively flat, that is, where the rise of the roof is under 12 inches per foot of horizontal distance, the **snow load** is estimated at 12 pounds per square foot; for roofs that have a steep slope, or a rise of more than 12 inches per foot of horizontal distance, it is good practice to assume the snow load to be 8 pounds per square foot.

44. Wind pressure on roofs is always assumed as acting normal (that is, perpendicular) to the slope. In Fig. 26, the outline *a b c* of a roof is shown; the force *d* is normal to the slope *a b*, and represents the assumed pressure of the wind on the roof. The wind generally acts in a horizontal direction, as shown by the full arrow *e*. The maximum horizontal pressure of the wind is always considered to be 40 pounds per square foot; this pressure represents a wind velocity of from 80 to 100 miles per hour, which is a *violent hurricane* in intensity, and as this velocity is seldom realized, and never exceeded except in cyclonic storms, the assumption may be considered reasonably safe. The wind, blowing with a horizontal pressure of 40 pounds, strikes the roof at an angle; consequently, the pressure *d*, normal to the slope, is considerably less than 40 pounds, unless the slope of the roof is very steep. Referring to Figs. 27 and 28, it is clear that the horizontal force *e* of the wind on the slope of the roof, shown in Fig. 27, is almost as intense as on a vertical surface; on the extremely flat roof in Fig. 28, however, the wind exerts hardly any force at all normal to the slope, because it strikes the slope at such an acute angle, and

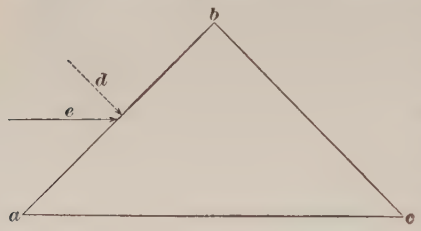


FIG. 26.

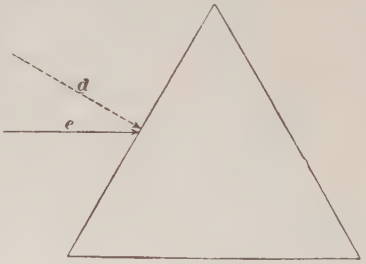


FIG. 27.

FIG. 28.

therefore has a tendency to slide along and off it. The more acute the angle between the lines *e* and *d*, the greater the pressure normal to the slope; whereas, the greater the distance they are apart or the greater the angle, the less the pressure normal to the slope, until they form a right angle with each other, where the pressure on the roof may be disregarded. On the basis of a horizontal wind pressure of 40 pounds, the pressure normal to the slope has been reckoned by a formula known as *Hutton's formula.* This formula, being trigonometric, is not given here, but results derived from it are given in the following table:

TABLE 5.

WIND PRESSURE NORMAL TO THE SLOPE OF ROOF.

Rise.	Angle of Slope with Horizontal.	Pitch, Proportion of Rise to Span.	Wind Pressure Normal to Slope in Pounds.
4 inches per foot horizontal..	18° 25′	$\frac{1}{6}$	16.8
6 inches per foot horizontal..	26° 33′	$\frac{1}{4}$	23.7
8 inches per foot horizontal..	33° 42′	$\frac{1}{3}$	29.1
12 inches per foot horizontal..	45° 0′	$\frac{1}{2}$	36.1
16 inches per foot horizontal..	53° 7′	$\frac{2}{3}$	38.7
18 inches per foot horizontal..	56° 20′	$\frac{3}{4}$	39.3
24 inches per foot horizontal..	63° 27′	1	40.0

45. In order to more fully explain Table 5, refer to Fig. 29. The rise in the slope *a b* is 6 inches for every 12 inches on the horizontal line *a c*; for instance, at 4 feet from *a* on the horizontal line *a c*, the rise is 4 times 6 inches, or 2 feet, the angle included between the line of slope *a b* and the horizontal base line *a c* is 26° 33′, and the pressure normal to the slope, according to Table 5, is assumed at 23.7 pounds per square foot. Since the rise at the center is equal to

one-half the length of one-half the span, the total rise is one-quarter of the span. Under these conditions, the pitch of the roof, that is, the ratio of the rise to the span, is $\frac{1}{4}$, and the roof is said to be $\frac{1}{4}$ *pitch.*

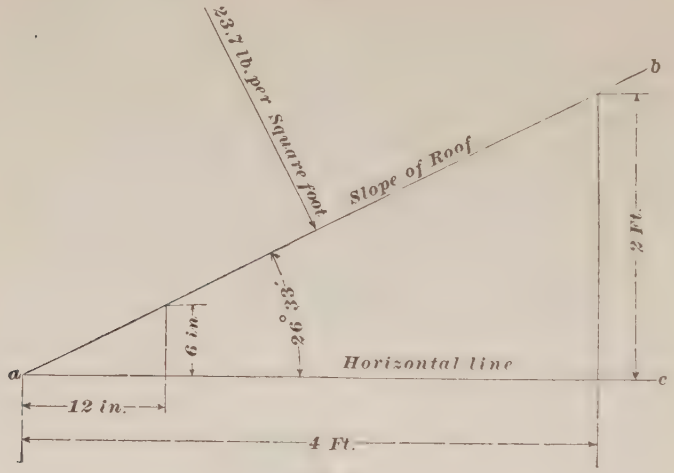


FIG. 29.

EXAMPLE.—(*a*) What will be the dead load per square foot of roof surface, on a roof with a 12-inch rise, the span of the trusses being 50 feet, the roof covering 1 inch white-pine sheathing, 2 layers of Neponset roofing felt, and $\frac{1}{8}$-inch slate 3-inch lap? (*b*) What will be the wind pressure per square foot normal to the slope? (*c*) If the roof trusses are placed 12 feet apart, what will be the entire dead load on one truss? Fig. 30 shows a plan with elevation and detail section of the roof.

SOLUTION.—(*a*) By referring to Table 3, it is seen that the approximate weight of a roof truss with a span of 50 feet is 5 pounds for every square foot of area covered. It is first necessary to obtain the length of the line of slope *a b*; this is done by calculating the hypotenuse of the triangle, or by laying the figure out to scale and measuring. In this case it is found that *a b* measures about 35 feet 4 inches, equal to 35.33 feet. The area covered by the roof supported on one truss is $12 \times 50 = 600$ square feet. The area of the roof supported by one truss is $2 \times 35.33 \times 12 = 847.92$ square feet. Then, $600 \div 847.92 = .70$, which means that the weight of the truss per square foot of roof surface is .70 times 5 pounds, or $5 \times .70 = 3.5$ pounds. The dead load per square foot of roof surface is, then, as follows:

Weight of supporting truss	3.5 lb. per sq. ft.
Weight of white-pine sheathing 1 inch thick .	2.5 lb. per sq. ft.
Weight of 2 layers of Neponset roofing paper	.5 lb. per sq. ft.
Weight of slate ($\frac{1}{8}$ inch thick)	4.5 lb. per sq. ft.
Total	11.0 lb. per sq. ft. Ans.

The weight of the purlins supporting the sheathing has not been estimated in the above, it being safe in this case to assume that the

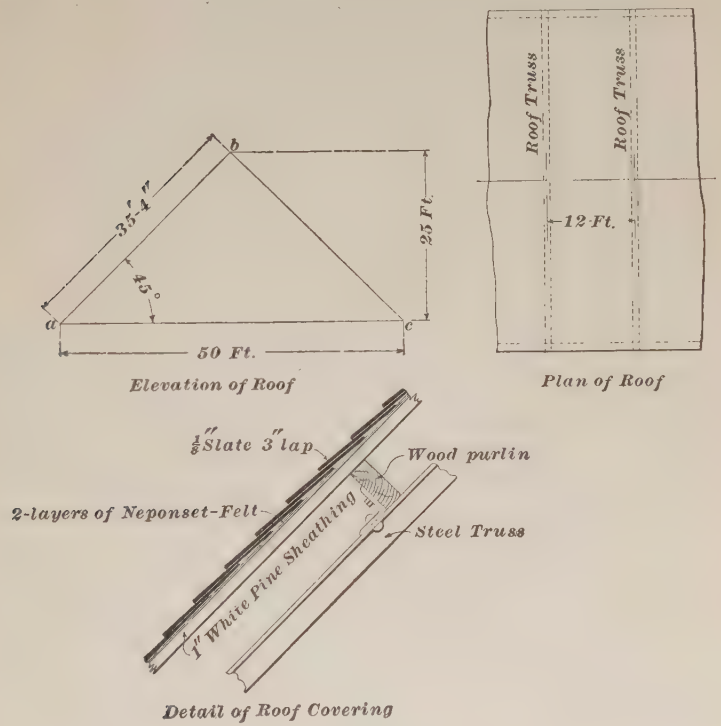


FIG. 30.

weight used for the principals, or trusses, is sufficient to cover this item. A snow and accidental load of 12 pounds per square foot of roof surface should also be added to the dead load to get the entire vertical load upon the roof.

(*b*) The wind pressure normal to the slope of this roof, according to Table 5, for a one-half pitch roof is 36.1 pounds, say 36 pounds per square foot. Ans.

(*c*) The area of the roof supported by one truss is, as previously found, 847.92 square feet, and the dead load is 11 pounds per square foot. Then, $847.92 \times 11 = 9{,}327.12$ pounds to be supported by one truss, not including the snow load. Ans.

46. Engineering is not, it must be remembered, an *exact science*, the results obtained depending more or less upon the judgment and experience of the designer. When, for instance, the wind is blowing a hurricane, snow never lodges on a roof, the slates, shingles, and sheathing being themselves, in such a case, exposed to sudden removal. If, therefore, the full wind pressure be assumed, the snow load may, in most cases, be omitted, especially if the desire be to build an economical roof. It is, however, not well for the student to make such assumptions until his experience and judgment is sufficiently developed to enable him to make true deductions.

47. Careful designers sometimes make allowance for the **accidental load** caused by a heavy body falling upon the floor, or by a mass of snow dropping from one roof to another. But this load may usually be ignored, because it is taken care of in the factor of safety, within the limit of which every member in a structure is designed.

EXAMPLES FOR PRACTICE.

1. The area of one slope of a one-half pitch roof is 800 square feet. What is the entire pressure upon the slope of the roof, provided the maximum horizontal wind pressure is taken at 40 pounds per square foot?

Ans. 28,800 lb.

2. In a quarter-pitch roof the trusses are 20 feet apart, and the length of the roof slope is 40 feet. What wind load is there upon each roof truss, if the horizontal pressure is 40 pounds per square foot?

Ans. 18,960 lb.

3. The purlins supporting a $\frac{3}{4}$-pitch roof are placed 6 feet apart, and the trusses are 12 feet from center to center. What is the maximum load due to the wind upon each purlin, providing the greatest horizontal pressure is 40 pounds per square foot? Ans. 2,830 lb.

STRESSES AND STRAINS.

DEFINITIONS.

48. It has been shown that the weight of the materials composing a building and its contents produces forces that must be resisted by the different members of the structure; the action of these forces has a tendency to change the relative position of the particles composing the members, and this tendency is, in turn, resisted by the cohesive force in the materials, which acts to hold the particles together.

The internal resistance with which the force of cohesion opposes the tendency of an external force to change the relative position of the particles of any body subjected to a load is called a **stress.** Or, stress may be defined as the load per square inch which produces a fractional alteration in the form of a body, and this fractional alteration of form is called the *strain.*

49. In accordance with the direction in which the forces act with reference to a body, the stress produced may be either **tensile, compressive,** or **shearing.**

50. Tensile stress is the effect produced when the external forces act in such a direction that they tend to stretch a body; that is, to pull the particles away from each other. A rope by which a weight is suspended is an example of a body subjected to a tensile stress.

51. Compressive stress is the effect produced when the tendency of the forces is to compress the body or to push the particles closer together. A post or the column of a building is an example of a body subjected to a compressive stress.

52. Shearing stress is the effect produced when the forces act as in a shear, so as to produce a tendency for the particles in one section of a body to slide over the particles of the adjacent section. When a steel plate is acted on by a punch or the knives of a shear, or where a load acts on a

beam, as shown in Fig. 31, the plate or beam is subjected to a shearing stress.

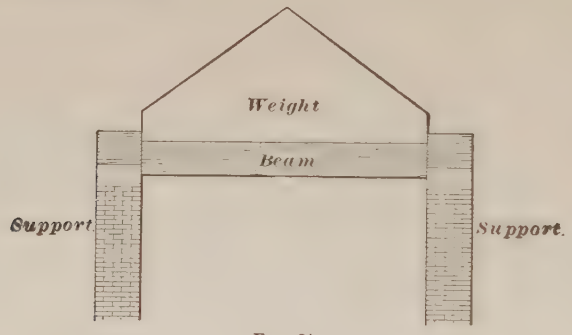


FIG. 31.

53. When a beam is loaded in such a manner that there is in it a tendency to bend, as shown in Fig. 32, it is subjected to a **transverse,** or **bending, stress.** There is, in this case, a combination of the three above mentioned

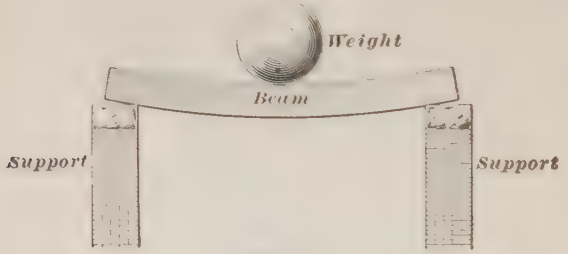


FIG. 32.

stresses (tension, compression, and shear) in different parts of the beam.

54. There is still another type of stress called **torsion,** which, however, is comparatively seldom met with in building construction. An example of a body subjected to a torsional, or *twisting*, stress is a shaft which carries two pulleys, one of which is acted on by the driving force of a belt. The force transmitted from the driven pulley through the shaft produces a tendency to twist the shaft. The effect of this twisting action is a tendency to slide the particles in

any two adjacent sections of the shaft over each other. Torsion may, consequently, be included under the head of shearing stress.

55. The **unit stress** (called, also, the **intensity** of stress) is the name given to the stress per unit of area; or, it is the total stress divided by the area of the cross-section. Thus, if a weight of 1,000 pounds is supported by an iron rod whose area is 4 square inches, the unit stress is $\frac{1000}{4} = 250$ pounds per square inch. If, with the same load, the area is $\frac{1}{2}$ square inch, the unit stress is $\frac{1000}{.5} = 2{,}000$ pounds per square inch.

Let P = total stress in pounds;
A = area of cross-section in square inches;
S = unit stress in pounds per square inch.

Then, $$S = \frac{P}{A}, \text{ or } P = A\,S. \qquad (1.)$$

That is, *the total stress is equal to the area of the section multiplied by the unit stress.*

EXAMPLE.—An iron rod, 2 inches in diameter, sustains a load of 90,000 pounds; what is the unit stress?

SOLUTION.—Using formula **1**,

$$S = \frac{P}{A} = \frac{90{,}000}{2^2 \times .7854} = 28{,}647.8 \text{ lb. per sq. in.} \quad \text{Ans.}$$

56. When a body is stretched, shortened, or in any way deformed through the action of a force, the deformation is called a **strain.** Thus, if the rod before mentioned had been elongated $\frac{1}{10}$ inch by the load of 1,000 pounds, the strain would have been $\frac{1}{10}$ inch. Within certain limits, to be given hereafter, strains are proportional to the stresses producing them.

57. The **unit strain** is the strain per unit of length or of area, but is usually taken per unit of length and called the *elongation* per unit of length. If we consider the unit of length as 1 inch, the unit strain is equal to the total strain divided by the length of the body in inches.

Let l = length of body in inches;
e = elongation in inches;
s = unit strain.

Then, $$s = \frac{e}{l}, \text{ or } e = ls. \qquad (2.)$$

EXAMPLES FOR PRACTICE.

1. A wrought-iron tension member in a roof truss has a load upon it of 27,000 pounds. If it has been figured to sustain a working stress of 6,000 pounds, what will be its diameter? Ans. $2\frac{1}{2}$ in., nearly.

2. The sectional dimensions of a wooden compression member are 8 in. $\times$ 10 in.; if the load upon it is 64,000 pounds, what is the unit stress upon the material? Ans. 800 lb.

3. What is the total load upon a hollow cast-iron column 10 inches outside diameter; the thickness of the metal is 1 inch, and the unit stress upon the column is 20,000 pounds? Ans. 565,600 lb.

4. Two steel angles form the tension member in a roof truss and have a combined sectional area of $3\frac{1}{2}$ square inches; they are subjected to excessive stress, and are stretched $\frac{1}{4}$ inch. What is the unit strain in them if they are 10 feet long? Ans. .00208 in.

STRENGTH OF BUILDING MATERIALS.

58. The **ultimate strength** of any material is that unit stress which is just sufficient to break it.

59. The **ultimate elongation** is the total elongation produced in a unit of length of the material having a unit of area, by a stress equal to the ultimate strength of the material.

60. Modulus of Rupture.—The fibers in a beam subjected to transverse stresses are either in compression or tension, depending whether they are above or below the neutral axis. It has, however, been determined that the strength of the extreme fibers in a beam neither agree with their compressive or tensile strength. Hence, in beams of uniform cross-section above and below the neutral axis it is usual to use a constant, which has been determined by actual tests. This constant is called the *modulus of rupture*, and is generally expressed in pounds per square inch.

61. Table 6 gives values of the strength of building materials commonly used, when subjected to different stresses.

TABLE 6.

STRENGTH OF MATERIALS, IN POUNDS PER SQUARE INCH.

Material.	Ultimate Tensile.	Ultimate Compression Parallel to the Grain.	Allowable Compression Perpendicular to the Grain.	Ultimate Shearing.		Modulus of Rupture.
				Parallel to the Grain.	Perpendicular to the Grain.	
White pine	6,000	3,000	250	300	2,500	4,800
Hemlock	4,000	2,000	250	250	2,500	3,600
Spruce	6,000	3,000	300	300	3,000	4,800
Yellow pine	8,000	4,400	600	400	4,500	7,300
Oak	10,000	3,600	700	600	5,000	6,000
Wrought iron	50,000	44,000		44,000		48,000
Shape iron	48,000					
Structural steel	60,000 to 65,000	52,000		52,000		60,000
						Allowable, 5,000
Cast iron	18,000	81,000		25,000		45,000
Granite		15,000				1,800
Limestone		7,000				1,500
Sandstone		5,000				1,200 to 700
Good sandstone		10,000				1,700

NOTE.—The terms "parallel to the grain" and "perpendicular to the grain" apply to wood only.

62. The values given above, under their several headings, being conservative, are on the safe side. The column in the table headed "Ultimate Compression Parallel to the

Grain" will be found useful in computing the strength of columns. The values in the column headed "Allowable Compression Perpendicular to the Grain" are used in cases

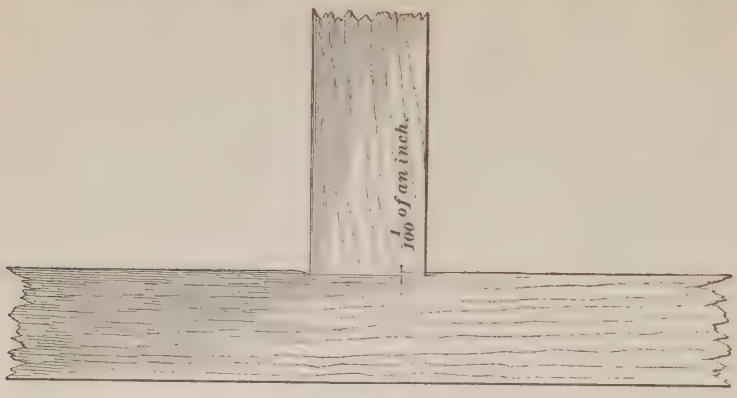


FIG. 33.

similar to Fig. 33, and are such as will not produce an indenture of more than $\frac{1}{100}$ of an inch in the surface of the timber, a value well within the safe limit. The values under the heading of "Ultimate Shearing Parallel to the Grain" are used in computing the strength of the end of

FIG. 34.

the tie-beam at the heel of the main rafter in a roof truss, as shown in Fig. 34. The tendency is to shear off the piece *h* parallel to the grain along the line *a b*. The figures in the column headed "Modulus of Rupture" are the constants, or values, used when computing the strength of beams. When a simple beam breaks, the fibers at the top

side are in *compression*, and those at the bottom side in *tension*, as shown in Fig. 35. By actual tests, it has been found that though some of the different fibers of materials under transverse stresses are in compression and some in tension, the ultimate resistance of the material does not agree with the ultimate resistances of the fibers to either tension or compression. Though many attempts have been made to account for it, this fact remains; hence, it becomes necessary to obtain some constant, or value, more closely agreeing with the strength of materials under transverse stresses. It is usual, therefore, where the cross-section of the beam is uniform, to obtain, by actual tests, the constants, or values, for each material, and these values are called the *modulus of rupture* and are generally expressed in pounds per square inch.

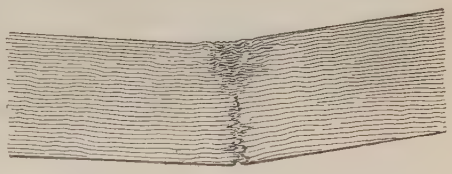


FIG. 35.

EXAMPLE 1.—What pull will be required to break a 2-inch diameter rod of wrought iron?

SOLUTION.—The area of the rod is equal to the area of a 2-inch diameter circle, which is $2^2 \times .7854 = 3.14$ square inches; the ultimate tensile, or breaking, strength of wrought iron, according to Table 6, is about 50,000 pounds per square inch. Therefore, the ultimate strength of the rod in question is about $3.14 \times 50{,}000 = 157{,}000$ pounds. Ans.

EXAMPLE 2.—What length of wrought-iron bar, if hung by one end, will break of its own weight?

SOLUTION.—Assume any size of bar; say $1\frac{1}{8}$ inches in diameter. The area of this bar is .99 square inch, which may, for convenience, be called 1 square inch. Wrought iron, according to Table 1, weighs .277 pound per cubic inch. Now, as there is just 1 cubic inch in each lineal inch in the rod, a length of 1 foot weighs $.277 \times 12 = 3.32$ pounds. The tensile strength of wrought iron being 50,000 pounds per square inch, and 1 foot of its length weighing 3.32 pounds, the length of rod required is $\frac{50{,}000}{3.32} = 15{,}060$ feet. Ans.

EXAMPLE 3.—In Fig. 36 is shown the splice of a tie-beam in a wood roof truss composed of yellow pine. What is the strength of the splice, disregarding the bolts *a*, *a* entirely?

SOLUTION.—The strength of the splice depends on the tensile strength of the wood at the net section *e f* and upon the tensile strength of the net section of the two splice plates. It also depends upon the tendency of the splice plates to shear along the lines *s t* and *s′ t′*, and upon the tendency of the tie to shear along the line *m n* and *m′ n′*. Assume the areas of the net sections to be sufficient to make their strength greater than that of the sections which will fail by shearing; then referring to Fig. 36, it will be seen that the line of shear on the splice plate at *s t*

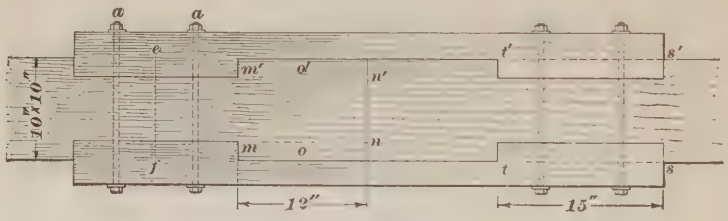


FIG. 36.

and *s′ t′* is longer than that of the tie member at *m n* and *m′ n′*; therefore, the strength of the tie along the line *m n* and *m′ n′* only need be considered in computing the strength of the splice. The pieces *o* and *o′* tend to slide or shear off of the main tie along the lines *m n* and *m′ n′*. The area along these lines is $12 \times 10 \times 2 = 240$ square inches. Referring to Table 6 under the column headed "Ultimate Shear Parallel to the Grain," the value for yellow pine is found to be 400 pounds per square inch. Hence, $240 \times 400 = 96{,}000$ pounds, the ultimate strength of the splice, disregarding the bolts *a*, *a*.

63. The **factor of safety,** or, as some call it, the **safety factor,** is the ratio of the breaking strength of the structure to the load which, under usual conditions, it is called upon to carry. Suppose the load required to break, dismember, or crush a structure is 5,000 pounds, and the load it is called upon to carry is 1,000 pounds, then the factor of safety may be obtained by dividing the 5,000 pounds by the 1,000 pounds, or $\frac{5000}{1000} = 5$, the factor of safety in this structure.

The safety factor depends upon the conditions, circumstances, or materials used; in other words, it is the *factor of ignorance.* When a piece of steel, wood, or cast iron is used in a building, the engineer does not know the exact strength of that particular piece of steel, wood, or cast iron. From his own experience, and that of others, he knows the approximate tensile strength of structural steel to be 60,000

pounds per square inch, and that it varies more or less from this value. In regard to timber, the uncertainty is much greater, because of knots, shakes, and interior rot, not always evident on the surface. Cast iron is even more unreliable, on account of almost indeterminable blowholes, flaws, and imperfections in the castings.

64. Deterioration.—Another factor to be considered is deterioration in the material, due to various causes. In metals there is corrosion on account of moisture and gases in the atmosphere, especially noticeable in the steel trusses over railroad sheds, where the sulphur fumes from the stacks of the locomotives unite with the moisture in the air, forming free sulphuric acid, which attacks the steel vigorously, and demands constant painting, to prevent its entire destruction. Wood is subject to decay from either dry or wet rot, caused by local conditions; it may, like iron and steel, be subjected to *fatigue*, produced by constant stress due to the load it may have to sustain. Cast iron does not deteriorate to any great extent, its corrosion not being as rapid, possibly, as that of steel or wrought iron. There are, however, internal strains produced in cast iron by the irregular cooling of the metal in the mold. Castings under the slightest blow will sometimes, owing to these internal stresses, snap and break in a number of places.

These reasons are, in truth, sufficiently cogent to require the factor of safety now adopted in all engineering work. Table 7 gives the factor of safety recognized by conservative and judicious constructors, for various materials.

TABLE 7.

SAFETY FACTORS FOR DIFFERENT MATERIALS USED IN CONSTRUCTION.

Structural steel and wrought iron	3 to 4.
Wood	4 to 5.
Cast iron	6 to 10.
Stone	10 at least.

In the above table the factor of safety generally used for structural steel is 3 to 4, which simply means that the steel structure should not break until it bears a load 3 or 4 times greater than it is expected to carry.

EXAMPLE.—If the breaking strength of a cast-iron column is 200,000 pounds, what safe load will the column sustain if a factor of safety of 6 is used?

SOLUTION.— $200{,}000 \div 6 = 33{,}333$ pounds. Ans.

EXAMPLES FOR PRACTICE.

1. Providing a factor of 3 is adopted, what will be the safe working stress on a 2-inch diameter tension rod of structural steel?

Ans. 62,800 lb.

2. The pull upon a 2-inch eyebolt, passing through a piece of yellow pine, is 40,000 pounds. What should be the diameter of the washer, if the bolt hole through it is $2\frac{1}{8}$ inches in diameter? Ans. $9\frac{1}{2}$ in.

3. What will be the crushing strength of a granite capstone 24 inches square? Ans. 8,640,000 lb.

4. The bottom of the notch in a yellow-pine timber 10 inches wide and 12 inches deep, forming the tie member in a roof truss, is 18 inches from the end. What resistance will the end of the tie offer to the thrust of the rafter? Ans. 72,000 lb.

5. A short block of yellow pine, 10 in. $\times$ 10 in. in section, standing on end, supports 50,000 pounds. What is its factor of safety? Ans. 8.8.

FOUNDATIONS.

STRENGTH OF FOUNDATION MATERIALS.

65. It is useful, before considering the strength of columns or compression members in buildings, to take up the compressive strength of brickwork and masonry, these being the mediums by which the columns or posts in a building transmit their compressive resistance to the ground. As the bearing strength of the soil, that is, its capacity to support a greater or less load, determines the spread of the foundation, we here submit bearing values for brickwork and stonework and the several kinds of soils likely to be encountered in building operations.

Table 8 gives the conservative bearing values of brickwork, masonry, and soils.

TABLE 8.

THE SAFE BEARING VALUES OF BRICKWORK, MASONRY, AND SOILS.

BRICKWORK.	
Brickwork, hard bricks, dried lime mortar	100 lb. per sq. in.
Brickwork, hard bricks, dried Portland cement mortar	200 lb. per sq. in.
Brickwork, hard bricks, dried Rosendale cement mortar	150 lb. per sq. in.
MASONRY.	
Granite, capstone, in lime mortar	700 lb. per sq. in.
Sandstone, capstone, in lime mortar	350 lb. per sq. in.
Bluestone (a sandstone)	500 to 700 lb. per sq. in.
Limestone, capstone, in lime mortar	500 lb. per sq. in.
Granite, square stone masonry, in lime mortar	350 lb. per sq. in.
Sandstone, square stone masonry, in lime mortar	175 lb. per sq. in.
Limestone, square stone masonry, in lime mortar	250 lb. per sq. in.
Rubble masonry, in lime mortar	80 lb. per sq. in.
Rubble masonry, in Portland cement mortar	150 lb. per sq. in.
Concrete, Portland cement (1 of cement, 2 of sand, 5 of broken stone)	150 lb. per sq. in.
SOIL.	
Rock foundation	20 tons per sq. ft.
Gravel and sand (compact)	6 to 10 tons per sq. ft.
Gravel and sand (mixed with dry clay)	4 to 6 tons per sq. ft.
Stiff clay, blue clay	2.5 tons per sq. ft.
Chicago clay	1 to 1.5 tons per sq. ft.

In the values given for masonry, the height of the wall should not be over 16 times the thickness.

DESIGN OF FOUNDATIONS.

66. Proper designs for foundations are of the utmost importance. The maximum load carried by the foundation must first be obtained. The loads to be considered in buildings are of two kinds, the dead and live loads, previously explained. The live load is variable. In office buildings, parts of the floor may be loaded to their full capacity, but the probability of the entire structure being so loaded is remote. In breweries, storage warehouses, factories, and buildings for similar purposes, all the floors may be, however, fully loaded. The maximum of both dead and live loads must be considered and the area of the footings for the foundations such that the greatest pressure on different soils does not exceed that given in Table 8.

In designing foundation footings or piers for the support of columns, certain proportions are considered best, and are therefore generally adopted. Fig. 37 presents a

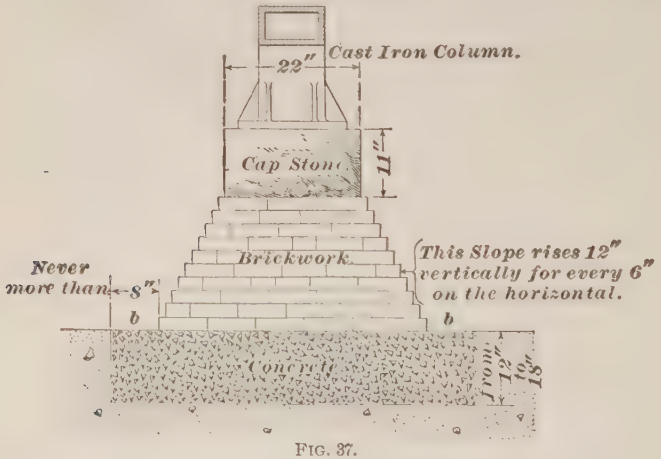


FIG. 37.

diagram of a properly proportioned foundation pier for a column. The thickness of the capstone should be about one-half the width of the side. The concrete offset, as shown at *b*, should never be more than 8 inches. The layer of concrete may be from 12 to 18 inches thick, and the batter of the main body of the foundation 1 to 2; that is, for every foot of rise it should incline 6 inches horizontally.

67. As an example of the method of designing a foundation pier for a column, let us assume that the load upon a cast-iron base supporting a wood column is 200,000 pounds, and that we are required to design a foundation pier for this column, the pier being made of brickwork in cement mortar with concrete base and granite cap.

The soil under the pier, being compact gravel and sand, can safely sustain 6 tons per square foot. The load upon the soil, transmitted through the pier, is 200,000 pounds, or 100 tons.

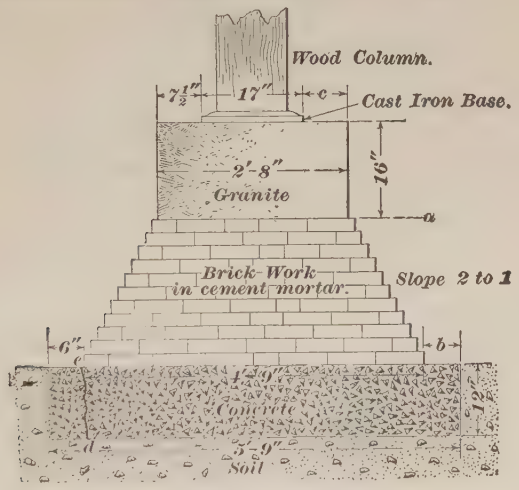


FIG. 38.

Assuming a maximum load on the soil of only one-half of its safe bearing value, we have $100 \div 3 = 33\frac{1}{3}$ square feet that the base of the concrete must cover. Therefore, the concrete base should be about 5 feet 9 inches square. Next, it is required to find how large the brick pier should be at the top, or at the surface marked a, in Fig. 38. The bearing value of brickwork in Portland cement mortar, according to Table 8, is 200 pounds per square inch. The load coming upon the pier is 200,000 pounds. Then, $200{,}000 \div 200 = 1{,}000$ square inches, the required area at the surface a; $1{,}000 \div 144 = 6.93$ square feet. The upper surface a of the brickwork being a square, the length of one side should be $\sqrt{6.93}$, or about 2 feet 8 inches. It is now required to

determine the area of the brickwork at the bottom, or where it rests upon the concrete base. The concrete base, according to Table 8, will with safety support 150 pounds per square inch. As the pressure upon it is 200,000 pounds, the area in square inches at this point must be $200{,}000 \div 150 = 1{,}333$ square inches, or 9.25 square feet. A square whose sides are 3 feet 1 inch has an area of 9.50 square feet, which would, in theory, be about the area of the brick base next to the concrete. In the case before us, represented by Fig. 38, it happens, however, that the area of the concrete base required is 5 feet 9 inches on a side, while the greatest limit to which it may extend beyond the brick pier is, according to good practice, about 8 inches, due to its liability to break off at the line de; adoption of the theoretical area of the pier at this point is, therefore, inconsistent with good practice, the edge of the brick pier being carried out to within 8 inches of the edge of the concrete, regardless of dimensions obtained in calculating the required area for brick piers bearing upon the concrete bases. As the granite cap bears upon the brickwork at the surface a, its area is governed by the bearing strength of the brickwork, and is required to be, as previously found, 2 feet 8 inches square. The area of the cast-iron base is governed by the permissible unit pressure on the granite capstone, which is 700 pounds per square inch. Therefore, $200{,}000 \div 700 = 285$ square inches required to be covered, which means a cast-iron base about 17 inches square. The distance that the capstone extends beyond the base should not be over one-half of the thickness of the capstone. The capstone being in thickness one-half the width of the side, its thickness in this is $\frac{1}{2}$ of 2 feet 8 inches = 1 foot 4 inches, or 16 inches. The distance c in this cap is, then, with safety placed at $\frac{1}{2}$ of 16, or 8 inches. The cast-iron base being 17 inches square, and the cap 2 feet 8 inches, or 32 inches square, the distance c, in this case, would be $32 - 17 = 15$, which divided by 2 $= 7\frac{1}{2}$ inches, a figure well within the limit.

Fig. 38 shows this pier foundation drawn to scale, according to the figures reached by the above calculation, in which

the weight of the pier was not considered, such exactitude not being generally looked for in ordinary building construction.

EXAMPLES FOR PRACTICE.

1. What safe load, uniformly applied, will a $3' \times 3'$ brick pier laid in Rosendale cement mortar sustain, the height of the pier being 10 feet? Ans. 259,200 lb.

2. The load transmitted by a cast-iron column to a foundation pier is 200,000 pounds. What should be the size of the base of the pier, providing the soil is compact gravel and sand? Ans. 4 ft. $\frac{1}{2}$ in. square.

3. What size of granite capstone, resting upon a brick pier laid in Portland cement mortar, will be required under a cast-iron column transmitting a load of 22,000 pounds? Ans. $10\frac{1}{2}$ in. square.

4. A brick pier rests upon stiff clay. If the footing is 5 feet square, what load will it safely sustain? Ans. 125,000 lb.

COLUMNS.

SHORT COLUMNS.

68. The column may, in its first stage of development, be considered a cubical or rectangular block, shown in Fig. 39. As long as the column does not exceed from 6 to about 10 times the width of the least side, the load it can safely carry may be estimated by multiplying its sectional area in square inches by the safe resistance to compression of the material parallel to the grain. To get the safe resistance of a short column or block, divide the ultimate resistance to compression of the material parallel to the grain by the

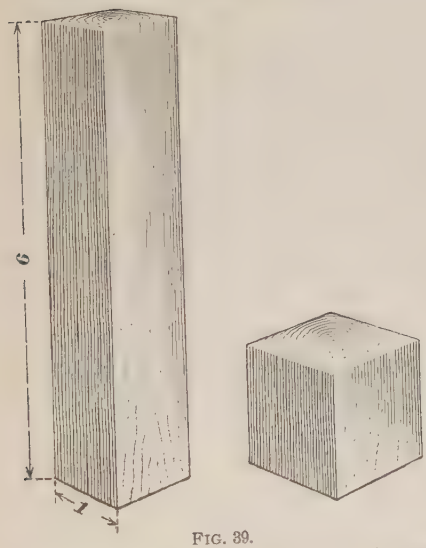


FIG. 39.

factor of safety, which, for wood columns, according to Table 7, is from 4 to 5, though it may be, in some instances, good practice to use 6. It is all a matter of judgment, governed by the conditions to be met with. After having obtained the safe resistance of the material to compression, multiply by the sectional area of the column in square inches, and the result will be the safe resistance of the short column to compression.

EXAMPLE.—What safe load will a short yellow-pine block, 12 inches square and 6 feet long, standing on end support, using a safety factor of 5?

SOLUTION.—According to Table 6, the ultimate compression of yellow pine parallel to the grain, per square inch, is 4,400 pounds; using a factor of safety of 5, the safe load per square inch on this short column would be $4{,}400 \div 5 = 880$ pounds per square inch. The area of the column is 12 in. $\times$ 12 in. = 144 square inches, and, therefore, $144 \times 880 = 126{,}720$ pounds, the safe resistance to compression of the short column. Ans.

LONG COLUMNS.

69. The statements of the preceding paragraphs do not apply to columns of over 10 times the width of the least side. When long columns are under compression, and not secured against yielding sideways, it is evident they are liable to bend before breaking. To ascertain the exact stress in such pieces is, sometimes, quite difficult. Hence, we must have a formula making due allowance for this tendency in the column to bend, or to split and spread from the center, as shown in Fig. 40.

FIG. 40.

70. Formula for Wood Columns.—The formula mostly used for long, square, or rectangular wood columns, with square ends, is the following, deduced from elaborate tests made on full-length columns at the Watertown arsenal:

$$S = U - \left(\frac{U \times L}{100D}\right), \qquad (3.)$$

in which S = ultimate compressive strength per square inch of sectional area of the column;
U = ultimate compressive strength of the material per square inch parallel to the grain;
L = length of column in inches;
D = length of least side of column in inches.

EXAMPLE.—What safe load will a white-pine column, 10 inches square and 20 feet long, support, using a factor of safety of 6?

SOLUTION.—The ultimate compressive strength of white pine parallel to the grain is, according to Table 6, 3,000 pounds per square inch. Therefore, by substituting in the formula, we have

$$S = 3{,}000 - \left(\frac{3{,}000 \times 240}{100 \times 10}\right) = 2{,}280 \text{ pounds},$$

the ultimate bearing value of the column per square inch. As the factor of safety required is 6, the safe bearing value per square inch of sectional area is $2{,}280 \div 6 = 380$ pounds. The area of the column being 100 square inches, the safe load is $100 \times 380 = 38{,}000$ pounds. Ans.

The column formulas in general use do not give a direct method of calculating the dimensions of a column to carry a given load. The usual method of procedure, when it is required to find the dimensions of a column of a given form which will safely support a given load, is to assume a value for the size of the column, substitute this value in the formula, together with the other values given, and solve for the compressive stress S. If the assumed size gives a value of S that is satisfactory for the given conditions, it is correct; if, however, the resulting value of S is too great, the assumed size is too small and it will be necessary to assume a larger size and try again; if, on the contrary, the value of S is less than the allowable stress, a smaller size of column may be assumed and a new value of S obtained. After a few trials a size will be found which gives a satisfactory unit stress for the given conditions.

71. Importance of the Method of Securing the Ends of Columns.—Materials in compression develop more strength if the bearing surfaces are true and level, for the tendency is then for all the material in the piece, or member, to resist compression equally, and not crush in one place,

before the balance of the material can be brought under compression, as would be the case if the bearing surfaces were uneven and rough.

The manner of securing the ends of columns has, also, an appreciable effect upon their strength. Columns fixed so firmly at the ends as to be liable to fail in the body before rupturing their end connections, develop greater strength than columns connected by means of pins through the ends. Columns with square ends develop less ultimate strength than if the ends are firmly fixed, but greater than if the ends are pin-connected, that is, fastened by pins which permit them to swing freely.

The above given formula for wood columns and the following formula for cast-iron columns apply only to those columns having flat ends, the usual condition met with in building construction.

72. Cast-iron columns are most frequently used in buildings of moderate height, but have been, in some cases, used in buildings of sixteen stories and even more. The best practice has, during the last few years, so uniformly declared in favor of steel columns that the employment of cast iron is now generally confined to buildings of ordinary height, say, four or five stories, or to special cases, where advantages are to be gained in the use, for instance, of a number of ornamental cast columns.

It is true that cast-iron columns are cheaper per pound and perhaps easier of erection than steel, though the declining price of structural steel is rapidly removing this consideration in favor of cast-iron columns.

The uncertain strength of cast iron has compelled the adoption of a very low unit stress, in other words, a very high factor of safety. The uniform strength of structural steel is, on the other hand, so well understood that cast iron is, for columns, and especially girders, falling into disuse. Considerations of economy may, however, in some cases, still justify its employment.

One disadvantage in the use of cast-iron columns is that

when fracture occurs, it comes without warning. In high buildings, erected entirely upon cast-iron columns, the danger from wind pressure is very much increased on account of lack of stiffness in the joints of the connections at the several floors. In fact, buildings have been blown 10 inches out of plumb, owing to this lack of rigidity in the connections.

73. Formula for Cast-Iron Columns.—The strength of a cast-iron column with square ends may be calculated by the following rule:

Rule.—*To find the ultimate strength per square inch of sectional area of a cast-iron column with square ends, divide the ultimate compressive strength per square inch of the material composing the column by 1 plus the quotient obtained by dividing the square of the length of the column in inches, by 3,600 times the square of the radius of gyration of the section of the column.*

This rule is expressed by the formula

$$S = \frac{U}{1 + \frac{L^2}{3{,}600\,R^2}}, \qquad \textbf{(4.)}$$

in which S = ultimate strength per square inch of cross-section;

U = ultimate compressive strength per square inch of the material composing the column (for cast iron U may be taken as 81,000);

L = length of column in inches;

R^2 = square of the radius of gyration.

The term **radius of gyration,** which will be more fully explained in a later article, is a mathematical expression much used in calculating the strength of columns. Table 9 is a collection of formulas for finding the value of R^2 (the square of the radius of gyration) for the forms of cross-section most often used for cast-iron columns.

TABLE 9.

TABLE OF THE SQUARE OF THE LEAST RADIUS OF GYRATION FOR THE DIFFERENT SECTIONS OF CAST-IRON COLUMNS.

Solid Square.		$R^2 = \frac{D^2}{12}$.	(5.)
Solid Rectangle.		$R^2 = \frac{D^2}{12}$.	(6.)
Solid Circular.		$R^2 = \frac{D^2}{16}$.	(7.)
Hollow Square.		$R^2 = \frac{D^2 + A^2}{12}$.	(8.)
Hollow Circular.		$R^2 = \frac{D^2 + A^2}{16}$.	(9.)

EXAMPLE.—What is the square of the radius of gyration of a hollow circular column of 12 inches outside diameter, the thickness of the metal being 1 inch?

SOLUTION.—The formula for obtaining the square of the radius of gyration for a hollow cylindrical cast-iron column is, according to Table 9,

$$R^2 = \frac{D^2 + A^2}{16}.$$

Substituting the given values, we have

$$R^2 = \frac{12^2 + 10^2}{16} = \frac{244}{16} = 15.2. \quad \text{Ans.}$$

EXAMPLE.—Find the proper working load for a 10-inch square hollow cast-iron column, 20 feet long, using a factor of safety of 6, the thickness of the metal being 1 inch.

SOLUTION.—The ultimate compressive strength U of cast-iron per square inch, according to Table 6, is 81,000 pounds, the length L is $20 \times 12 = 240$ in.; and R^2 for a hollow square rectangular column is, according to Table 9,

$$\frac{D^2 + A^2}{12} = \frac{10^2 + 8^2}{12} = 13.67.$$

Substituting these values in formula **4**, we have

$$S = \frac{U}{1+\frac{L^2}{3{,}600R^2}} = \frac{81{,}000}{1+\frac{240^2}{3{,}600\times 13.67}} = \frac{81{,}000}{2.17} = 37{,}327,$$

the breaking strength of the column in pounds per square inch of section. With a factor of safety of 6, the safe bearing value of the column is $37{,}327 \div 6 = 6{,}221$ pounds per square inch. The net area of the section of the column is $10^2 - 8^2 = 100 - 64 = 36$ square inches. The entire load that it will support with safety is, therefore. $36 \times 6{,}221 = 223{,}956$ pounds. Ans.

DESIGN OF CAST-IRON COLUMNS.

74. Fig. 41 shows a design for a **circular** cast-iron column. *B* shows the elevation for the cap and brackets

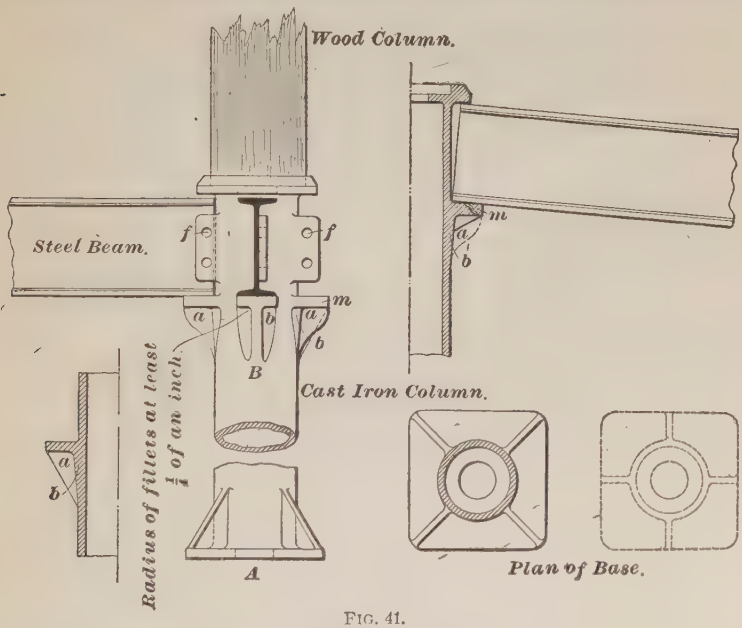


FIG. 41.

supporting steel floorbeams. Attention should be paid to the design of the bracket *a* and the web made as shown at *B*, care being taken that it is brought to the edge of the plate *m*, upon which the steel beams rest. Otherwise, if the beam takes a bearing on the edge of the plate *m* as

shown at *C*, the tendency will be to fracture the edge of the bracket. The web of the bracket should, if possible, form an angle of 60° with the horizontal plate *m*, as shown in Fig. 41.

The bolt holes *f* should be always drilled either in the casting or in the steel beams after these beams are in place, because if the holes were cored in the casting and the holes punched in the beams at the mill, it would more than likely be found in the course of erection at the building that the beams were supported entirely by the shear of the bolts and would not bear upon the supporting brackets. It is well to have at least $\frac{1}{4}$-inch fillets, as shown, but larger if possible, in all the corners of the casting. It is good practice to thicken up the metal in the column, where the brackets are cast upon it, as shown at *b*.

In forming the bolt holes *f*, it must be remembered that the bolt should fit the hole as closely as possible; that it should, in fact, be a machine fit. It is well, indeed, to drill the holes both in the beams and the cast-iron flanges, to insure a true and accurate bolt hole, so that there may be a minimum amount of play in the connection and that it may be as rigid as possible.

In designing the base for this column, it is well to place the ribs strengthening the web in such position that they may be most effective, which, in this case, is on the diagonals, for, if placed parallel to the sides, the corners will have a tendency to break off, and part of the bearing surface at the base of the column may prove ineffective.

Designs for the base and cap of cast-iron columns are as numerous as various conditions may require them. It would be almost impossible to give examples of the different connections demanded in actual building practice. The foregoing remarks will, if kept in mind, apply to every case, and, if followed, insure good design as well as secure construction.

75. Inspection.—In examining castings used in building construction, to ascertain their quality and soundness, several points are to be considered. The edges should be struck with a light hammer. If the blow makes a slight

impression, the iron is probably of good quality, providing it be uniform throughout. If fragments fly off and no sensible indentation be made, the iron is hard and brittle. Air bubbles and blowholes are a common and dangerous source of weakness. They should be searched for by tapping the surface of the casting all over with a hammer. Bubbles, or flaws, filled in with sand from the mold, or purposely stopped with loam, cause a dullness in the sound, leading to their detection. The metal of a casting should be free from bubbles, core nails, or flaws of any kind. The exterior surface should be smooth and clean, and the edges of the casting should be sharp and perfect. An uneven or wavy surface indicates unequal shrinkage, caused by want of uniformity in the texture of the iron.

The surface of a fracture, examined before becoming rusty, should present a fine-grained texture, of a uniform bluish-gray color and high metallic luster.

In inspecting cast-iron columns, care should be taken to see that they are straight and cored directly through the center, and that the metal is of the same thickness throughout. It is not unusual to find, in molding cast-iron columns, that the core has not been placed in the mold in a central position, or that, having been insecurely fastened, it has floated over to one side. Hence, the column which should have been, say, $\frac{3}{4}$ of an inch in thickness throughout, may be $1\frac{1}{4}$ inches on one side and $\frac{1}{4}$ of an inch on the other. The base and the cap of a cast-iron column should be turned accurately, being true and perpendicular to the center line of the column.

76. Danger from Fire.—One of the great objections to the use of cast-iron columns is that they are liable to be broken by sudden contraction, due to water being played upon them in case of fire.

EXAMPLES FOR PRACTICE.

1. A yellow-pine column 20 feet long is required to sustain a load of 100,000 pounds; provided a factor of safety of 5 is used, what must be the size of this column? Ans. 12 in. $\times$ 12 in.

2. What will be the allowable load upon a $4'' \times 10''$ spruce column, 12 feet long, the factor of safety being 6? Ans. 12,800 lb.

3. It is required that a short, round yellow-pine column shall carry 173,000 pounds. What must be the diameter of the column, the safe unit compressive stress upon the material being 1,000 pounds? Ans. 15 in.

4. The section of a hollow cast-iron column is 16 inches square, outside measurement; the thickness of the material in the column is 1 inch; what is the radius of gyration of this column? Ans. 6.13.

5. What will be the breaking load of a cast-iron column 20 feet long, 12 inches in diameter outside, made of 1-inch metal? Ans. 1,372,200 lb.

6. The thickness of the metal in a cast-iron column is $\frac{1}{2}$ inch; if a factor of safety of 6 is used, and the length of the column is 18 feet, what must be the outside diameter of the column to support a load of 133,000 pounds? Ans. 10 in.

7. Find the ultimate crushing strength of a 10-inch square, outside measurement, cast-iron column; the thickness of the metal is 1 inch, and the length of the column is 20 feet. Ans. 1,343,700 lb.

BEAMS.

DEFINITIONS.

77. Any bar resting upon supports, and liable to transverse stresses, is called a **beam.**

A **simple beam** is a beam resting upon two supports very near its ends.

A **cantilever** is a beam resting upon one support in its middle, or which has one end fixed (as in a wall) and the other end free.

A **fixed beam** is one which has both ends firmly secured (as a plate riveted to its supports at both ends).

A **continuous beam** is one which rests upon more than two supports.

The **span** of a simple beam is the distance between its supports.

REACTIONS.

78. Since one condition of equilibrium requires that the sum of all the forces acting on a body in one direction must be balanced by an equal set of forces acting in the opposite

direction, it follows that, in order that any body may be kept from falling, there must be an upward pressure, or thrust, against it, just equal to the downward pressure due to its weight; this upward thrust is called a **reaction.**

In accordance with this principle, it is evident that the simple beam shown in Fig. 42 is supported by the sum of

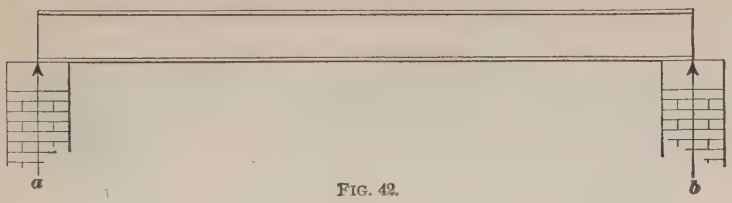


FIG. 42.

the upward pressures exerted on it by the two brick piers on which it rests; also that this sum is equal to the weight of the beam plus the weight of any load it may carry. This is expressed by the statement: *The sum of the reactions at the supports of any beam is equal to the sum of the loads.*

79. Relation Between the Reactions.—If the load on a simple beam is either uniformly distributed, applied at the center of the span, or symmetrically placed on each side of the center of the span, the reaction at each support is equal to one-half of the total load. When, however, the loads are not symmetrically placed, the reactions are unequal and must be determined before the first step towards obtaining the strength of the beam can be taken. To determine the reactions at the points of support of a beam loaded with a number of loads, irregularly placed, we apply the principle of moments, as shown in the following illustrative examples:

80. Two men *a* and *b*, 15 feet apart, carry a 50-pound weight between them on a plank, as shown in Fig. 43. What part of the load does each man carry?

If the load had been placed midway between them, it is quite evident that each man would have half the weight of the plank and load to support. But, since the load is moved towards *a* until within 5 feet of him, he must support a greater proportion of the load than *b*. If *b* raises his end of

the plank, as shown in dotted lines, it is evident that a simply acts as a hinge while b raises the weight with a lever 15 feet long. The weight of 50 pounds acts down with a leverage of 5 feet; its moment about a as a center is, therefore, $50 \times 5 = 250$ foot-pounds. That there may be equilibrium of

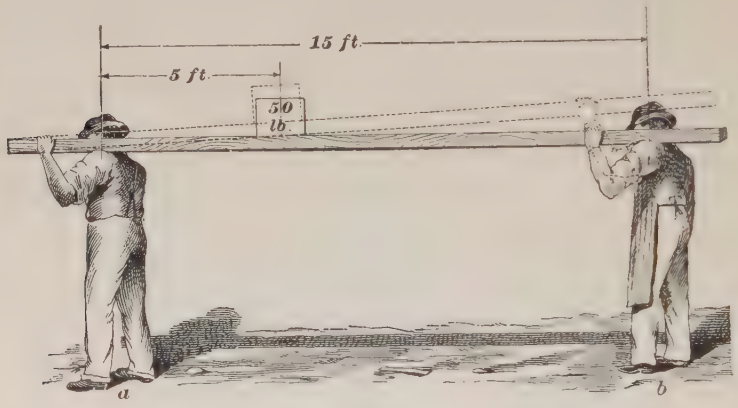


FIG. 43.

moments, it is evident that the man at b must exert an upward force whose moment with a lever arm of 15 feet equals 250 foot-pounds; that is, he must exert a force of $250 \div 15 = 16\frac{2}{3}$ pounds to support his share of the weight. Since the sum of the reactions must equal the sum of the loads, it follows that if b supports $16\frac{2}{3}$ pounds, a must support the difference between the load of 50 pounds and $16\frac{2}{3}$ pounds, or $33\frac{1}{3}$ pounds.

81. Fig. 44 shows the men a and b supporting three loads of 50, 40, and 80 pounds, respectively. It is desired to estimate the force that each must exert to sustain the weights, leaving the weight of the plank out of the question. Assuming the center of moments at a, find the resultant moment of all the weights about this point, as follows:

$$\begin{aligned} 50 \times\ 3 &= 150 \text{ ft.-lb.} \\ 40 \times\ 8 &= 320 \text{ ft.-lb.} \\ 80 \times 12 &= 960 \text{ ft.-lb.} \\ \text{Total,}\quad & 1{,}430 \text{ ft.-lb.} \end{aligned}$$

This is the moment of all the loads upon the beam about the point a as a center. Hence, the force that b must exert, in

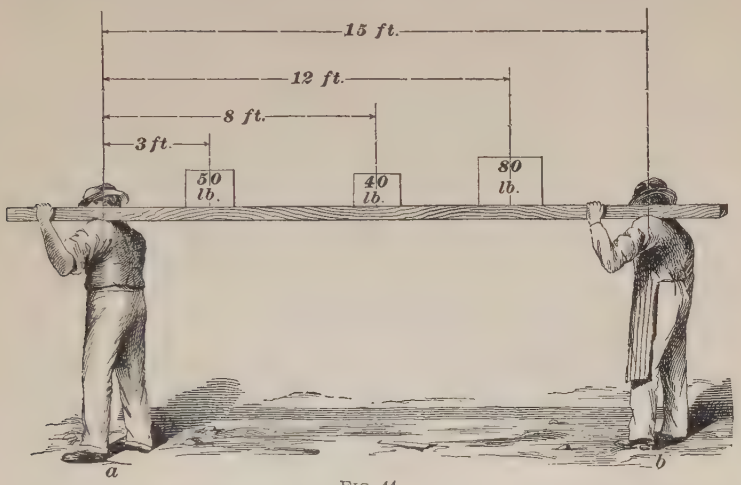


FIG. 44.

order to produce equilibrium, is $1{,}430 \div 15 = 95\frac{1}{3}$ pounds. The part of the load which a supports is the difference between the total load, $50 + 40 + 80 = 170$ pounds, and the part of the load supported by b, that is, $170 - 95\frac{1}{3} = 74\frac{2}{3}$ pounds.

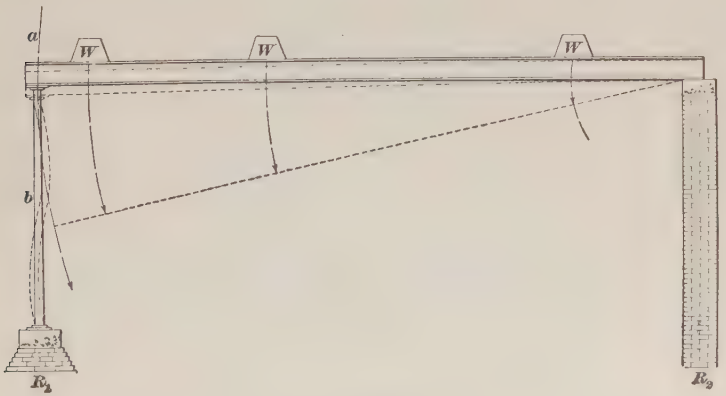


FIG. 45.

82. Take a more practical example. In Fig. 45 let it be required to find the reactions R_1 and R_2. (In all the

subjoined problems, R_1 and R_2 represent the reactions.) The center of moments may be taken at either R_1 or R_2. Taking R_2 as the center in this case, construct a diagram as in Fig.

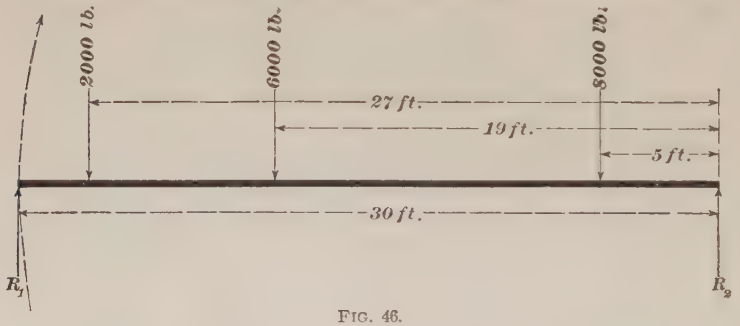


FIG. 46.

46. The three loads are forces acting in a downward direction; the sum of their moments with respect to the assumed center may be computed as follows:

$$\begin{aligned} 8{,}000 \times\ 5 &= \ \ 40{,}000 \text{ ft.-lb.} \\ 6{,}000 \times 19 &= 114{,}000 \text{ ft.-lb.} \\ 2{,}000 \times 27 &= \ \ 54{,}000 \text{ ft.-lb.} \\ \text{Total,} \qquad &\quad 208{,}000 \text{ ft.-lb.} \end{aligned}$$

The magnitude of the reaction R_1 acting in an upward direction with a lever arm of 30 feet is, therefore, $208{,}000 \div 30 = 6{,}933\frac{1}{3}$ pounds. The sum of all the loads is $2{,}000 + 6{,}000 + 8{,}000 = 16{,}000$ pounds. Then the reaction at R_2 is $16{,}000 - 6{,}933\frac{1}{3} = 9{,}066\frac{2}{3}$ pounds.

EXAMPLE 1.—What is the reaction at R_2 in Fig. 47?

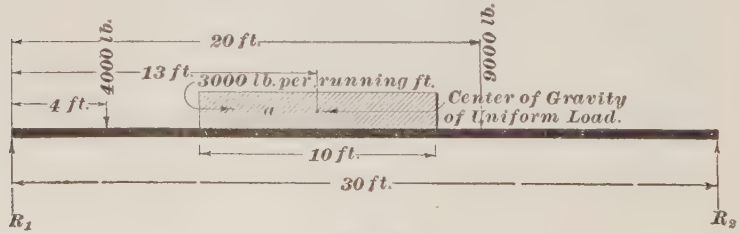


FIG. 47.

SOLUTION.—In computing the moment due to a uniform or evenly distributed load, as at a, the lever arm is always considered as the distance from the center of moments to the center of gravity of the load.

The amount of the uniform load a is $3{,}000 \times 10 = 30{,}000$ pounds, and the distance of its center of gravity from R_1 is 13 feet. The moments of the loads upon this beam about R_1 are as follows:

$$\begin{aligned} 30{,}000 \times 13 &= 390{,}000 \text{ ft.-lb.} \\ 4{,}000 \times 4 &= 16{,}000 \text{ ft.-lb.} \\ 9{,}000 \times 20 &= 180{,}000 \text{ ft.-lb.} \\ \text{Total,} \quad & 586{,}000 \text{ ft.-lb.} \end{aligned}$$

This is the sum of the moments of all the loads about R_1 as a center. The leverage of the reaction R_2 is 30 feet. Hence, the reaction at R_2 is $586{,}000 \div 30 = 19{,}533\frac{1}{3}$ pounds. Ans.

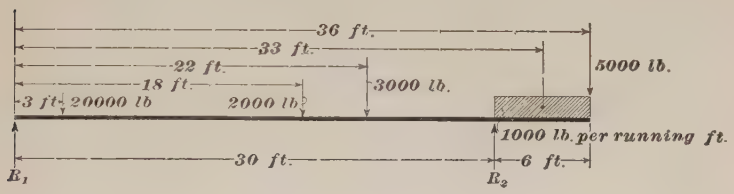


FIG. 48.

EXAMPLE 2.—A beam is loaded as shown in Fig. 48. Compute the reactions R_1 and R_2.

SOLUTION.—Consider R_1 as the center of moments. Then the moments of the loads about R_1 are:

$$\begin{aligned} 20{,}000 \times 3 &= 60{,}000 \text{ ft.-lb.} \\ 2{,}000 \times 18 &= 36{,}000 \text{ ft.-lb.} \\ 3{,}000 \times 22 &= 66{,}000 \text{ ft.-lb.} \\ 5{,}000 \times 36 &= 180{,}000 \text{ ft.-lb.} \\ 1{,}000 \times 6 \times 33 &= 198{,}000 \text{ ft.-lb.} \\ \text{Total,} \quad & 540{,}000 \text{ ft.-lb.} \end{aligned}$$

This divided by 30, the length of the lever arm of the reaction R_2, gives 18,000 pounds, the reaction at R_2. The sum of the loads is $20{,}000 + 2{,}000 + 3{,}000 + 5{,}000 + 6{,}000 = 36{,}000$ pounds; and $36{,}000 - 18{,}000 = 18{,}000$ pounds, the other reaction R_1. Ans.

EXAMPLE 3.—Compute the reactions at the supports R_1 and R_2 in a beam loaded as shown in Fig. 49.

SOLUTION.—Again, letting R_1 be the center of moments, the moments of the loads are:

$$\begin{aligned} 5{,}000 \times 10 &= 50{,}000 \text{ ft.-lb.} \\ 10{,}000 \times 20 &= 200{,}000 \text{ ft.-lb.} \\ 30{,}000 \times 40 &= 1{,}200{,}000 \text{ ft.-lb.} \\ \text{Total,} \quad & 1{,}450{,}000 \text{ ft.-lb.} \end{aligned}$$

Now, $1{,}450{,}000 \div 30$, the distance between the supports $= 48{,}333\frac{1}{3}$ pounds, the required reaction at R_2. The sum of the loads is $5{,}000 + 10{,}000 + 30{,}000 = 45{,}000$ pounds; therefore, the reaction R_2 is greater than the sum of the loads. This shows that the force at R_1 must act in a downward direction in order that the sum of the downward forces may equal the upward force at R_2. Since this is opposite to the usual direction, the reaction at R_1 is called negative or minus. In other words, instead of an upward reaction at R_1, there must be a downward force at this point, or the beam will, as shown by the dotted lines, rotate around

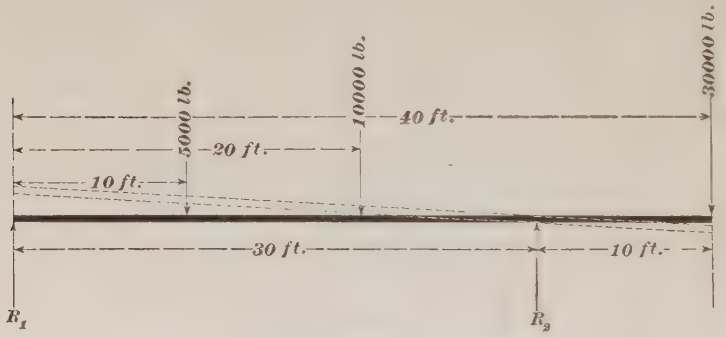


FIG. 49.

the support R_2. The magnitude of this downward force is the difference between the upward reaction at R_2 and the sum of the downward pressures due to the loads; that is, $48{,}333\frac{1}{3} - 45{,}000 = 3{,}333\frac{1}{3}$ pounds.

We will now compute the reaction at R_1 by taking the center of moments at R_2, and applying the rule in Art. **35** to find the magnitude and direction of action of the force at R_2 whose moment is the resultant of the moments of the loads on the beam. The load of 30,000 pounds tends to produce right-hand rotation around the center R_2; hence, its moment, $30{,}000 \times 10 = 300{,}000$ foot-pounds, is positive. The 10,000-pound load is 10 feet to the left of R_2, and its tendency is to produce left-hand rotation about R_2; consequently, its moment is negative and equal to $10{,}000 \times 10 = 100{,}000$ foot-pounds. We find, in a similar manner, the moment of the 5,000-pound load to be negative and equal to $5{,}000 \times 20 = 100{,}000$ foot-pounds. These results may be collected thus:

Positive moment:

$30{,}000 \times 10 =$ . . . . . $+300{,}000$ ft.-lb.

Negative moments:

$10{,}000 \times 10 = 100{,}000$ ft.-lb.

$5{,}000 \times 20 = 100{,}000$ ft.-lb.

$-200{,}000$ ft.-lb.

Algebraic sum, $+100{,}000$ ft.-lb.,

the resultant of the moments of the three loads. Since the positive moment is greater than the sum of the negative moments, we see that to produce equilibrium the force at R_1 must tend to produce left-hand rotation; that is, it must act downwards; its lever arm being 30 feet long, its magnitude must be $100{,}000 \div 30 = 3{,}333\frac{1}{3}$ pounds, the same result as was obtained before.

EXAMPLES FOR PRACTICE.

1. The span of a simple beam is 25 feet; at distances of 9 feet, 16 feet, and 18 feet from the left-hand end are placed concentrated loads of 8,000, 4,000, and 16,000 pounds, respectively. What is the magnitude of the left reaction? Ans. 11,040 lb.

2. The two reactions supporting a beam are 2,500 and 3,000 pounds; what is the single concentrated load necessary to produce these reactions? Ans. 5,500 lb.

3. The length of a beam is 30 feet, and it overhangs the right-hand support 6 feet. From the overhanging end there is a weight of 6,000 pounds; 10 feet, 12 feet, and 18 feet from the left-hand support are loads of 8,000, 6,200, and 7,800 pounds, respectively. What is the magnitude of the right-hand reaction? Ans. $19{,}783\frac{1}{3}$ lb.

4. If for a distance of 10 feet from the left-hand end of a beam is distributed a load of 1,000 pounds per running foot, and at the center of the beam is located the concentrated load of 16,500 pounds, what is the magnitude of the left-hand reaction, providing the beam is supported at both ends and is 30 feet long? Ans. $16{,}583\frac{1}{3}$ lb.

STRESSES IN BEAMS.

83. We have seen that a beam is a body acted on by various external forces so related as to be in a condition of equilibrium; so far, however, we have not considered the effect of these forces on the beam itself.

In a body subjected to a direct pull or thrust, as a rope or a column, the external forces are directly opposed to each other, and the resultant stresses in all sections are of the same kind, tension or compression. In a beam, however, the external forces, while they generally act in parallel lines, are not directly opposed to each other, and it is the function of the beam to transfer these forces from one

line of action to another. Take, for example, the case of a weight suspended from a pin driven in a wall, as shown in Fig. 50. The downward force of 20 pounds due to the action of the weight is balanced by the upward pressure or reaction of the pin on the rope; the rope is thus subjected to the action of two directly opposing forces, the result being a tensile stress which is the same for each section of the rope between the weight and pin. The pin acts as a cantilever beam which transfers horizontally to the wall, where it is balanced by an equal, upward pressure or reaction, the downward pressure due to the pull of the rope. The pin is thus subjected to two opposing forces which, however, act in different lines; these forces produce a set of opposing forces, or stresses, in the pin itself, which are different in kind for different parts of the pin, and vary in magnitude for each section between the rope and the wall.

FIG. 50.

SHEAR.

84. By an inspection of Fig. 50, we see that if we pass a vertical plane through any point between the rope and wall, the part of the pin between this plane and the rope will be acted on by a downward force due to the pull of the rope, while the other is subjected to an equal upward force due to the reaction of the wall; the action of these two forces tends to slide the two parts of the pin past each other, along the section formed by the cutting plane. The pin is thus subject to a stress which, from its similarity to a shearing action, is called **shear.**

85. Shear in a Simple Beam.—Consider now the simple beam shown in Fig. 51. Since the loads are symmetrically applied, each reaction is equal to 40 pounds, one-half of the total load on the beam. Beginning at the left reaction R_1, there is an upward force of 40 pounds acting on

the beam; since the forces are in equilibrium, this upward force is balanced by an equal downward force, which is the vertical resultant of the loads and the reaction R_2. Considering, therefore, any section of the beam between R_1 and the point of application of the load n, we see that the part of the beam at the left of this section is subjected to an upward thrust of 40 pounds, while the part at the right is subjected to an equal downward thrust; the result is a shearing stress on this section, whose magnitude is equal to the reaction R_1.

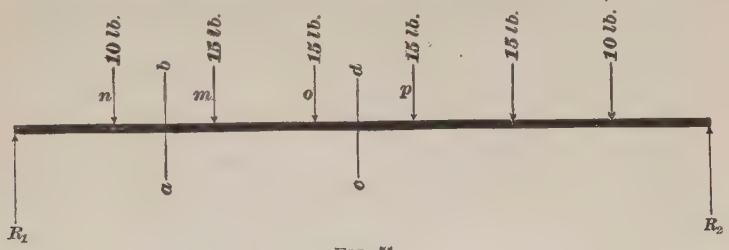


FIG. 51.

When the point of application of n is reached, the effect of the upward force R_1 is partly balanced by the downward force of 10 pounds due to the load n; considering, therefore, any section of the beam, as $a\,b$, between the points of application of the loads n and m, we see that the part of the beam at the left is acted on by the vertical resultant of the reaction R_1 and the load n, that is, by an upward force of $40 - 10 = 30$ pounds, while the part at the right is acted on by an equal downward force, the vertical resultant of the remaining loads and the reaction R_2. Any section between the points of application of n and m is, therefore, subject to a shearing stress equal to the difference between the reaction R_1 and the load n, that is, to $40 - 10 = 30$ pounds. In the same way, it follows that the shearing stress for any section between m and o is $40 - (10 + 15) = 15$ pounds. For any section, as $c\,d$, between the points of application of o and p, the shearing stress is $40 - (10 + 15 + 15) = 0$; in other words, on each side of this section the downward forces and the reactions are equal, and their resultant is zero; it is, therefore, a section in which there is no shear.

For convenience, it is customary to call the reactions, or forces, acting in an upward direction, positive, and the loads, or downward forces, negative; since the difference between the sums of the positive and negative numbers representing a given set of values is called their algebraic sum, it follows that *the shear for any section of a beam is equal to the algebraic sum of either reaction and the loads between this reaction and the given section.*

In nearly all cases the external forces—loads and reactions—act on a beam along vertical lines; the shearing stress just considered being the resultant of these forces along a section formed by an imaginary *vertical* cutting plane, is often called the **vertical shear.**

86. Maximum Shear.—From what has been said, it is evident that the shear in any simple beam is always greatest between the reactions and the nearest loads, and that in any case the maximum shear is equal to the greater reaction.

87. Positive and Negative Shear.—If we take a section of the beam near the left reaction and consider the forces acting on the part of the beam at the left, we see that their resultant is positive; the shear at this section is, therefore, called **positive shear.** If, however, we take a section near the right reaction, the resultant of the forces at the left is found to be negative, and in consequence the shear is called **negative.** It is also evident that there is a section between the two, where the resultant of the forces changes from positive to negative; at such a section the shear is said to **change sign.**

EXAMPLE 1.—(*a*) What is the maximum shear on the beam shown in Fig. 52? (*b*) What is the shear at a point 9 feet from the right support? and (*c*) what is the shear at a point 18 feet from the right support?

SOLUTION.—(*a*) First estimate the reactions as follows: Taking the center of moments at the left support, the moments of the loads are:

$$\begin{aligned} 2{,}000 \times 3 &= 6{,}000 \text{ ft.-lb.} \\ 6{,}000 \times 11 &= 66{,}000 \text{ ft.-lb.} \\ 8{,}000 \times 25 &= 200{,}000 \text{ ft.-lb.} \\ \text{Total,} \quad & 272{,}000 \text{ ft.-lb.} \end{aligned}$$

$272{,}000 \div 30 = 9{,}066\frac{2}{3}$ pounds, the reaction at R_2. The sum of the loads equals $2{,}000 + 6{,}000 + 8{,}000 = 16{,}000$ pounds; $16{,}000 - 9{,}066\frac{2}{3} = 6{,}933\frac{1}{3}$ pounds, the reaction at R_1. The maximum shear is therefore $9{,}066\frac{2}{3}$ pounds. Ans. (*b*) As the reaction R_2 at the right support is equal to $9{,}066\frac{2}{3}$ pounds, and as there is only the one load *c* of 8,000 pounds,

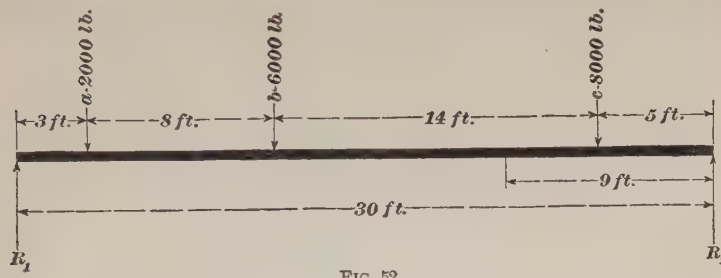


FIG. 52.

between R_2 and a point 9 feet away, the shear at this point must equal $9{,}066\frac{2}{3} - 8{,}000 = 1{,}066\frac{2}{3}$ pounds. Ans. (*c*) The shear at 18 feet from the reaction R_2 is also $1{,}066\frac{2}{3}$ pounds, because there is no other weight occurring between this point and R_2.

EXAMPLE 2.—At what point in the beam loaded as shown in Fig. 53 does the shear change sign?

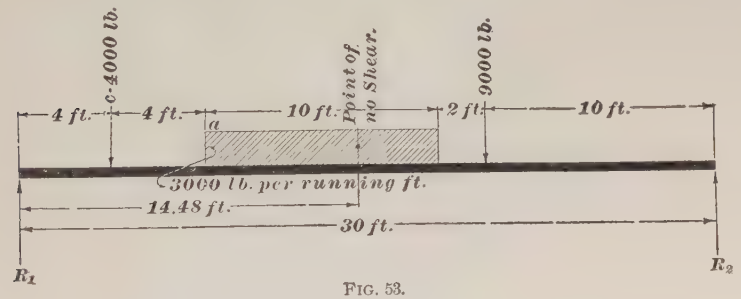


FIG. 53.

SOLUTION.—Compute the reaction R_1 as follows: With the center of moments at R_2 the moments of the loads are:

$$\begin{aligned} 9{,}000 \times 10 &= \quad 9\,0{,}0\,0\,0 \text{ ft.-lb.} \\ 4{,}000 \times 26 &= 1\,0\,4{,}0\,0\,0 \text{ ft.-lb.} \\ 3{,}000 \times 10 \times 17 &= 5\,1\,0{,}0\,0\,0 \text{ ft.-lb.} \\ \text{Total,} \quad &\overline{7\,0\,4{,}0\,0\,0 \text{ ft.-lb.}} \end{aligned}$$

$704{,}000 \div 30 = 23{,}466\frac{2}{3}$ pounds, the reaction at R_1. The first load that occurs, working out on the beam from R_1, is *c* of 4,000 pounds. Then, $23{,}466\frac{2}{3} - 4{,}000 = 19{,}466\frac{2}{3}$ pounds. The next load that occurs on the beam is the uniform load of 3,000 pounds per running foot. There

being altogether 30,000 pounds in this load, it is evident that it will more than absorb the remaining amount of the reaction R_1; the point where the change of sign occurs must consequently be somewhere in that part of the beam covered by the uniform load. The load being 3,000 pounds per running foot, if the remaining part of the reaction, $19{,}466\frac{2}{3}$ pounds, be divided by the 3,000 pounds, the result will be the number of feet of the uniform load required to absorb the remaining part of the reaction, and this will give the distance of the section, beyond which the resultant of the forces at the left becomes negative, from the edge of the uniform load at a; thus, $19{,}466\frac{2}{3} \div 3{,}000 = 6.48$ feet. The distance from R_1 to the edge of the uniform load is 8 feet. The entire distance to the section of change of sign of the shear is, therefore, $8 + 6.48 = 14.48$ feet from R_1. Ans.

EXAMPLES FOR PRACTICE.

1. The uniformly distributed load upon a beam supported at both ends is 40,000 pounds. What is the maximum shear upon the beam?

Ans. 20,000 lb.

2. A beam is loaded with three concentrated loads: A of 2,000 pounds, B of 6,000 pounds, and C of 8,000 pounds; they are located 10 feet, 12 feet, and 18 feet, respectively, from the left-hand end of the beam, the span of which is 40 feet. What is the shear between the loads C and B?

Ans. 2,100 lb.

3. The span of a beam is 20 feet, and there is a uniformly distributed load on three-quarters of this distance from the left-hand support of 9,000 pounds. At distances of 8 feet and 12 feet from the right-hand support are located concentrated loads of 5,000 pounds and 6,000 pounds, respectively. At what distance from the left-hand end of the beam does the shear change sign?

Ans. 8 ft. $8\frac{1}{2}$ in.

BENDING STRESSES.

88. Bending Moment.—If, in a cantilever loaded as in Fig. 54, we take any point x on the center line $a\,b$, as a center of moments, and consider a section made by a vertical plane $c\,d$ through this center, it is evident the moment of the force due to the downward thrust of the load tends to turn the end of the beam to the right of $c\,d$, around the center x; the measure of this tendency is the product of the weight W multiplied by its distance from $c\,d$, and,

since it is the moment of a force which tends to bend the beam, it is called the **bending moment.**

89. Resisting Moment.—A further inspection of Fig. 54 shows that if the end of the beam turns around the center x until it takes the position shown by the dotted lines, the parts of the two surfaces formed by the cutting plane $c\,d$ which are above the center x, must be pulled away from each other, while those below are pushed closer together. We thus see that, if we consider a vertical section through any point on the center line $a\,b$, between the load and the point of support, the tendency of the load is to separate the particles in this section above the center line, and to push those below the center line closer together; in other words, through the bending action of the load, the upper part of the beam is subjected to a tensile stress, while the lower is subjected to a compressive stress.

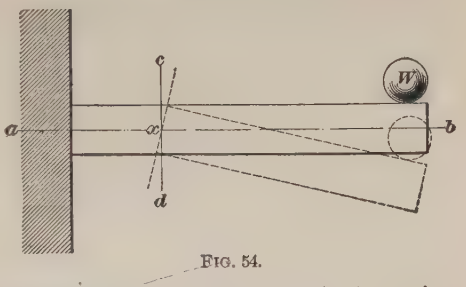


FIG. 54.

Fig. 54 also shows that the greater the distance of the particles in the assumed section above or below the center x, the greater will be their displacement; since the stress in a loaded body is directly proportional to the strain, or relative displacement of the particles, it follows that the stress in a particle of any section is proportional to its distance from the center line, and that the greatest stress is in the particles composing the upper and lower surfaces of the beam.

In accordance with the conditions of equilibrium, the algebraic sum of the moments of all the forces tending to produce rotation around a given center must be zero; we have seen that the weight of the load is a force which tends to produce right-hand rotation around the center x; therefore, if the beam does not break under the action of the

load, there must be forces acting whose moments, with respect to the center x, balance the moment of the load. These forces are the resistances with which the particles of the beam oppose any effort to change their relative positions. The tensile stresses in the particles above the center x, and the compressive stresses in those below it, are a set of forces which resist the tendency of the load to turn the end of the beam, and, when the effect of the load is just balanced by the effect of these forces, it is evident that the sum of the moments of these resisting stresses is equal to the moment of the load. *The sum of the moments of the stresses of all the particles composing any section of a beam is called the* **resisting moment** *or* **moment of resistance** *of that section.*

90. Neutral Axis.—In Fig. 55, let $A\,B\,C\,D$ represent a cantilever. A force F acts upon it, at its extremity A; the principles developed in Art. **89** show that the force will tend to bend the beam into the shape shown by $A'\,B\,C\,D'$.

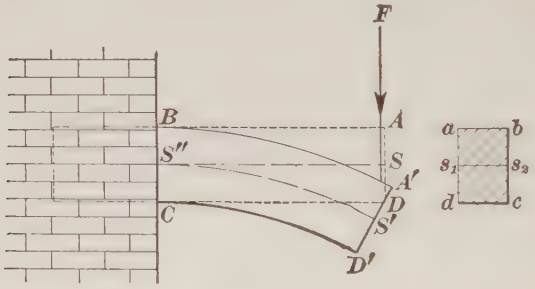


FIG. 55.

It is evident from what has preceded that the upper part $A'\,B$ is now longer than it was before the force was applied; i. e., $A'\,B$ is longer than $A\,B$. It is also evident that $D'\,C$ is shorter than $D\,C$. Hence, the effect of the force F in bending the beam is to lengthen the upper fibers and to shorten the lower ones. Further consideration will show that there must be a fiber, $S\,S''$, which is neither lengthened nor shortened when the beam is bent, i. e., $S\,S'' = S'\,S''$.

When the beam is straight, the fiber $S\,S''$, which is neither lengthened nor shortened when the beam is bent, is called the **neutral line.**

The neutral line corresponds to the center line $a\,b$, Fig. 54, on which the center of moments x was taken.

91. The relations between the effect of a load and the resulting stresses in a beam have been thoroughly proved, both by mathematical investigations and numerous experiments. The results of these experiments on beams may be briefly expressed by the following

Experimental Law.—*When a beam is bent, the horizontal elongation (or compression) of any fiber is directly proportional to its distance from the neutral surface, and, since the strains are directly proportional to the horizontal stresses in each fiber, they are also directly proportional to their distances from the neutral surface, provided the elastic limit is not exceeded.*

The line $s_1\,s_2$, which passes through any section, as $a\,b\,c\,d$, Fig. 55, perpendicular to the neutral line, is called the **neutral axis,** and the surface $S_1\,S'\,S''\,S_2$, Fig. 56, which

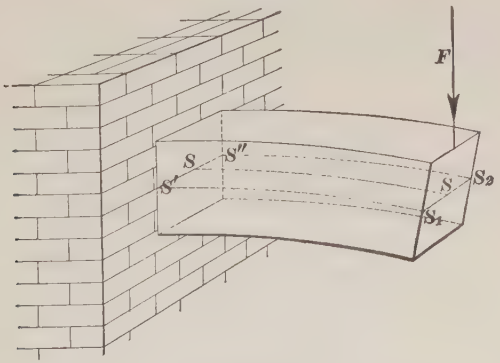


FIG. 56.

separates all the parts of the beam above any neutral line, or neutral axis, from those below, is called the **neutral surface.** The neutral axis, then, is the line of intersection

of a cross-section with the neutral surface. It is shown in works on mechanics that *the neutral axis always passes through the center of gravity of the cross-section of the beam.*

92. Bending Moments in Simple Beams.—Referring to the simple beam shown in Fig. 52, let us take the center of moments on the neutral axis directly under the load a, and consider the effect produced on a vertical section of the beam through this center, by the reaction R_1. It was shown in Art. **87** that the reaction R_1 is an upward force of $6{,}933\frac{1}{3}$ pounds; it therefore has a tendency to turn the end of the beam upwards around the assumed center with a moment of $6{,}933\frac{1}{3} \times 3 = 20{,}800$ foot-pounds. It is evident that, to prevent this turning from actually taking place, the positive moment of the reaction must be balanced by a negative moment, which can be produced only by a set of internal stresses. The condition that the moment of the stresses must be negative makes it plain that the upper fibers must be in compression and the lower in tension, a result exactly opposite to the effect produced by the bending moment on the fibers in the cantilever.

93. Effect of the Moments Due to Loads.—The only force acting on the beam at the left of the section considered in Art. **92** was the reaction R_1. The load a acted downwards directly through this section, but its lever arm, and consequently its moment, with respect to the assumed center, was zero. Take now a point on the center line of the beam directly under the positive load b. The reaction has a moment with respect to this center of $6{,}933\frac{1}{3} \times 11 = 76{,}266\frac{2}{3}$ foot-pounds, while the load a, which acts downwards with a lever arm of 8 feet, has a negative moment of $2{,}000 \times 8 = 16{,}000$ foot-pounds. The bending moment at the assumed section is the algebraic sum of these moments, that is, $76{,}266\frac{2}{3} - 16{,}000 = 60{,}266\frac{2}{3}$ foot-pounds. Again, taking the center of moments on a section 9 feet from the right reaction R_2, the moments are as follows:

Positive moment:

Reaction R_1, $6{,}933\frac{1}{3} \times 21 =$ 145,600 ft.-lb.

Negative moments:

Load a, $2{,}000 \times 18 = 36{,}000$ ft.-lb.

Load b, $6{,}000 \times 10 = 60{,}000$ ft.-lb. 96,000 ft.-lb.

Difference . . . . . 49,600 ft.-lb.

This resultant moment is the bending moment at the given section.

94. The illustrations show that the bending moment varies from point to point in a beam, and depends on the length of the beam and on the size as well as position of the loads. Since the stresses in the beam, and consequently its ability to carry its loads, depend directly on the bending moment, it follows that it is important to find, not only the bending moment for any assumed section, but also the section where the bending moment is greatest. It is, in this connection, useful to note the relation between the bending moment and the shear.

95. The shear in a simple beam is always greatest at the greater reaction, being equal to that reaction. In passing along the beam from either reaction, there is no change in the shear until a load is reached. At each point where a load is added, the shear is diminished by an amount equal to the load. At the point where the sum of the added loads equals or exceeds the reaction, the shear is said to change sign. The section where the change in sign in the shear takes place, depends on the method of loading. With a uniformly distributed load, the shear diminishes uniformly from each reaction, and the section where the sign changes is the section of the beam midway between the supports. With a single concentrated load, the shear is equal to each reaction at all sections between that reaction and the point where the load is applied, and the section where the shear changes sign is directly under the load. With any system of loading, the section where the shear changes sign can be found by adding together the successive loads from either reaction towards the center of the beam, until a sum is obtained

which equals or exceeds the reaction; the section where the shear changes sign is under the point of application of the last load added.

96. The bending moment in a simple beam increases as the shear decreases; it is zero at either reaction and increases towards the center, becoming greatest at the section where the shear changes sign. With a uniformly distributed load, the greatest bending moment is at the section of the beam midway between the supports; with a single concentrated load, the greatest bending moment is directly under the load; and with any system of loading, the greatest bending moment occurs at the section where the shear changes sign. Having located the section of the greatest bending moment, the magnitude of this moment can be readily computed by taking the center of moment on the section of greatest bending moment and computing the resultant moment of either reaction and all the loads between it and the center in question.

EXAMPLE.—A wooden beam, supported on two brick piers, is loaded as shown in Fig. 57. (*a*) What is the greatest shear? (*b*) Where does

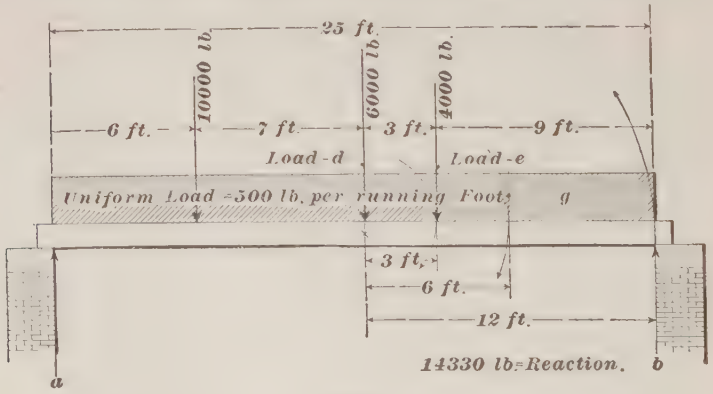


FIG. 57.

the shear change sign? (*c*) What is the greatest bending moment in inch-pounds?

SOLUTION.—(*a*) Since the greatest shear is equal to the greater reaction, we will first compute the reactions. Taking the center of moments

at the edge of pier *a*, and remembering that the moment of the uniform load is the same as the moment of an equal concentrated load acting at the center of gravity of the uniform load, the moments of the loads are:

10,000-pound load	$10{,}000 \times 6$	=	60,000 ft.-lb.
Load *d*	$6{,}000 \times 13$	=	78,000 ft.-lb.
Load *e*	$4{,}000 \times 16$	=	64,000 ft.-lb.
Uniformly distributed load	$12{,}500 \times 12\frac{1}{2}$	=	156,250 ft.-lb.
Total			358,250 ft.-lb.

This also equals the moment of the reaction of the pier *b*. The reaction at *b* is, therefore, $358{,}250 \div 25 = 14{,}330$ pounds. The total load is $10{,}000 + 6{,}000 + 4{,}000 + 12{,}500 = 32{,}500$ pounds; the reaction at *a* is, therefore, $32{,}500 - 14{,}330 = 18{,}170$ pounds. This, being the greater reaction, is the greatest shear. Ans.

(*b*) Beginning at the left reaction and adding the loads in succession towards the right, we find that the load of 10,000 pounds plus the uniformly distributed load between the reaction and the point of application of the load *d*, is $10{,}000 + 500 \times 13 = 16{,}500$ pounds. This is less than the left reaction, but when we add the load *d*, the sum of the loads is greater than the reaction; consequently, the shear changes sign under the load *d*. Ans.

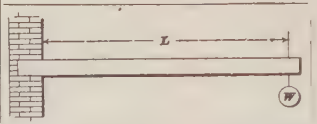
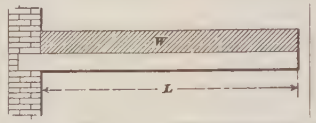
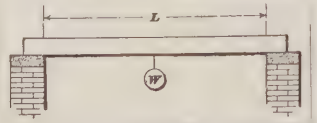
(*c*) Taking the center of moments, under the load *d*, and considering the forces at the left, we have

Positive moment:		
$18{,}170 \times 13$ =		236,210 ft.-lb.
Negative moments:		
$6{,}500 \times 6\frac{1}{2}$ =	42,250 ft.-lb.	
$10{,}000 \times 7$ =	70,000 ft.-lb.	
		112,250 ft.-lb.
Difference (bending moment in ft.-lb.)		123,960 ft.-lb.

The greatest bending moment in inch-pounds is, therefore, $123{,}960 \times 12 = 1{,}487{,}520$ inch-pounds. Ans.

97. General Formulas for Bending Moments. – Where a cantilever or a simple beam is symmetrically loaded, it is not necessary to calculate the moments of the forces acting upon the beam in order to find the reactions and bending moments. The rules and formulas of Table 10 are available in computing the bending moment M in the five simple cases of beam loading set forth.

TABLE 10.

Case.	Method of Loading.	Rule.	Formula.
I		To obtain the bending moment in foot-pounds on this beam: Multiply the weight W in pounds by the distance L in feet.	$M = WL.$ **(10.)**
II		To obtain the bending moment in foot-pounds on this beam: Multiply the weight W in pounds by the distance L in feet and divide by 2.	$M = \frac{WL}{2}.$ **(11.)**
III		To obtain the bending moment in foot-pounds on this beam: Multiply the weight W in pounds by the distance L in feet and divide by 4.	$M = \frac{WL}{4}.$ **(12.)**
IV		To obtain the bending moment in foot-pounds on this beam: Multiply the weight W in pounds by the distance L in feet and divide by 8.	$M = \frac{WL}{8}.$ **(13.)**
V		To obtain the bending moment in foot-pounds on this beam: Multiply the weight W in pounds by the distance L in feet and divide by 6.	$M = \frac{WL}{6}.$ **(14.)**

98. The rule given in Case IV is that most used, as it applies to a beam uniformly loaded, such as floor joists, girders, and, in some cases, the rafters of a roof. The rule in Case V is convenient in calculating the bending moment on lintels supporting brickwork or masonry over openings. If, in Case III, the beam is firmly fixed or fastened at both ends, the bending moment under the same load will be only half as much. If, in Case IV, the ends of the beam are firmly fixed, instead of dividing by 8 a constant of 12 is to be used. It is seldom advisable, in ordinary building practice, to consider the ends of a beam fixed, it being good practice to assume the ends of the beams as simply bearing on the wall, using the rules and formulas in Table 10. It should be, however, understood that all these rules and formulas apply to static loads. The same load suddenly applied produces a stress in the beam twice as great as that of a static load. The safe sudden load is, therefore, only half as much.

EXAMPLE.—What will be the bending moment in inch-pounds on a wood girder supporting a floor area of 150 square feet, the dead and live load being 100 pounds per square foot, and the span of the girder 20 feet?

SOLUTION.—The total uniformly distributed load is $150 \times 100 = 15{,}000$ pounds; therefore, by applying the formula in Case IV, Table 10, we have the bending moment $M = \frac{WL}{8} = \frac{15{,}000 \times 20 \times 12}{8}$ = 450,000 inch-pounds. Ans.

EXAMPLES FOR PRACTICE.

1. A beam has a span of 20 feet, and is loaded with a uniformly distributed load of 2,500 pounds per lineal foot. What is the greatest bending moment in inch-pounds upon the beam? Ans. 1,500,000 in.-lb.

2. What is the bending moment in foot-pounds on a cantilever beam securely fastened into a wall, extending from the point of support 10 feet, and loaded with a uniformly distributed load of 1,000 pounds per lineal foot? Ans. 50,000 ft.-lb.

3. What is the bending moment in inch-pounds on a girder having a span of 30 feet, if there is a uniformly distributed load of 1,500 pounds per lineal foot, and a load of 20,000 pounds concentrated at the center? Ans. 3,825,000 in.-lb.

4. A plate girder in a building is required to support a uniformly distributed load of 2,000 pounds per lineal foot, extending 20 feet each side of the center of the girder; in addition, it is required to support a load of 30,600 pounds, concentrated 10 feet from one end of the girder, and another load of 43,000 pounds, located 22 feet from the same end. What will be the greatest bending moment on the girder, in foot-pounds, if the span is 60 feet? Ans. 1,528,933 ft.-lb.

STRENGTH OF BEAMS.

99. Resisting Moment.—It was stated in Art. **89,** that the resisting moment of any section of a beam is the sum of the moments of all the stresses produced in that section by the bending moment; it was also shown that the resisting moment must equal the bending moment. By higher mathematics it is proved that the resisting moment is equal to the product of the greatest unit stress in any part of a section multiplied by a factor, called the **section modulus,** or the **resisting inches,** which depends on the shape of the section.

100. If we assume the greatest unit stress to be the modulus of rupture of the material composing a beam, we have the following rule:

Rule.—*To find the ultimate resisting moment of a beam, multiply the section modulus by the modulus of rupture of the material of which the beam is composed.*

The modulus of rupture for the materials used in building construction may be obtained from Table 6.

101. Section Modulus.—The general method for finding the section modulus of any section of a beam will not be discussed here, but a number of rules, formulas, and tables are given from which to find the section modulus of the sections used in ordinary building operations.

Rule.—*To obtain the section modulus of any rectangular beam, multiply the square of its depth in inches by its width in inches and divide by 6.*

This rule is expressed by the formula

$$K = \frac{b\,d^2}{6}, \qquad \textbf{(15.)}$$

in which K = section modulus;
d = depth of beam in inches;
b = breadth or width of beam in inches.

EXAMPLE 1.—(*a*) What will be the section modulus of a yellow-pine beam, 10 inches wide by 12 inches in depth? (*b*) What will be the ultimate resisting moment of this beam?

SOLUTION.—(*a*) Here $d = 12$ inches;
$b = 10$ inches.

Therefore, by applying the formula, we have

$$K = \frac{b\,d^2}{6} = \frac{10 \times 12^2}{6} = \frac{1{,}440}{6} = 240. \quad \text{Ans.}$$

(*b*) According to Table 6, the modulus of rupture for yellow pine is 7,300 pounds per square inch; hence, the resisting moment of the $10'' \times 12''$ yellow-pine beam is $240 \times 7{,}300 = 1{,}752{,}000$ inch-pounds. Ans.

EXAMPLE 2.—What size of spruce girder is required to support a uniformly distributed load of 500 pounds per lineal foot, the span of the girder being 22 feet, and the factor of safety 4?

SOLUTION.—First find the bending moment in inch-pounds. The total load on the girder is $500 \times 22 = 11{,}000$ pounds. The bending moment due to a uniformly distributed load, according to Table 10, is $\frac{W\,L}{8}$. Then $\frac{11{,}000 \times 22}{8} = 30{,}250$ foot-pounds, which multiplied by 12 gives 363,000, the bending moment in inch-pounds. To find the size of a spruce beam having a safe resisting moment equal to this bending moment, take a $10'' \times 14''$ beam—the section modulus from formula **15** being

$$K = \frac{b\,d^2}{6} = \frac{10 \times 14^2}{6} = 326.$$

The modulus of rupture of spruce being 4,800, the ultimate resisting moment of the beam is $326 \times 4{,}800 = 1{,}564{,}800$ inch-pounds. If a factor of safety of 4 is used, the safe resisting moment of the beam is $1{,}564{,}800 \div 4 = 391{,}200$ inch-pounds. Since the bending moment is only 363,000 inch-pounds, the beam is amply strong. Ans.

102. To determine the uniformly distributed load that will break a beam whose size is known, take the formula

$$B = \frac{\left(\frac{K \times S}{12}\right)8}{L} = \frac{2\,K\,S}{3\,L}, \qquad \textbf{(16.)}$$

where K = the section modulus of the beam;
S = modulus of rupture of the material;
L = span of beam in feet;
B = breaking load of rectangular beam.

This formula may be stated by the following rule:

Rule.—*To determine the breaking load in pounds of any rectangular beam, multiply the sectional modulus of the beam by the modulus of rupture of the material; multiply this product by 2 and divide the result by 3 times the span of the beam in feet.*

EXAMPLE.—(*a*) What uniformly distributed load will break a hemlock joist, 3 inches by 14 inches, the span being 25 feet? (*b*) What will be the safe load if a factor of safety of 4 is used?

SOLUTION.—(*a*) Section modulus $= \frac{b\,d^2}{6} = \frac{3 \times 14^2}{6} = 98$. The modulus of rupture for hemlock, according to Table 6, is 3,600 pounds per square inch. Substituting these values in the formula, we have

$$B = \frac{2 \times 98 \times 3{,}600}{3 \times 25} = 9{,}408 \text{ pounds.} \quad \text{Ans.}$$

(*b*) If a factor of safety of 4 is used, the safe uniformly distributed load that this beam will carry is 9,408 pounds $\div$ 4 = 2,352 pounds. Ans.

103. Steel Beams.—Steel beams used in building construction may be either **channels, I beams, angles,** or **tees.** For floorbeams, channels and I beams are principally used, while angles and tees are used in roof trusses and like construction. The strength of channels and I beams only are here to be considered.

In engineering, the two rolled shapes, channels and I beams, are generally called channels and beams. When the word *beam* is used, it is generally understood that an I beam is meant. For instance, a 12-inch, 40-pound beam would mean an I beam 12 inches in depth, weighing 40 pounds to the lineal foot; while a channel of the same size would be expressed as a 12-inch, 40-pound channel. In designating rolled shapes on working drawings, various systems of abbreviations are used. A 12-inch, 40-pound beam may be expressed as 12″ I 40♯, or a channel as 12″ ⊏ 40♯.

This is entirely a matter of judgment with the draftsman, or is governed by the practice used in the particular drafting room. As long as the size, character, and weight of the beam are given, it matters little how expressed, if intelligibly written.

104. Strength of Steel Beams.—In calculating the strength of steel beams, it is first necessary to find the bending moment, using the methods and rules already laid down. Then the section modulus required in the beam may be obtained by dividing the bending moment in inch-pounds by the quotient obtained by dividing the modulus of rupture by the factor of safety. Assume, for example, the bending moment on a beam to be 50,000 foot-pounds. Reduce it to inch-pounds by multiplying it by 12, which gives 600,000 inch-pounds. The modulus of rupture for structural steel is 60,000 pounds. If a factor of safety of 4 is used, the safe working value of this material will be $60{,}000 \div 4 = 15{,}000$ per square inch. Then $600{,}000 \div 15{,}000 = 40$, the section modulus required.

105. The approximate section modulus of an **I** beam or a channel may be found by the following rules:

Rule I.—*To obtain the approximate section modulus of an* **I** *beam, multiply the sectional area of the beam in square inches by the depth in inches, and divide by the constant 3.2.*

Rule II.—*To obtain the approximate section modulus of a channel, multiply the sectional area of the channel in square inches by the depth in inches, and divide by the constant 3.67.*

Letting a = the sectional area of an **I** beam or a channel in square inches, and h its depth in inches, the approximate section modulus of an **I** beam may be found from the formula

$$K_i = \frac{a\,h}{3.2}, \qquad \textbf{(17.)}$$

and of a channel, $$K_c = \frac{a\,h}{3.67}. \qquad \textbf{(18.)}$$

EXAMPLE.—What is the section modulus of a 12-inch **I** beam, the sectional area of which is 9.01 square inches?

SOLUTION.—Applying the formula, we have

$$K_t = \frac{ah}{3.2} = \frac{9.01 \times 12}{3.2} = 33.7. \quad \text{Ans.}$$

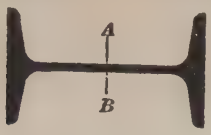
106. Tables 11 and 12 give the principal elements of **I** beams and channels, and obviate the necessity of calculating the section modulus, this being given under the column headed "Section Modulus on Axis *A B*."

The **moment of inertia** on the axis *A B* is also given in these tables. The section modulus of any beam may be obtained by dividing the moment of inertia of the beam section by one-half the depth of the beam in inches. In Table 11, for instance, "Elements of **I** Beams," a 15-inch, 69.2-pound beam has a moment of inertia of 710. If this is divided by one-half the depth of the beam in inches, in this case $7\frac{1}{2}$, the result obtained will be $94.66\frac{2}{3}$, which, as may be seen by referring to the column headed "Section Modulus on *A B*," is the correct section modulus of this beam.

107. To illustrate the method of calculating the dimensions of a steel beam, let it be required to find what size of steel **I** beams is necessary to support the floor of an office building, this floor resting on brick arches sprung between the beams and weighing complete 110 pounds per square foot. The building is designed to carry a live load of 40 pounds per square foot. The span of the beams is 20 feet, spaced 5 feet on centers. The owner requires that the building have a large factor of safety, and suggests that for the floorbeams a safety factor of 5 be used. The total dead and live load on the floor is 110 lb. + 40 lb. = 150 pounds per square foot. The floor area supported by one beam is $20 \times 5 = 100$ square feet. Then the total load on one beam is $100 \times 150 = 15{,}000$ pounds. The load being uniformly distributed, the formula for the bending moment, according to Table 10, is $M = \frac{WL}{8}$; substituting the values for W and L, $M = \frac{15{,}000 \times 20}{8} = 37{,}500$, the bending moment in foot-pounds, which, being multiplied by 12, gives 450,000 inch-pounds.

TABLE 11.

ELEMENTS OF I BEAMS.



Size in Inches.	Area in Square Inches.	Weight in Pounds per Foot.	Moments of Inertia on Axis *A B*.	Section Modulus on Axis *A B*.
3	1.56	5.3	2.41	1.61
3	2.01	6.8	2.75	1.83
4	1.81	6.2	5.02	2.51
4	2.17	7.4	5.82	2,91
5	2.76	9.4	11.58	4.63
5	3.61	12.3	13.34	5.34
6	3.51	11.9	21.14	7.05
6	4.47	15.2	24.02	8.01
6	9.49	32.3	52.53	17.51
6	10.99	37.4	57.03	19.01
6	12.06	41.0	64.07	21.36
6	13.56	46.1	68.57	22.86
7	4.31	14.6	35.43	10.12
7	5.29	17.9	39.43	11.27
8	5.12	17.4	54.31	13.58
8	6.24	21.2	60.28	15.07
9	6.04	20.5	80.78	17.95
9	7.48	25.4	90.50	20.11
10	6.91	23.5	112.42	22.48
10	8.78	29.8	141.44	28.29
10	8.91	30.3	129.08	25.82
10	10.28	34.9	153.94	30.79
12	9.01	30.6	207.90	34.65
12	11.29	38.4	233.80	38.97
12	11.60	39.4	268.30	44.72
12	14.00	47.6	299.76	49.96
12	16.32	55.5	362.88	60.48
12	19.68	66.9	403.38	67.23
15	12.11	41.2	433.00	57.73
15	14.49	49.3	518.61	69.15
15	15.56	52.9	497.68	66.36
15	16.74	56.9	560.79	74.77
15	16.95	57.6	583.78	77.84
15	20.38	69.2	710.00	94.67
15	20.70	70.4	654.09	87.21
15	25.03	85.1	789.24	105.23
20	19.04	64.8	1,145.79	114 58
20	22.94	78.0	1,367.37	136.74
20	24.04	81.7	1,312.46	131.24
20	28.94	98.4	1,567.37	156.74

TABLE 12.

ELEMENTS OF CHANNELS.

Size in Inches.	Area in Square Inches.	Weight in Pounds per Foot.	Moments of Inertia on Axis *A B*.	Section Modulus on Axis *A B*.
$1\frac{3}{4}$	0.33	1.1	0.14	0.16
2	0.87	2.97	0.48	0.48
2	1.06	3.60	0.52	0.52
$2\frac{1}{4}$	1.12	3.8	0.80	0.35
3	1.51	5.1	2.04	1.36
3	1.78	6.1	2.24	1.49
4	1.58	5.4	3.67	1.84
4	2.30	7.8	4.64	2.32
5	1.93	6.6	6.64	2.66
5	2.83	9.6	8.52	3.41
6	2.27	7.7	11.62	3.87
6	3.24	11.0	17.54	5.84
6	3.35	11.4	14.86	4.95
6	5.46	18.6	24.20	8.06
7	2.67	9.1	19.39	5.54
7	3.75	12.8	27.14	7.75
7	3.86	13.1	24.25	6.93
7	7.11	24.2	40.86	11.67
8	3.22	10.9	29.76	7.44
8	4.29	14.6	39.76	9.94
8	4.42	15.0	36.16	9.04
8	6.05	20.6	49.15	12.28
9	3.94	13.4	46.48	10.33
9	5.17	17.6	59.89	13.31
9	5.74	19.5	58.63	13.03
9	7.96	27.1	78.72	17.49
10	4.84	16.5	71.09	14.22
10	6.17	20.9	88.01	17.60
10	7.24	24.6	91.09	18.62
10	10.37	35.3	123.01	24.60
12	5.81	19.8	118.32	19.72
12	6.07	20.6	123.38	20.56
12	9.17	31.2	157.20	26.20
12	9.24	31.4	189.93	31.65
12	9.43	32.1	163.71	27.28
12	16.32	55.5	274.89	45.81
15	9.62	32.7	286.84	38.25
15	14.64	49.8	435.72	58.09
15	14.87	50.6	385.28	51.30
15	20.34	69.2	542.59	72.34

The modulus of rupture for structural steel being 60,000 pounds per square inch, and, since a factor of safety of 5 is required, the safe working value will be $60{,}000 \div 5 = 12{,}000$ pounds per square inch. The bending moment in inch-pounds is 450,000, which divided by 12,000 gives a section modulus of 37.5.

Referring to Table 11, it is seen that the section modulus on the axis AB of a 12-inch, 39.4-pound beam is 44.72, which is ample, for a modulus of 37.5 only is required.

Referring, also, to the 15-inch beam in the same table, it is seen that the section modulus of a 15-inch beam, 41.2 pounds, is 57.73, and, as it weighs not 2 pounds more per foot, it might be advisable to use this beam, especially as the owner requires a large factor of strength.

In selecting beams from the table, care should be taken to obtain the deepest beams of the least weight with the required section modulus. Thus, by referring to Table 11, it is seen that the section modulus of a 10-inch beam, 30.3 pounds, is 25.82, while a 12-inch beam of 30.6 pounds, of nearly the same weight as the 10-inch beam, has a modulus of 34.65, and, in consequence, possesses nearly one-third more strength, making it, therefore, the more economical beam to use.

108. By way of general review of the subject of beams, we present the following practical example: Fig. 58 shows the transverse sectional elevation of a large department store. It will require two **I** beams to form the girder B. What will be the size and weight of these steel beams?

Before commencing the calculations, draw the outline diagram as shown in Fig. 59. These are called frame diagrams. The two supports for the girder are the wall W and the column C. The loads upon the girder are the two uniform loads g and h. The load h is due to the weight of the floor, girder, and the ceiling, together with the live load on the floor, due to the people, furniture, etc. This load has been assumed to amount to 500 pounds per running foot of the girder. The load g being due only to the ceiling and a portion

of the roof, and there being no floor load upon it, has been considered as amounting to 200 pounds per running foot.

The girder is also loaded with four concentrated loads: *a* of 10,000 pounds, due to the weight of the light wall and a portion of the roof; *d* of 20,000 pounds, due to the load coming down the small column from a portion of the roof; and two hanging loads *f* and *e*, of 3,000 and 2,000 pounds, respectively, from the weight of the stair landing or hall.

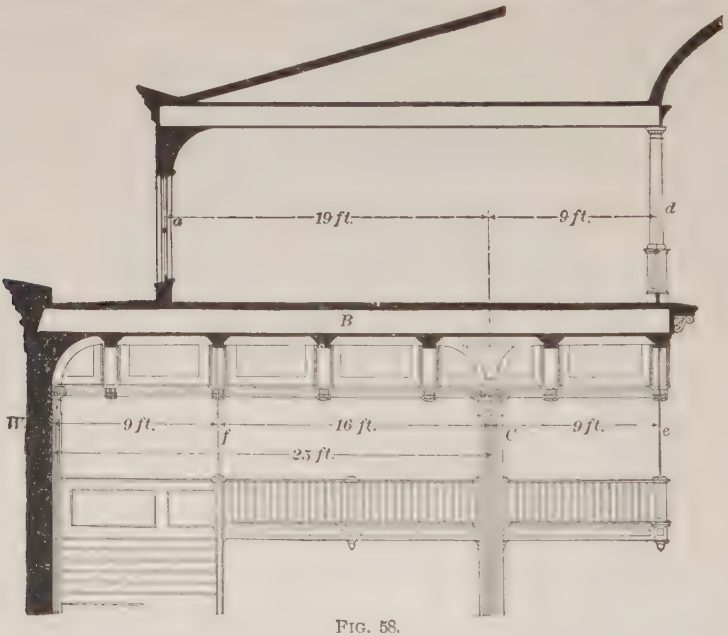


FIG. 58.

Let us now calculate the reactions. The moments about *W*, due to the various loads, are as follows:

		Ft.-Lb.
Load *g* (200 × 6 = 1,200 lb.)	1,200 × 3 =	3,600
Load *h* (500 × 28 = 14,000 lb.)	14,000 × 20 =	280,000
Load *a*	10,000 × 6 =	60,000
Load *f*	3,000 × 9 =	27,000
Load *d*	20,000 × 34 =	680,000
Load *e*	2,000 × 34 =	68,000
Total		1,118,600

This, divided by the distance between the supports, or the span, 25 feet, gives 44,744, the load in pounds coming on

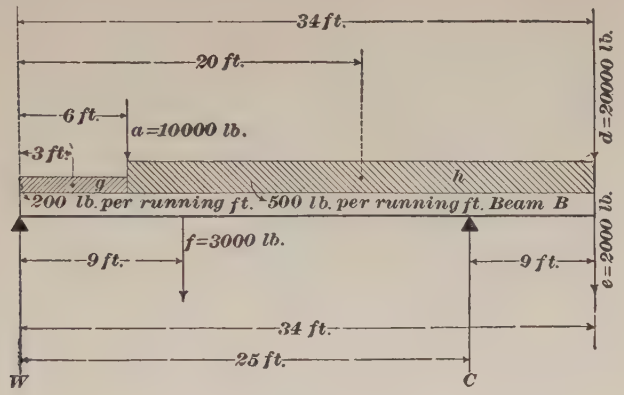


FIG. 59.

the column C; or, in other words, the reaction at C. The loads are as follows:

Load g =	1,2 0 0 lb.
Load h =	1 4,0 0 0 lb.
Load a =	1 0,0 0 0 lb.
Load f =	3,0 0 0 lb.
Load d =	2 0,0 0 0 lb.
Load e =	2,0 0 0 lb.
Total load =	5 0,2 0 0 lb.

Then, the reaction at W is $50{,}200 - 44{,}744 = 5{,}456$ pounds.

We next find the point between the two supports W and C, where the shear changes sign. Starting from W, the first load encountered and to be deducted from the reaction W is the uniform load g, equal to $200 \times 6 = 1{,}200$ pounds. Then, 5,456 (reaction at W) $-$ 1,200 (load g) $=$ 4,256 pounds. The next load on the beam is the concentrated load a of 10,000 pounds, which is much more than the remaining portion of the reaction W. The greatest bending moment occurring between the columns and the wall is, therefore, at the point a, and is equal to 5,456 (reaction at W) $\times 6$ $=$ 32,736 foot-pounds, less the moment of the load g of

$1,200 \times 3 = 3,600$ foot-pounds, or $32,736 - 3,600 = 29,136$ foot-pounds.

Again referring to the diagram, Fig. 59, it is seen that there is a large bending moment directly over the column *C*, due to the two concentrated loads *d* and *e* on the end of the beam and the portion of the uniform load *h* overhanging the support *C*. This portion of the beam may be considered as a cantilever; the bending moment at *C* is equal to the sum of the moments of all the loads on the overhanging portion of the beam, which are: Load *d*, $20,000 \times 9 = 180,000$ foot-pounds. Load *e*, $2,000 \times 9 = 18,000$ foot-pounds. Load *h* (overhanging portion $= 500 \times 9 = 4,500$ pounds) $= 4,500 \times 4.5 = 20,250$ foot-pounds. Total: 218,250 foot-pounds, or $218,250 \times 12 = 2,619,000$ inch-pounds. Since this bending moment is greater than that under the load *a*, it is used in determining the size of the beam. This bending moment divided by 20,000, the safe working value of structural steel (using the modulus of rupture of 60,000 pounds $\div 3$, the safety factor used in this case), gives 131, the required section modulus in the two beams. Then, the section modulus required in one of the beams is $131 \div 2 = 65.5$.

Referring to Table 11, it is seen that the section modulus of a 15-inch beam, 49.3 pounds, is 69.15. While this is in excess of the required amount, it is, in this case, the most economical beam to use, and two of this kind are required.

109. Relation Between Span and Depth of Beam.— In order to select beams that will not deflect too much under the load they are required to sustain, the depth of the beam in inches should never be less than half the span of the beam in feet. Thus, if the span of the beam be 20 feet, a beam not less than 10 inches in depth should be used, to avoid excessive deflection.

110. Separators for I Beams.—In building construction, it frequently happens that a single I beam is insufficient to carry the imposed load. Where heavy loads, such as brick walls, vaults, etc., are to be supported, a single I beam is

inadequate, and two or more beams are applied side by side, bolted together with cast-iron or steel **separators,** shown in Fig. 60. These separators hold the compression flanges of

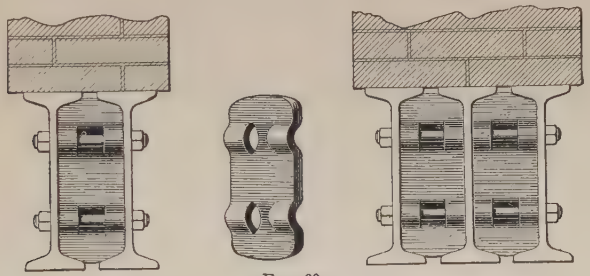
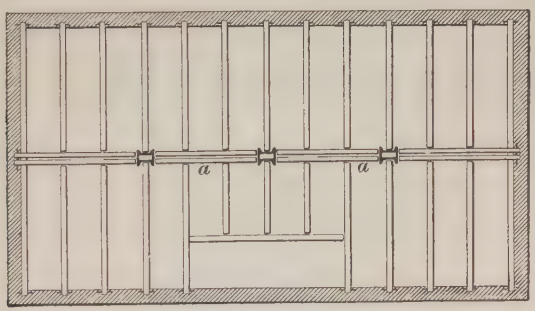


FIG. 60.

the beams in position, preventing deflection sideways, and also, in a measure, cause the beams to act together, distributing the load uniformly on both. Separators should be



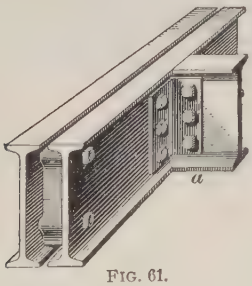


FIG. 61.

spaced from 6 to 7 feet throughout the length of the beam; they should also be provided at the supports and at points where heavy loads are concentrated.

111. Beam Girders.—In designing floors of buildings it is desirable to have a minimum number of interior supporting columns, consistent with economy. A beam girder consisting of a pair of I beams is frequently advantageous for supporting the steel floorbeams as shown at *a* in Fig. 61.

Girders composed of two or more I beams are commonly used to span openings in brick walls. If the wall to be supported is thoroughly seasoned and without openings, the weight carried by the girder can safely be assumed as the weight of a triangular piece of brickwork, whose altitude is one-third of the span of the girder. If the wall is newly built, or has openings for windows or other purposes, the girder must be designed to carry the entire wall above the girder between the supports.

EXAMPLE.—Required, the size of a steel I beam girder to carry a wall 12 inches thick, made of hard brick laid in lime mortar; there are no openings in the wall above the girder, nor does the wall support floor joists or roof beams, while the span of the opening is 24 feet.

SOLUTION.—Draw the diagram as shown in Fig. 62. The area of the triangular piece of brickwork is $24 \times 4 = 96$ square feet. The area

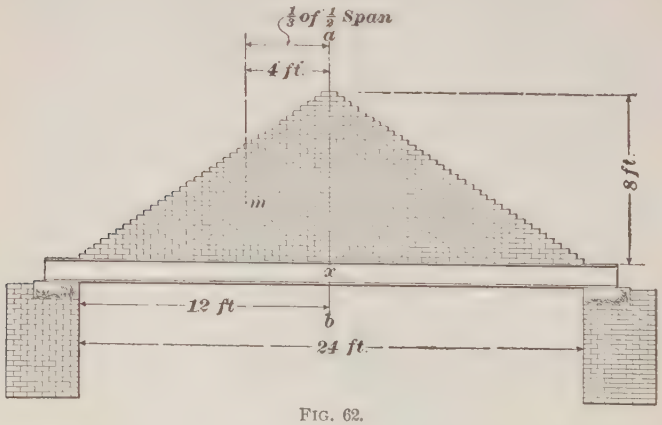


FIG. 62.

of a triangle is equal to one-half of the product of the base and altitude. As the wall is 1 foot thick, there are 96 cubic feet in this triangular piece. The weight of brickwork in lime mortar per cubic foot, according to Table 1, is 120 pounds. Then, the load on the girder is $96 \times 120 = 11{,}520$ pounds.

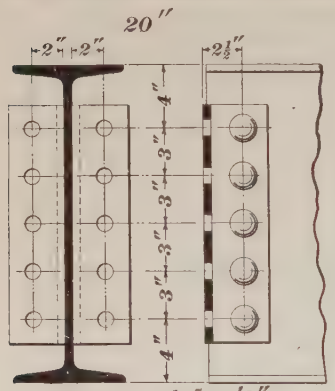
The bending moment may be determined by the formula or rule given in Table 10, for a beam carrying a triangular load, or it may be determined by calculating the moments, as follows: The reactions at the two supports are each equal to half the load, or $11{,}520 \div 2 = 5{,}760$ pounds. The greatest bending moment is at the center of the beam. Then the moment of the reaction about the point x is $5{,}760 \times 12 = 69{,}120$ foot-pounds. But counterbalancing this, and to be deducted from it, is the moment of the load at the left of x, equal to half of the triangular piece of brickwork. The moment of this load about the point x is equal to the product of its weight multiplied by the horizontal distance from a vertical line through its center of gravity to the point x. Take the line $a\,b$ as the base of a triangle, remembering that a line drawn parallel to the base line of a triangle, at a distance of one-third of the altitude from it, always passes through its center of gravity. Now the distance from the point x to the vertical line through the center of gravity m of the triangle is 4 feet, and the moment due to the triangular piece of brickwork to the left of the center is $5{,}760 \times 4 = 23{,}040$ foot-pounds. Deducting this from the moment of the reaction already found, the moment at the center is: $69{,}120 - 23{,}040 = 46{,}080$ foot-pounds, the bending moment on this beam or girder; or, $46{,}080 \times 12 = 552{,}960$ inch-pounds. This calculation may be checked by applying the formula $\frac{WL}{6}$. The bending moment in inch-pounds being 552,960, using a safe working value, or fiber stress, of 15,000 pounds, the section modulus required is $552{,}960 \div 15{,}000 = 36.8$. Referring to Table 11, it is seen that the section modulus of a 12-inch beam, 38.4 pounds, is 38.97, which gives the required strength in this case. It may be found to be better practice to use two channels instead of one I beam, for the top flange of the I beam may be too narrow to properly support the brick wall, while the two channels placed side by side, with separators between, could be made of the same thickness as the wall.

112. Connection Angles.—The standard connection angles for the principal sizes and weights of steel I beams are illustrated in Table 13. These connections are based upon shearing stresses of 10,000 pounds per square inch, bearing stresses of 20,000 pounds per square inch, and an extreme fiber stress of 16,000 pounds. The connections are properly designed for beams whose spans are not less than those given in Table 14.

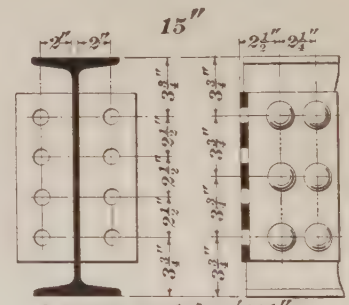
When beams are framed opposite one another, into another beam or girder, with a web less in thickness than $\frac{9}{16}$ inch, the minimum lengths of spans given in Table 14 ought to

TABLE 13.

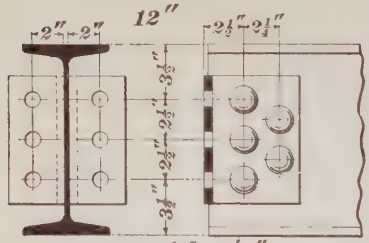
STANDARD FRAMING FOR I BEAM CONNECTIONS.



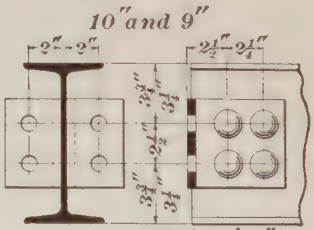
2 Angles 4×3½×$\frac{7}{16}$×1′3″



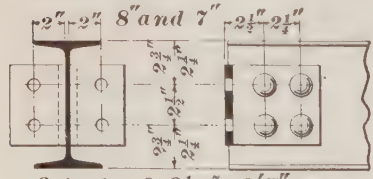
2 Angles 6×3½×$\frac{7}{16}$×0′10½″



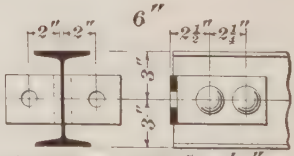
2 Angles 6×3½×$\frac{7}{16}$×0′8″



2 Angles 6×3½×$\frac{7}{16}$×0′5½″



2 Angles 6×3½×$\frac{7}{16}$×0′5″



2 Angles 6×3½×$\frac{7}{16}$×0′3″

2 Angles 6×3½×$\frac{7}{16}$×0′2½″

2 Angles 6×3½×$\frac{7}{16}$×0′2″

be increased in the same proportion that the thickness of the web is to $\frac{9}{16}$ inch.

The connections in Table 13 are designed for $\frac{13}{16}$-inch holes and $\frac{3}{4}$-inch diameter rivets or bolts. Connection angles may, if so specified, be riveted instead of bolted to the beams; but, unless otherwise ordered, bolted connections are generally used.

These connections are so proportional as to cover most cases occurring in ordinary practice; but where beams have short spans and are loaded to their full capacity, connections having a greater number of bolts than used in the standard connections may be found necessary.

TABLE 14.

Size of Beams.	Minimum Safe Span in Feet.	Size of Beams.	Minimum Safe Span in Feet.	Size of Beams.	Minimum Safe Span in Feet.
20	17.5	12	12.0	8	5.0
20	16.0	12	9.0	7	4.0
15	14.5	12	7.5	6	6.0
15	12.0	10	8.5	5	4.5
15	10.0	10	7.0	4	3.0
15	9.5	9	5.5	3	2.0

113. Stone Beams.—The strength of stone beams, or lintels, in which form stone beams are usually found, may be calculated in the same manner as rectangular beams of any material, except that it is usual to use a factor of safety of 10. First compute the bending moment on the beam in inch-pounds; then estimate the resisting moment of the beam, using the modulus of rupture given in Table 6, which should be divided by a safety factor of at least 10.

Another method, which may be found more convenient, is to use the formula

$$W = \frac{b\,d^2}{l} \times c, \qquad \textbf{(19.)}$$

in which W = safe uniformly distributed load in tons of 2,000 pounds;
b = breadth of beam in inches;
d = depth of beam in inches;
l = span of beam in inches;
c = a coefficient taken from the following table:

	Coefficient c.
Bluestone	0.18
Granite	0.12
Limestone	0.10
Sandstone	0.08
Slate	0.36

The formula may be expressed as follows:

Rule.—*To obtain the safe uniformly distributed load in tons that a stone lintel will support, square the depth of the beam in inches and multiply by the breadth in inches. Divide this product by the span of the lintel in inches; then multiply this last result by the coefficient given in the table for the particular stone used.*

EXAMPLE.—A limestone lintel is 20 inches wide and 14 inches thick, spanning an opening of 42 inches. What distributed load will this beam safely carry?

SOLUTION.—The safe distributed load in tons $= \frac{b\,d^2}{l} \times$ coefficient.

Then, $$W = \frac{20 \times 14 \times 14}{42} \times 0.10 = 9.33 \text{ tons},$$

or $$9.33 \times 2{,}000 = 18{,}660 \text{ pounds.}$$ Ans.

If the load is concentrated at the center of the span, the safe load will be one-half the safe uniform load given by the above rule.

EXAMPLES FOR PRACTICE.

1. What is the section modulus of a rectangular section 10 inches wide by 10 inches deep? Ans. $166\frac{2}{3}$.

2. Calculate the section modulus of a 10-inch I beam, the sectional area of which is 11.8 square inches. Ans. 36.87.

3. What must be the width of a yellow-pine beam to support a uniformly distributed load of 10,380 pounds, the span of the beam being 18 feet, its depth 12 inches, and the safety factor 5? Ans. 8 in.

4. A uniformly distributed load upon a steel I beam having a span of 24 feet is 25,000 pounds. What size of beam will be the most economical to use, the allowable unit stress on the material being 18,000 pounds? Ans. 15″; 41.2 lb.

5. It is desired to span an opening 20 feet wide in an 18-inch wall, laid in lime mortar, with two steel channels to support a solid brick

wall over the opening. What will be the size and weight of the channels if a safe fiber stress of 15,000 pounds is adopted? Ans. 12″; 19.8 lb.

6. A builder wishes to determine which will be the cheaper: to span a 20-foot opening in a 12-inch brick wall laid in cement mortar with two steel channels, or with yellow-pine timber; the steel beams being worth $1\frac{3}{4}$ c. per pound and the yellow-pine timber being worth $25 per M. The load to be supported is the solid brick wall above; a factor of safety of 4 will be required if the steel channels are used; if the wooden beam is used, a factor of safety of 6 will be adopted. If the builder adopts the cheaper, (*a*) which will be used? (*b*) how much will he save over the other?

Ans. { (*a*) Yellow-pine timber.
(*b*) $5.82.

7. What safe uniformly distributed load will a granite lintel, 16 inches deep and 24 inches wide, sustain, its span being 6 feet?

Ans. 10.24 tons.

8. A sandstone beam is required to support a uniformly distributed load of 4 tons; it is necessary that it should be 18 inches wide; what should be the depth of this beam, if the span is 8 feet? Ans. $16\frac{1}{3}$ in.

9. A heavy bluestone flag, 6 inches thick and 5 feet wide, spans a culvert which is 7 feet wide. What safe uniformly distributed load will this flag support? Ans. 4.6 tons.

TRUSSED BEAMS.

114. When wooden girders of great span are heavily loaded, it becomes necessary to strengthen them with iron or steel camber rods, as shown in Figs. 63 and 64. Fig. 63

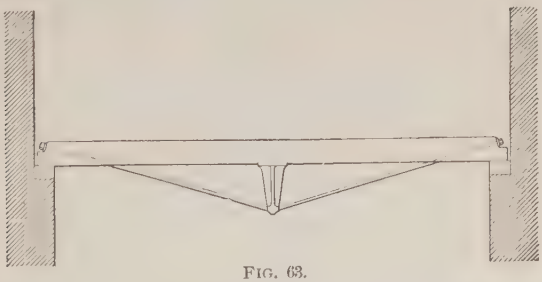


FIG. 63.

shows a girder with one support in the center, and Fig. 64 shows a girder with two supports. The span of the beam or girder may be considered, in each case, as the distance between the supports, the strength of the girder being thereby materially increased.

115. Stresses in a Beam With One Strut.—In Fig. 65, let W represent the load concentrated at D. Then the

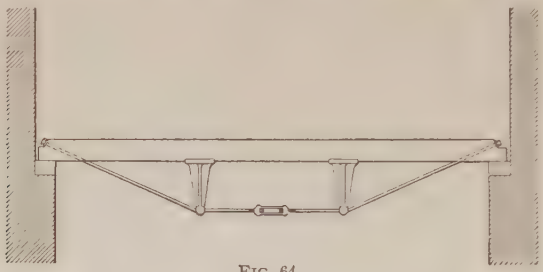


FIG. 64.

stress in the member $D\,C$ is equal to W. The stress in the other members may be found by applying the following

Rule.—I. *To find the stress in $A\,C$ or $B\,C$, divide the length of line $A\,C$ by the length of the line $D\,C$, and then multiply this result by one-half of the load W.*

II. *To find the stress in the beam $A\,B$, divide the length of the line $A\,D$ by the length of the line $D\,C$, and multiply by one-half of the load W.*

In the diagram, Fig. 65, the members represented by solid lines are in compression, and those shown dotted are in tension.

The length of the members in the above rules may be taken in feet or inches, but all lengths should be taken in

FIG. 65.

the same unit of measurement. The rules may be expressed by the formulas

$$\text{Stress in } D\,C = +W. \qquad \textbf{(20.)}$$

$$\text{Stress in } A\,C \text{ or } B\,C = -\frac{A\,C}{D\,C} \times \frac{W}{2}. \qquad \textbf{(21.)}$$

$$\text{Stress in } A\,B = +\frac{A\,D}{D\,C} \times \frac{W}{2}. \qquad \textbf{(22.)}$$

The + and − signs in the formulas indicate compression and tension, respectively. The + sign denotes that the result obtained is a *compressive* stress. The − sign means that the result is a *tensile* stress.

When a beam with one support at center has a uniform load, as in Fig. 66, the load W on the center strut to be used in formulas **20** to **22** is $\frac{5}{8}$ of the entire load.

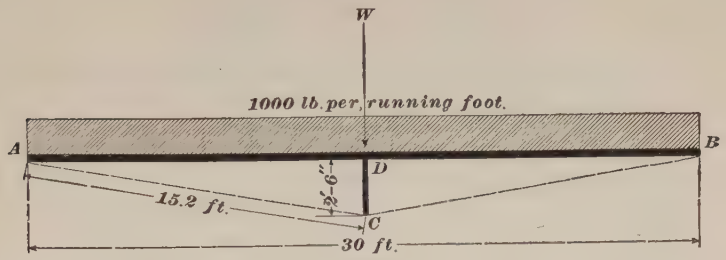


FIG. 66.

EXAMPLE.—What is (*a*) the tension in the camber rod, and (*b*) the compression on the trussed beam of the dimensions and loads shown in Fig. 66?

SOLUTION.—We must first compute the load W coming upon the strut $D\,C$. The load is, in this case, usually considered equal to one-half the entire load on the beam. But as the beam is composed of one length of timber, and is not hinged at D, being, in effect, a continuous beam, the load on the center strut is $\frac{5}{8}$ of the entire load on the beam. The entire load on the beam is equal to $30 \times 1{,}000 = 30{,}000$ pounds; $\frac{5}{8}$ of $30{,}000 = 18{,}750$ pounds, the load W acting on the beam directly over the strut $D\,C$.

Then (*a*) the tension in the camber rod $A\,C$ is equal to the length $A\,C \div D\,C$ multiplied by $\frac{1}{2}$ of W, or, substituting the given dimensions, $15.2 \div 2.5 = 6.08$; and $6.08 \times \frac{1}{2}$ of $18{,}750 = 57{,}000$ pounds, the tensile stress on rod $A\,C$. Ans.

(*b*) To determine the stress on beam $A\,B$, divide the length $A\,D$ by $D\,C$ and multiply by $\frac{1}{2}$ of W. Thus $15 \div 2.5 = 6$; $6 \times \frac{1}{2}$ of $18{,}750 = 56{,}250$ pounds, the compressive stress in the beam $A\,B$. Ans.

116. Stresses in Beams With Two Struts.—In Fig. 67, the calculations for the stresses in the various members are similar to those given for the trussed beam with one support. In the two-trussed beams, the stress in $B\,H$ or $C\,E = W$. The stresses in the other members may be expressed by rule, as follows:

Rule.—I. *To obtain the stress in A H or D E, divide the length of A H by the length of B H, and multiply this result by the load W.*

II. *To find the stress in A D or H E, divide the length of the line A B by the length of the line B H, and multiply this result by the load W.*

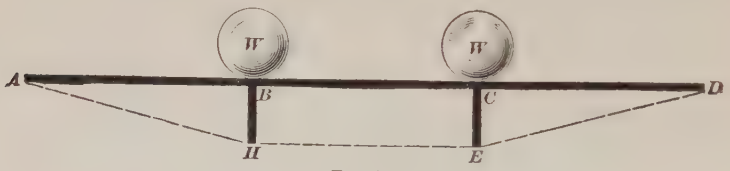


FIG. 67.

The above may be expressed in formulas:

$$\text{Stress in } B\,H \text{ or } C\,E = +W. \qquad \textbf{(23.)}$$

$$\text{Stress in } A\,H \text{ or } D\,E = -\frac{A\,H}{B\,H} \times W. \qquad \textbf{(24.)}$$

$$\text{Stress in } H\,E = -\frac{A\,B}{B\,H} \times W. \qquad \textbf{(25.)}$$

$$\text{Stress in } A\,D = +\frac{A\,B}{B\,H} \times W. \qquad \textbf{(26.)}$$

Compression is, as previously noted, indicated by the + sign and tension by the − sign.

When a beam has two supports $\frac{1}{5}$ of its length from each end, and is uniformly loaded, as in Fig. 68, the load W on each support is $\frac{11}{30}$ of the total load.

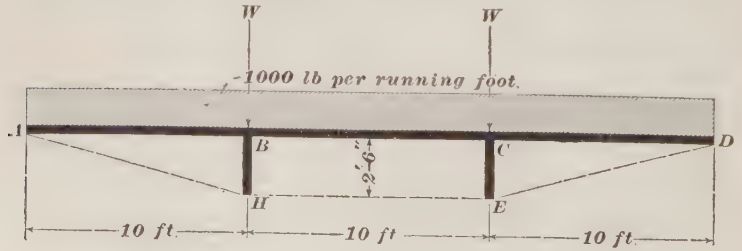


FIG. 68.

EXAMPLE.—A beam is trussed, as shown in Fig. 68; what is (*a*) the stress in camber rod *H E*, and (*b*) the compression in the beam *A D*?

SOLUTION.—The entire load on the beam $A\ D$ is $30 \times 1{,}000 = 30{,}000$ pounds. The beam being a continuous girder, as in the previous example, the loads W are each $\frac{11}{30}$ of the entire load. Hence, $W = \frac{11}{30} \times 30{,}000$, or 11,000 pounds.

(*a*) Applying formula **25**, we have

$$\text{Stress in } H\ E = \frac{10}{2.5} \times 11{,}000 = 44{,}000 \text{ pounds.} \quad \text{Ans.}$$

(*b*) Applying formula **26**, we have

$$\text{Stress in } A\ D = \frac{10}{2.5} \times 11{,}000 = 44{,}000 \text{ pounds.} \quad \text{Ans.}$$

117. Graphical Method of Computing Stress.—The stresses in the various members of a trussed beam may be obtained by means of a graphical method which is simply an application of the principles of the resolution of forces. (See Arts. **19** and **20.**) Although not as exact in its results as the mathematical method, it is probably more satisfactory, there being less chance of errors creeping into the calculation. This method is fully explained in the subjoined illustrative example:

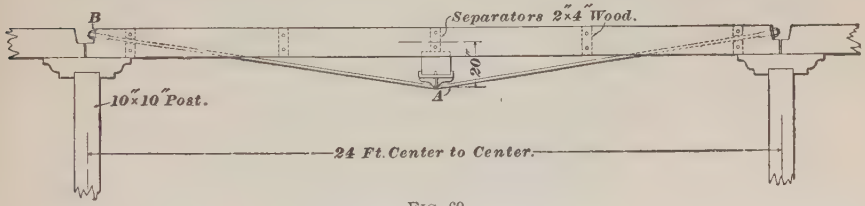


FIG. 69.

A floor is to be supported by yellow-pine girders, each composed of two $4'' \times 12''$ beams, trussed with a wrought-iron rod, as shown in Fig. 69. The span of the girders is 24 feet, and they are spaced 8 feet from center to center. The load is light, amounting to only 40 pounds per square foot of floor surface. Required, to determine whether the two yellow-pine beams are sufficiently strong, and what should be the size of the wrought-iron camber rod; also to design the detail construction for the parts A and B.

The floor area supported by each girder is $24 \times 8 = 192$ square feet; therefore, the total load on a girder is $192 \times 40 = 7{,}680$ pounds. To find the stress produced in the different

members of the truss by this load, first draw to some convenient scale, as in Fig. 70, making the lines *a b*, *a c*, *b c*, and *d c* correspond, respectively, to the center lines of the girder, the wrought-iron camber rod, and the strut; thus, the line *a b* represents the center line of the pine beams, its length being equal to 24 feet on the assumed scale; while *d c*, drawn perpendicular to *a b* at its middle point, represents, on the same scale, the length, 20 inches, of the strut.

FIG. 70.

In accordance with the statements of Art. **115,** the load carried by the strut may be taken as $\frac{5}{8}$ of the total load on the girder; therefore, the force *f*, Fig. 70, acting downwards on the frame, and borne directly by the strut *d c*, is 7,680 $\times \frac{5}{8} = 4{,}800$ pounds. This force is held in equilibrium by the stresses in the members of the truss, represented by the center lines *a d*, *d b*, *a c*, and *c b*, one-half of it, or 2,400 pounds, being held by each of the pairs *a d* and *a c*, *d b* and *c b*. Considering the half of the load carried by the pair *a d* and *a c*, we have a downward force of 2,400 pounds, and it is required to resolve it into two components, one acting along the line *a c* and the other along *a d*. Assuming a scale of forces, one, for example, in which a line 1 inch long represents a force of 800 pounds, draw the line *d c*, Fig. 71, parallel to the center line *d c*, Fig. 70, of the strut, and make its length correspond to a force of 2,400 pounds, the

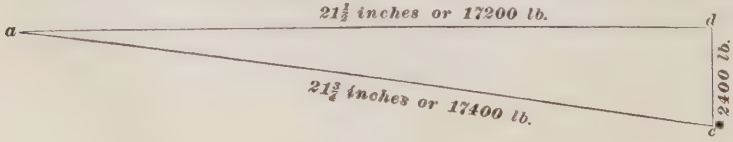


FIG. 71.

part of the total load on the strut which is borne by the members *a d* and *a c*. From the upper extremity of *d c*, Fig. 71, draw the line *d a* parallel to the line *d a* of Fig. 70,

and from the lower extremity draw the line ca parallel to ca of Fig. 70, prolonging these two lines until they meet in the point a. The lines da and ca of Fig. 71 represent, on the scale of forces to which the line dc was drawn, the stresses in the corresponding members of the girder. With the assumed scale of 1 inch = 800 pounds, the line dc must be $2{,}400 \div 800 = 3$ inches long; by measurement, the lines da and ca are found to be $21\frac{1}{2}$ and $21\frac{3}{4}$ inches long, respectively; therefore, the stress represented by the line da is $21\frac{1}{2} \times 800 = 17{,}200$ pounds, and that represented by ca is $21\frac{3}{4} \times 800 = 17{,}400$ pounds. The stress of 17,200 pounds is the total compressive stress produced in the two yellow-pine beams through the action of the downward thrust on the strut.

From Table 6, Art. **61,** it will be found that the ultimate resistance to compression of yellow pine per square inch is 4,400 pounds; and as wood is not so reliable as iron, it is considered advisable to use a factor of safety of 6, as against a factor of safety of 4 for the camber rods. Since the trussed girder is secured against lateral deflection by the floor joist, and as it is secured from deflection in a vertical direction at the center by the load upon the floor and by the camber rod and strut, the length of the wooden girder, which may be considered as a column under compressive stress, is only one-half the span, or 12 feet. Now, the sectional dimension of the girder is so great in comparison with its length, that it is not necessary to apply the column formula, and its strength may be considered as its resistance to direct compression. Hence, $4{,}400 \div 6 = 733$ pounds, which is the allowable compression strength of the girder per square inch of section. Then, 17,200 pounds (the compression) $\div$ 733 pounds (the allowable unit stress) = 23 square inches required to take care of the compressive stress. As the girder is known to be 12 inches in depth, it is readily seen that this compressive stress will require a section of the timber girder equal to 2 in. $\times$ 12 in.

There is, in addition to this, a transverse stress upon one-half of the girder produced by the uniformly distributed load. To find the amount of this bending stress, consider

the left-hand half of the girder as a simple beam, sustaining a uniformly distributed load equal to one-half of the total load upon the girder, that is, a load of $7,680 \div 2 = 3,840$ pounds.

Applying formula **13,** Art. **97,** the bending moment due to this load is $M = \frac{3,840 \times 12}{8} = 5,760$ foot-pounds; and $5,760 \times 12 = 69,120$ inch-pounds. Then the section modulus required may be obtained by the formula $K = \frac{M}{S}$, where K equals the section modulus, M the bending moment in inch-pounds, and S represents the allowable unit fiber stress of the material, which is equal, in this case, to 7,300 (the modulus of rupture of yellow pine) $\div 6$ (the factor of safety) $= 1,217$ pounds. Substituting the values in the above formula, the calculation will be $K = \frac{69,120}{1,217} = 56.8$, the section modulus required in the girder to successfully resist the transverse stress.

Since the section modulus of a rectangular beam may be obtained by formula **15,** Art. **101,** which is $K = \frac{b\,d^2}{6}$, b being the width of the beam in inches, and d the depth, and as K is already known to be 56.8 and the depth of the beam to be 12 inches, the width of the beam required to resist the transverse stress may be obtained by transposing the formula to $b = \frac{K \times 6}{d^2}$; the values substituted would give $b = \frac{56.8 \times 6}{12 \times 12}$, or 2.37 inches, which is the width of the required beam. Then adding the size of the timber required to resist compression and the size of timber required to resist the transverse stress, we have a timber 2 inches wide by 12 inches deep, added to a timber 2.37 or, say, $2\frac{1}{2}$ inches by 12 inches, which gives a piece $4\frac{1}{2}$ inches by 12 inches. In the girder there are two $4'' \times 12''$ timbers, and as only a single $4\frac{1}{2}'' \times 12''$ timber is required, it is evident the girder is nearly twice as strong as is necessary. It must, however, be borne in mind that the theoretical dimensions of members do not always agree with those required in practical rules; for

instance, in the above case it would not be good practice to make the combined sectional area of the girder equivalent to that of a $4\frac{1}{2}'' \times 12''$ timber, as obtained by the calculation as being correct, because this would make each timber a little larger than 2 in. × 12 in., and no timber or girder, especially where rafter or flooring is spiked to it, should be less than 3 inches wide.

From Table 6, the ultimate tensile strength of wrought iron is found to be 50,000 pounds per square inch; hence, if we use a factor of safety of 4, the safe working fiber stress in the rod must be $50{,}000 \div 4 = 12{,}500$ pounds per square inch.

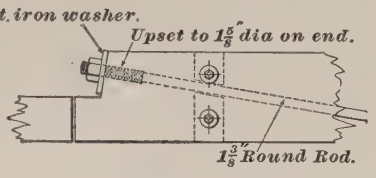


FIG. 72.

According to the results given by the diagram, the total stress in the rod is 17,400 pounds; therefore, the rod must have a section of $17{,}400 \div 12{,}500 = 1.39$ square inches. The area of a $1\frac{3}{8}$-inch round rod is 1.48 square inches, and as this is the nearest standard size having the required sectional area, it will be used. As the area at the bottom of the thread of a $1\frac{3}{8}$-inch bolt is, however, only 1.06 square inches, it will be necessary to **upset** or enlarge the ends of the rod to a diameter of $1\frac{5}{8}$ inches, in order to get the requisite strength in the threaded portion. The washer at *B*, Fig. 69, must be large enough to distribute the pressure due to the pull of the rod over a sufficient area of the end of the beams to prevent danger of crushing the wood. The allowable compressive strength of yellow pine, parallel to the grain, may be taken as 800 pounds per square inch; this requires a washer whose area is $17{,}400 \div 800 = 22$ square inches, nearly. Using a washer 6 inches wide,

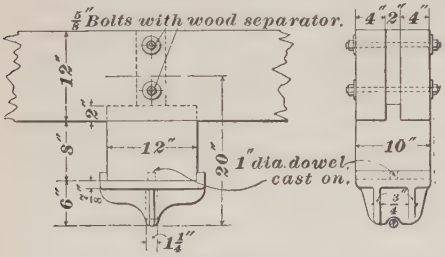


FIG. 73.

extending across the ends of the two beams, we get a bearing area of $2 \times 4 \times 6 = 48$ square inches. In order to resist the bending stress due to the pull of the rod, the washer should be from $\frac{3}{4}$ inch to 1 inch in thickness.

Figs. 69, 72, and 73, which are so clearly drawn as to require no further explanation, show excellent details for the different parts of the trussed stringer under consideration.

DEFLECTION OF FLOORBEAMS.

118. Beams used in floors should not only be strong enough to carry the superimposed loads, but also sufficiently rigid to prevent vibration. For beams carrying plastered ceilings, if the deflection exceeds $\frac{1}{360}$ of the distance between the supports, or $\frac{1}{30}$ of an inch per foot of span, there is danger of cracking the plaster. It is safe to assume that these deflections will not be exceeded in wood beams, and the sagging of the beam will not produce plaster cracks when the beam is loaded with its maximum safe load, if the depth of the beam is made $\frac{1}{17}$ of the span. That is, suppose the span of the beam is 20 feet, or 240 inches: $\frac{1}{17}$ of 240 in. = 14.1, say 15 in., the depth of the wood beam to be used for this span. If the calculation brings out a result with a decimal, like that just given, take the next stock size above the result thus obtained.

If steel beams have a depth in inches equal to one-half of the span in feet, their deflection may be considered well within the safe limit. Thus, if the span of a steel beam is 24 feet, its depth should not be less than 12 inches.

It must be remembered that the above rules are only approximate, and if there is any reason to suppose that the deflection of the beam will be excessive, its deflection should be computed by the rules and formulas subsequently given in the second section of this subject. The above rules may, however, be considered comparatively safe, and, if followed, are not likely to produce plaster cracks, provided the loads are statical and the floors not subject to sudden shocks or jars.

EXAMPLES FOR PRACTICE.

1. It is found necessary to truss the yellow-pine purlins supporting a roof, with a wrought-iron camber rod on each side of the purlin; the length of the purlin is 20 feet, the depth of the truss from the center of the rods to the center of the purlin is 14 inches, and the load upon the central strut is 1,600 pounds. What should be the diameter of the camber rods if the ends of the rods are upset, and a safety factor of 4 is desired? Ans. $\frac{5}{8}$ in. diam.

2. A girder of 24 feet span is trussed at the center by a camber rod and strut; the depth of the truss from the center of the girder to the center of the rod is 2 feet; if the beam is loaded with a uniformly distributed load of 2,000 pounds per lineal foot, (*a*) what is the stress on the rod? (*b*) what is the compressive stress on the beam? (*c*) what is the stress on the central strut?

Ans. { (*a*) 91,200 lb. (*b*) 90,000 lb. (*c*) 30,000 lb.

GRAPHICAL STATICS.

119. The application of the principles of the triangle of forces, in finding the stresses in the different members of a simple trussed stringer, have been illustrated in Art. **117.** We will now show how this process may be extended, by the use of the method of the polygon of forces (see Art. **17**), so as to make it available in computing the stresses in the members of the most complicated frames and trusses found in ordinary engineering practice. The application of the polygon of forces to the computation of stresses is called **graphical statics.**

In the use of graphical statics to determine the stresses on the various members entering into the construction of a frame, not only may the magnitude of the stresses upon the members, but the direction in which they act, be determined.

The representation to the eye of the forces existing in the several parts of a frame structure, possesses many advantages over their determination by calculation. Graphical analysis being founded on correct principles, the diagrams give results depending for accuracy on the exactness with which the lines have been drawn, and upon the scale by which they are measured. With ordinary care, the different forces may

be obtained much more accurately than the several parts of the frame can be proportioned.

120. Frame and Stress Diagrams.—In Fig. 74, the weight W of 1,000 pounds is supported at c by the two branching cords $c\,a$ and $c\,b$, fastened to the pins a and b. The figure, which is drawn to scale and accurately represents the outline of the structure, is called a **frame diagram.** Now, to obtain the stress upon the cords $c\,a$ and $c\,b$, draw the vertical line *1–2* in Fig. 75; this line must represent the direct pull on the rope, or cord, $c\,W$, so let each inch represent 400 pounds; then, to represent a force of 1,000 pounds, the length of the line *1–2* shall be $1{,}000 \div 400 = 2\frac{1}{2}$ inches. From *1* draw the line *1–3* parallel to $a\,c$, and from *2* draw the line *2–3* parallel to the line $c\,b$ of Fig. 74; they will intersect at the point *3* and form a triangle. If the lines *1–3* and *2–3* are measured with the 1-inch scale, the stresses $c\,a$ and $c\,b$ can be determined.

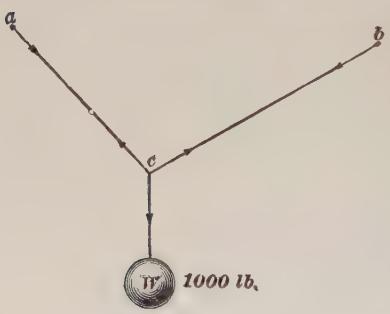


FIG. 74.

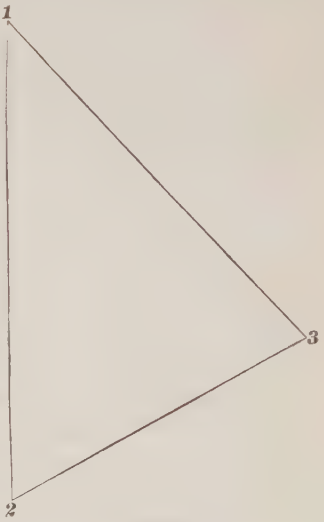


FIG. 75.

The diagram, Fig. 75, is called the **stress diagram.** In working out the stresses for roof truss, it is first necessary to make the frame diagram, drawn accurately to any scale and representing the outline of the truss and the members of which it is composed. It is then necessary to draw a stress diagram for the dead load on the roof, which sometimes includes the snow load, and another diagram to

represent the stresses produced by the action of the wind on the roof.

121. Lettering the Diagrams.—In laying out the frame and stress diagram, it is useful to adopt a system of lettering, so that the relative position of the different members in the frame and stress diagram may be seen at a glance, as in Figs. 76 and 77. In Fig. 76, the reactions at the walls are R_1 and R_2.

It must be remembered that a roof truss is nothing more nor less than a beam, and, consequently, the sum of the reactions must be equal to the sum of the loads.

The loads, or forces, acting upon a roof truss are always considered as concentrated at the **panel points,** that is, where several members in the structure join each other. In

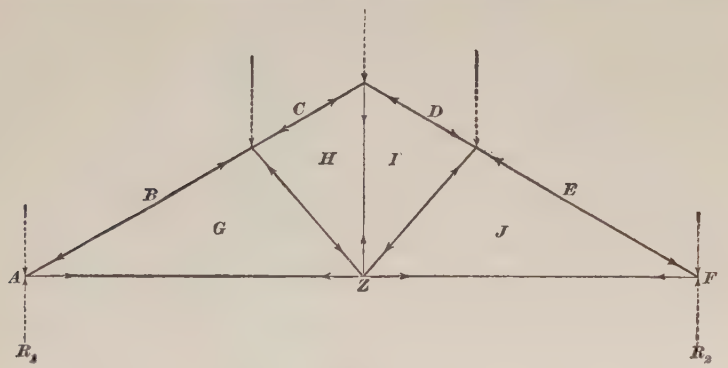


FIG. 76.

Fig. 76, the loads and reactions on the truss are shown by dotted lines and arrowheads.

The spaces outside of the truss, between the loads, together with each triangle inside of the truss, should be lettered with capitals, as in Fig. 76. It is well to begin at the left-hand reaction of the truss with the letter A, working around the outside of the truss in alphabetical order, until the right-hand reaction is reached. Then start with the first triangular space at the left-hand end of the truss, following with the next letter in the alphabet, continuing in alphabetical order,

it being well to mark the space between the two reactions, at the center of the truss, *Z*.

It is not absolutely necessary to use this system of lettering, for any letters or numbers may be employed, as long as no two spaces in the truss are marked alike. It is, however, well to have some definite system in engineering work, the above being as convenient as any that can be suggested.

Having marked the truss with letters, as in Fig. 76, the various members and forces may be designated by those letters between which they are located. The left-hand reaction, for instance, will be *Z A*, the first load on the truss *A B*, the second load *B C*, the load at the apex of the truss *C D*, next in succession the loads *D E* and *E F*—always following the truss in the same direction. The last external force on the truss is the right-hand reaction *F Z*.

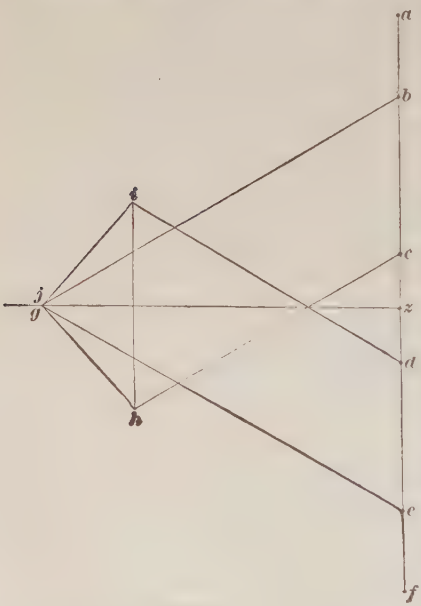


FIG. 77.

The lower portion of the left-hand rafter is *B G*, the upper portion *C H*, the left-hand portion of the tie at the base of the truss is *G Z*, the left-hand compression member in the truss is *H G*, and the central tie *I H*, and so on. In designating the various members of the truss, the letters are given in the order in which they occur, always following the joints in the same direction, which in this case is that in which the hands of a watch travel. This being important, it is fully explained further on.

122. The stress diagram for the truss shown in Fig. 76, is represented by Fig. 77. The stress diagram is always

drawn to some scale in which the unit measurement represents a certain number of pounds. Thus, suppose 1 inch represents 1,000 pounds, and suppose that in the stress diagram the line $c\,h$ is 4 inches long; then the stress in the member $C\,H$ in Fig. 76, is equal to 4,000 pounds. It will be noticed that the small letters are used in the stress diagram and that the letters at the end of a line designate that line. Thus, the length of the line $d\,i$ in the stress diagram, Fig. 77, measures the stress in the corresponding member $D\,I$ of the frame diagram, Fig. 76. The distance from z to a measures the magnitude of the reaction $A\,Z$; the length of the line $i\,h$ in the stress diagram measures the stress in the corresponding member $I\,H$ of the frame diagram.

123. Order of Letters.—It is, as previously mentioned, very important always to read the letters in the same direction, and in proper order. If, for instance, the joint at the middle of the left-hand rafter (Fig. 78) is examined, the members and stresses must be read off in their proper order, as $B\,C$, $C\,H$, $H\,G$, and back again to $G\,B$. Leaps should not be made from $B\,C$ to $G\,B$, etc., as this would lead to errors, and prevent the drawing of the stress diagram. Still more important is it to read around the joints in one direction, as in Fig. 78, i. e., in the direction of the arrow. If you reverse the reading of the pieces, you must reverse the direction of the stresses in the diagram. If, for instance, in Fig. 76, $G\,H$ was read, and its corresponding line $g\,h$ found in the stress diagram, its direction would be downwards, pulling away from the joint at the left-hand rafter and making $G\,H$ a tie-rod, whereas it is well known as a strut. If, however, it had been read correctly $h\,g$, it would indicate a push towards the joint, which is, of course, the correct action of a strut.

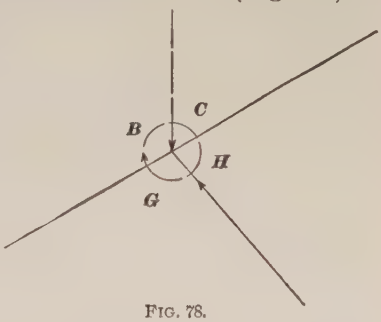


FIG. 78.

When the joint Z, Fig. 76, is examined, the reverse of this occurs, and here the correct reading is $g\,h$, which is the same

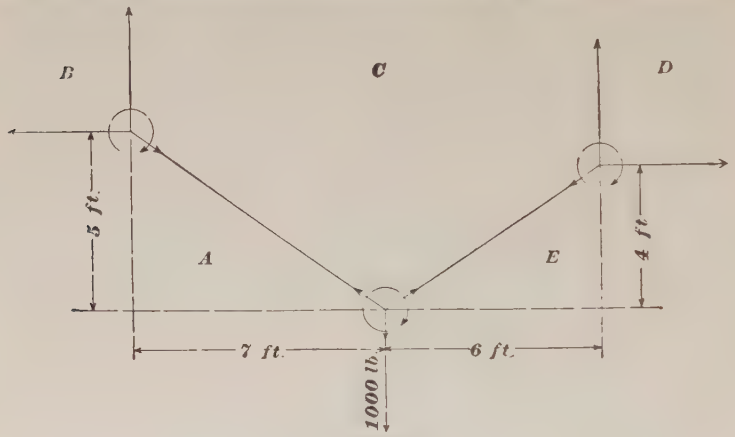


FIG. 79.

relative direction, for the point Z, as was $h\,g$ for the point at the center of the left-hand rafter.

124. Diagram for Simple Frames.—Fig. 79 shows a force, or load, of 1,000 pounds pulling upon the cord $E\,A$, and held in position by the two cords $A\,C$ and $C\,E$. Find by graphical statics the stress in these two cords, also the magnitudes of the forces ($A\,B$, $B\,C$) ($C\,D$, $D\,E$) required to act at the ends of the cords.

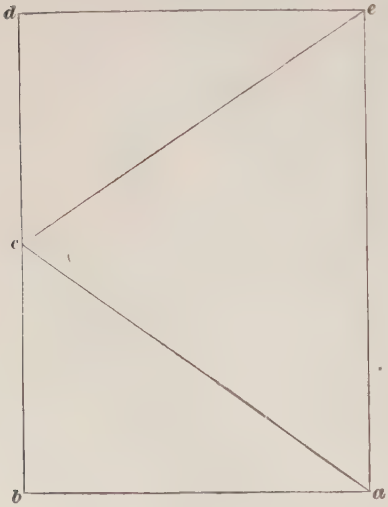


FIG. 80.

Draw the frame diagram, Fig. 79, accurately, say to a scale of $\frac{3}{8}$ inch to 1 foot. Then start to draw the stress diagram, Fig. 80. The force $E\,A$, in the frame diagram, being already known, take some scale, say 400 pounds to 1 inch, and draw $e\,a$ in the

stress diagram. It must be drawn parallel to the line along which the force or load $E A$ acts. This force $E A$ being 1,000 pounds, and the scale to which the stress diagram is drawn being 400 pounds to each 1 inch, the line $e a$ must be $2\frac{1}{2}$ inches long. Having drawn $e a$, work around the joint in the direction of the arrow. From a in the stress diagram draw a line parallel to $A C$, and from e in the stress diagram draw a line parallel to $C E$ until it intersects the line $a c$ at the point c.

In going around the stress diagram, the same direction is to be followed. Thus, in going around the joint $A E$, it is read, in the stress diagram, from e to a, from a to c, from c to e, always arriving at the same point from which the start was made. As the stresses are read, their direction should be marked with arrows on the frame diagram, Fig. 79. This shows the direction of the stress and designates whether it is compression or tension. Forces acting away from a joint are always tensile stresses; those acting towards a joint are always compressive stresses.

If there is tension at one end of a member, it is evident there must be an equal amount of tension at the other end; if there is compression at one end of a member, there is an equal amount at the other.

An easy way to remember whether the arrows designate compression or tension by their direction, as shown on the members in a frame diagram, may be seen by referring to Figs. 81 and 82. The member $A B$, Fig. 81, is a tension member; the arrows point away from the joints and towards each other, and resemble the form of an elastic material, stretched, as shown at a, Fig. 82; while in the compression member $A C$, the arrows act against the joints, or away from each other, and resemble the form that a plastic material assumes on being compressed, as shown at b, Fig. 82.

To continue the solution of the problem in Fig. 79: Having gone around the joint $E A C$, proceed to go around the joint $A B C$ in the same direction. Having the point a, draw from it a line parallel to $A B$ in the frame diagram, then draw a line from c parallel to the line $B C$ in the frame

diagram; the point where the two lines intersect is *b*. The polygon of forces may be read from *c* to *a*, from *a* to *b*, from *b* to *c*, always moving in the same direction and arriving at the starting point. The next joint to work around is the

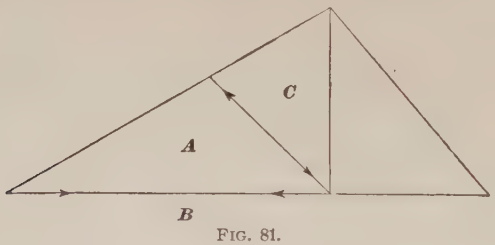


FIG. 81.

joint *C D E*. Starting at the point *c* already determined, draw a line parallel to the direction of *C D*, and from *e* draw a line parallel to *D E*; the polygon of forces is from *a* to *c*, *c* to *d*, and *d* to *e*, the last point in the diagram, and the point from which the whole stress diagram was started. Then by measuring the lines (with the 1-inch scale, wherein every inch represents 400 pounds) in the stress diagram, the

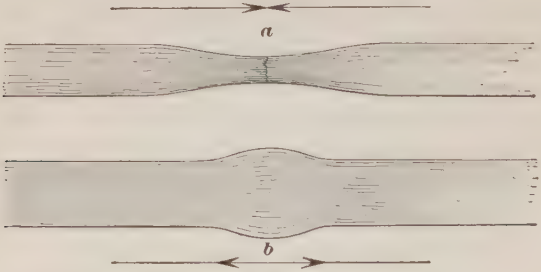


FIG. 82.

stresses in the different members of the frame diagram may be found. If, for instance, it is desired to obtain the stress in the cords *A C* and *C E*, measure the lines *a c* and *c e* in the stress diagram; likewise, to obtain the forces *A B*, *B C*, *C D*, and *D E*, measure the length of the lines *a b*, *b c*, *c d*, and *d e* in the stress diagram.

125. In Fig. 83 is shown a case similar to that in the preceding article. The compression member *B D*, and the

tension members $A\,D$ and $D\,C$ form a triangular frame which supports the downward pull of 1,000 pounds. The

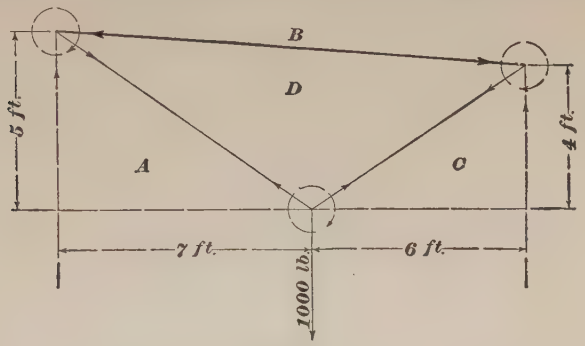


FIG. 83.

triangular frame is supported, in turn, by the reactions $A\,B$ and $B\,C$. Draw the stress diagram to determine the stress in the various members.

Take a scale, in this case 100 pounds to $\frac{1}{4}$ inch, and draw the vertical line $c\,a$, Fig. 84, equal to 1,000 pounds. This line represents the force $C\,A$ in the frame diagram, Fig. 83. Start to work around the joint $A\,D\,C$, in the direction of the arrow. The first member encountered is $A\,D$. Hence, from a in the stress diagram draw a line parallel to $A\,D$ in the frame diagram. Then, $D\,C$ being the next member met with, from c in the stress diagram draw a line parallel to $D\,C$. The point of intersection of the two lines just drawn is d. This done, go around the joint again, to see that none of the members have been omitted, and also to get the direction in which the stresses act. Starting at

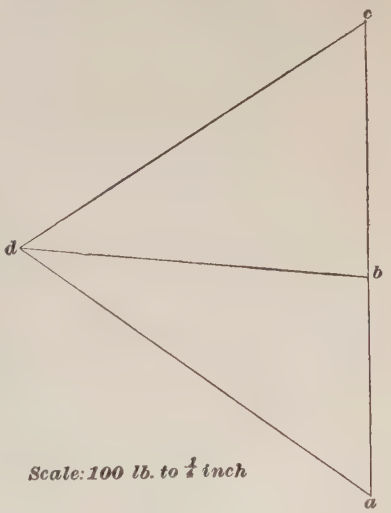


FIG. 84.

c in the stress diagram, and going around the joint $C\,A\,D$, the polygon of forces is as follows: from c to a, from a to d, and from d back again to c, thus arriving at the point from which the start was made. The next joint in the frame diagram is $A\,B\,D$. The point b on the line $c\,a$ is not known, but may be determined by calculating the reactions $A\,B$ and $B\,C$ in the same manner as for a beam. Thus, the load of 1,000 pounds is placed upon the assumed beam, 6 feet from the reaction $B\,C$. The moment about $C\,B$ is $1{,}000 \times 6 = 6{,}000$ foot-pounds, while the reaction at $A\,B$ equals 6,000

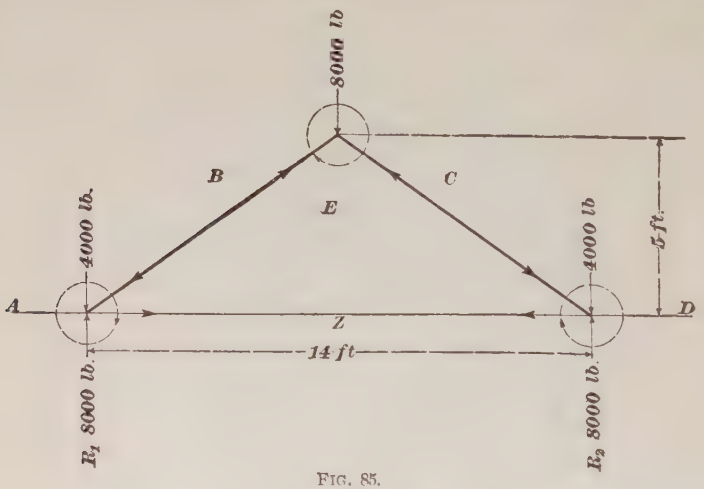


FIG. 85.

$\div 13 = 461$ pounds. Knowing that the force $A\,B$ is 461 pounds and that it acts upwards, the point b can easily be located by measuring from a on the line $a\,c$ in the stress diagram; then the line $b\,d$ may be drawn and if found parallel to the member $B\,D$ in the frame diagram, the stress diagram is correct. In this case, however, it is not necessary to calculate the reactions $A\,B$ and $B\,C$, the point d having been already determined, and since we know that the line $d\,b$ must be parallel to $D\,B$, all that is needed to locate the point b is to draw a line from d parallel to $D\,B$, and the point where it cuts the line $c\,a$ is b. Having found the point b and drawn the line $d\,b$, go around the joints

A B D and *C D B*, marking the direction of the stress by the arrowheads, as shown in the frame diagram, Fig. 83. Around the joint *A B D* the polygon of forces is from *a* to *b*, from *b* to *d*, and from *d* back again to *a*. Working around the joint *C D B*, the polygon of forces is from *c* to *d*, from *d* to *b*, and from *b* back again to *c*. This completes the stress diagram; the magnitude of the stresses in the several members of the frame diagram is found by measuring the corresponding lines in the stress diagram.

126. Diagram for a Small Roof Truss.—Fig. 85 is the frame diagram for a small roof truss. The two rafter members *E B* and *C E* are connected at their foot by the tension member *E Z*. The loads and their reactions are as shown in the frame diagram. Determine the stresses in the several members composing the truss.

Draw the vertical line *a d*, shown in the stress diagram, Fig. 86. Lay off to any scale, say, in this case 2,000 pounds to $\frac{1}{2}$ inch, the load *a b*; then, to the same scale, the loads *b c* and *c d*. From the point *d*, the reaction *d z*, 8,000 pounds, acts upwards, which determines the point *z*. Now go around the joint *A B E Z*. The reaction *Z A* acts upwards and *A B* downwards. Then, from the point *b* in the stress diagram draw the line *b e* parallel to *B E* in the frame diagram, and from *z* draw the line *e z* parallel to the member *E Z* in the frame diagram. The point of intersection will be the point *e*. Having gone thus far, again go around the joint, to get the direction of

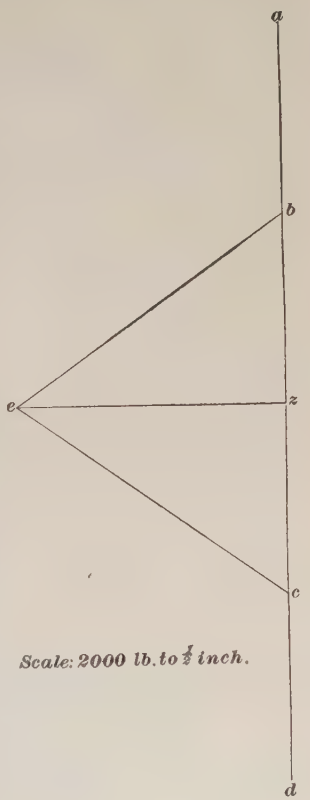


FIG. 86.

the stress in the members and to see whether the polygon of forces is correctly drawn. Go, for instance, from z to a upwards; a to b downwards; then from b to e, and from e back again to z, the starting point. The next joint in the frame diagram is $B\,C\,E$. The force $b\,c$ being already determined, from point c draw the line $c\,e$, parallel to the member $C\,E$ in the frame diagram; this line passes through the point e if the diagram has been drawn correctly. The polygon of forces at $E\,B\,C$ is from b to c downwards, then from c to e, and back again from e to b, the starting point. The next joint in the frame diagram is $E\,C\,D\,Z$, and the polygon of forces in the stress diagram is from e to c already drawn, from c to d, d to z, and then from z back to e.

The stress diagram completed, all that remains is to measure the various lines in the stress diagram which represent

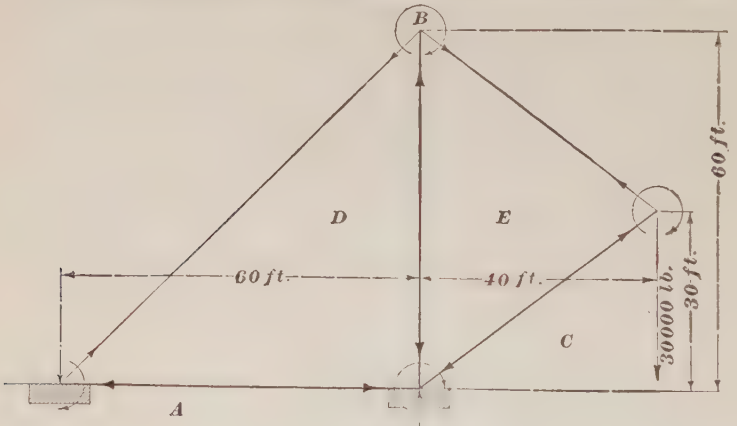


FIG. 87.

the corresponding members in the frame diagram. Thus, $c\,b$ measures $1\frac{3}{4}$ inches, the scale being 2,000 pounds to $\frac{1}{2}$ inch; hence, the stress in this member is 7,000 pounds; the line $e\,z$ measures about $1\frac{3}{8}$ inches, and the stress in the member $e\,z$ is 5,500 pounds. In this manner, the stress in any member may be determined.

127. Diagram for a Jib Crane.—A jib crane proportioned as in Fig. 87 has a load of 30,000 pounds suspended

at the end of the jib; what are the stresses in the guy ropes and in the different members of the crane, and what are the reactions CA and DA?

In the stress diagram, Fig. 88, draw the vertical line bc equal to 30,000 pounds, and from the point c draw the line ce parallel to CE in the frame diagram, Fig. 87. Then from b draw the line eb parallel to EB in the frame diagram. Again going around the joint to check the polygon of forces, they are found to be from c to e, from e to b, and from b back again to c. The next joint encountered is EDB. Hence, from e draw ed upwards parallel to ED, and from b draw db parallel to DB, the point where these two lines intersect being d. The polygon of forces about the joint EDB is from b to e, from e to d, and from d back again to b, the starting point. Next, go around the joint $CADE$, and draw ca upwards; then from d draw ad parallel to AD in the frame diagram; where the lines just drawn intersect will be the point a; de and ec have already been drawn. The remaining joint to work around is ABD. On looking at the stress diagram, it may be seen that the forces around this joint have already been determined, completing the stress diagram.

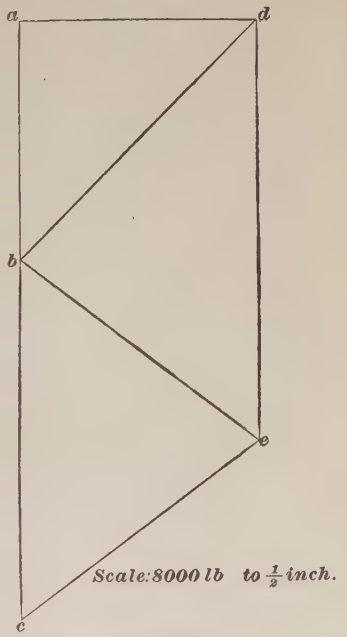


FIG. 88.

The stresses in the members may be determined, as already stated, by measuring the lines corresponding to them in the stress diagram, with the scale to which the diagram has been drawn.

128. Roof Truss With a 40-Foot Span.—Fig. 89 shows the frame diagram for a 40-foot span roof truss. The

loads are as shown, the compression members being indicated by heavy lines, and the tension members by a light line. Required, to draw the stress diagram for this truss.

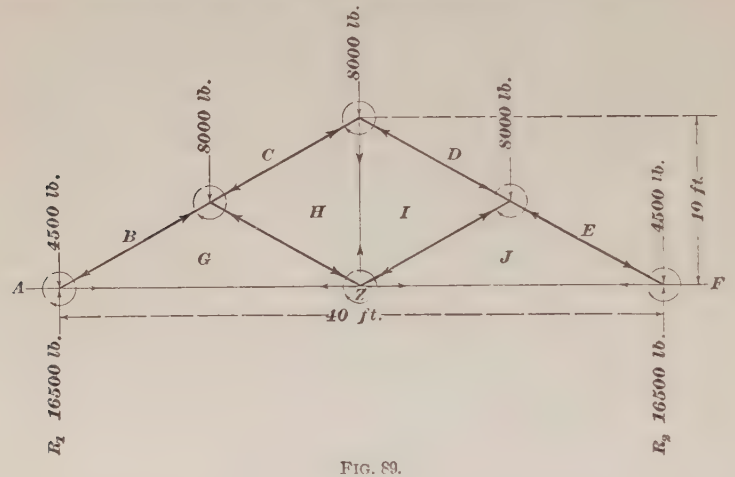


FIG. 89.

First draw the vertical line af as shown in the stress diagram, Fig. 90; mark the point a and lay off on this vertical line, to any scale, using, in this case, 4,000 pounds to $\frac{1}{2}$ inch, the loads ab, bc, cd, de, and ef, corresponding to the loads AB, BC, CD, DE, and EF in the frame diagram. The truss being symmetrically loaded, the loads are the same in amount on the two sides of the center line. The reactions R_1 and R_2 are, therefore, each equal to one-half of the load, in this case, 16,500 pounds. Hence, za may be laid off on the vertical line, and as R_1 equals R_2, za must equal fz; consequently, z is located centrally between a and f, or between c and d. The point z having been determined, proceed with the diagram by going around the joint $ABGZ$. Draw bg in the stress diagram parallel to BG in the frame diagram; then from z draw gz parallel to GZ, the point where the two lines intersect being g. The next joint is $BCHG$. As be in the stress diagram is already known, draw ch parallel to CH and hg parallel to HG. Then the polygon of forces around this joint will be

from b to c, from c to h, from h to g, and from g back again to b. It is now expedient to analyze the joint $C\,D\,I\,H$. In the stress diagram, $h\,c$ and $c\,d$ have already been obtained; then, from d, draw $d\,i$ parallel to $D\,I$, and from the point h, already known, draw $i\,h$ parallel to $I\,H$. The polygon of

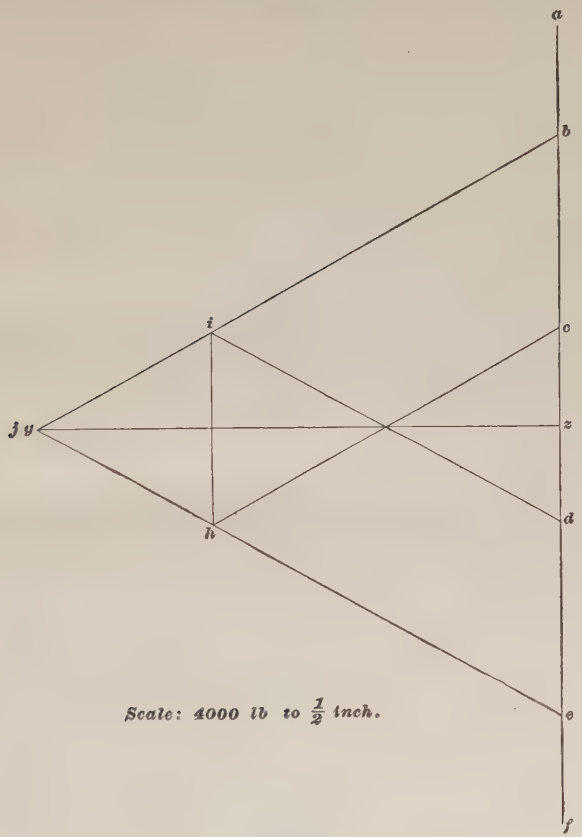


FIG. 90.

forces around this joint will be from c to d, from d to i, from i to h, and h to c, the starting point, arrowheads always marking the direction in which the stresses act in the stress diagram, upon the members in the frame diagram. Now analyze the forces around the joint $G\,H\,I\,J\,Z$ in which the forces $z\,g$, $g\,h$, and $h\,i$ have been obtained. From the point

i, ij is drawn parallel to IJ; and from z, jz is drawn parallel to JZ; the point j is found to fall on the point g, and the polygon of forces around this joint is from z to g, from g to h, from h to i, from i to j, and from j back again to z, the starting point. The stress in the members around the joint $IDEJ$ should next be determined, ji, id, and de being already known. From e draw ej parallel to EJ. The polygon of forces around this joint will then be from i to d, from d to e, from e to j, and from j back again to i.

The only remaining joint to go around is $JEFZ$. By referring to the stress diagram, it is seen that the stresses in these members have been determined, while the polygon of forces around this joint is from j to e, from e to f, from f to z, and back again from z to j.

The stress diagram completed, the magnitudes of the stresses may be determined by measuring the various lines with the scale to which the diagram has been drawn, in this case 4,000 pounds to $\frac{1}{2}$ of an inch.

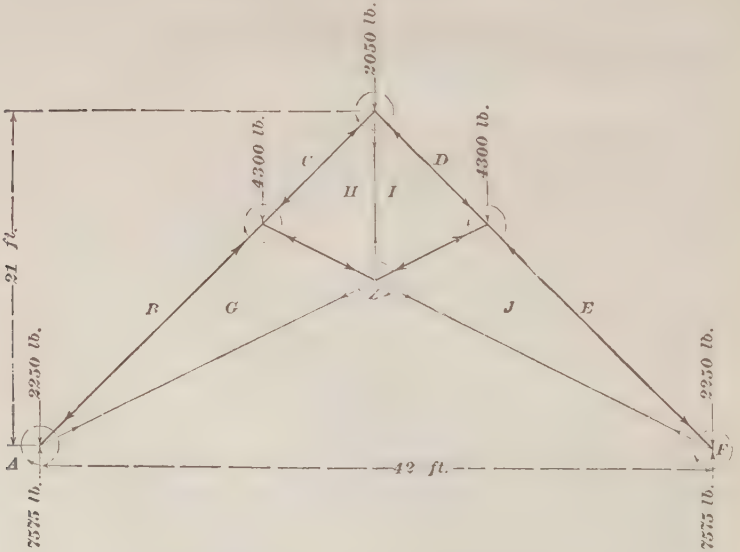


FIG. 91.

129. Truss for a Church Roof.—Fig. 91 is the frame diagram of a form of truss sometimes used to support a

church roof. Determine the stress diagram for the dead load and also the stress diagram for the wind pressure on this roof.

First draw the stress diagram (Fig. 92) for the dead load. As the dead loads upon the truss are symmetrical both in

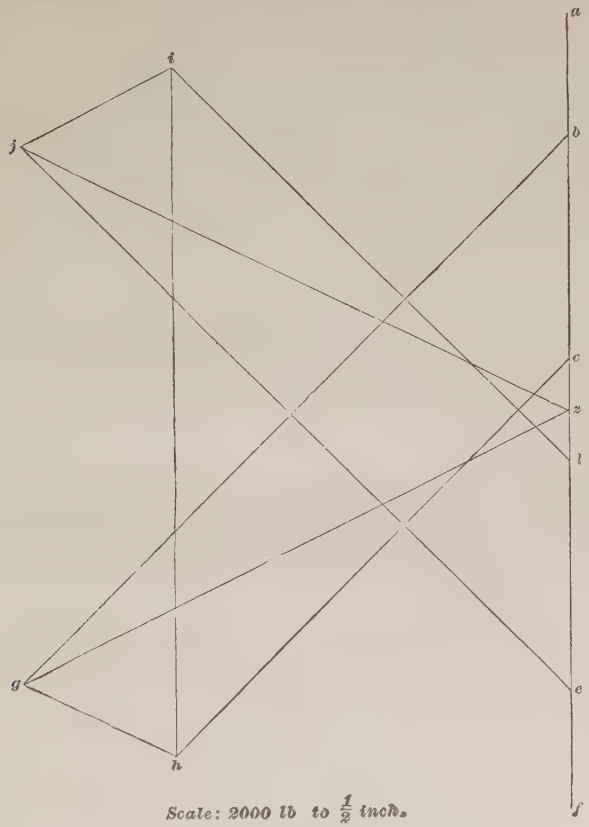


FIG. 92.

amount and location with regard to the center line of the truss, the reactions are the same at either end of the truss, and each one is equal in amount to one-half of the load, in this case 7,575 pounds.

Draw the vertical line af in the stress diagram; then, starting at the point a, lay off the scale of, say, 2,000 pounds

to every ½ inch, the force $a\,b$ equal to $A\,B$ in the frame diagram; then lay off $b\,c$ equal to $B\,C$, $c\,d$ equal to $C\,D$, $d\,e$ equal to $D\,E$, and $e\,f$ equal to $E\,F$. Then, as the truss is symmetrically loaded, the point z is located midway between the points a and f. If the truss was not symmetrically loaded, the reactions would have to be calculated in the same manner as in a beam, already explained.

Having located the loads and their reactions upon the vertical line $a\,f$, obtain the stresses in the members around the joint $A\,B\,G\,Z$ from the point b, by drawing a line $b\,g$, parallel to $B\,G$ in the stress diagram; then from z draw the line $g\,z$ parallel to $G\,Z$, and the intersection of these two lines will be the point g. The polygon of forces around this joint is from b to g, from g to z, from z to a, and then from a back to b, the starting point. Bear in mind that the forces in the stress diagram representing the reactions must have the same direction as the reactions in the frame diagram. The lines determining the stresses around the joint $B\,C\,H\,G$ should next be drawn; $b\,c$ having been determined, from c draw a line $c\,h$ parallel to $C\,H$, and from g draw a line $g\,h$ parallel to $H\,G$, the intersection of these two lines determining the point h; the polygon of forces is from b to c, from c to h, from h to g, and back again from g to b.

Now work around the joint $C\,D\,I\,H$; $c\,d$ being already known, from the point d draw the line $d\,i$ parallel to $D\,I$, and from the point h draw the line $h\,i$ parallel to $I\,H$, the intersection of the two lines being the point i. In going around the joint $G\,H\,I\,J\,Z$, the stresses in the members $z\,g$, $g\,h$, $h\,i$ have already been determined and drawn in the stress diagram. Then from the point i draw the line $i\,j$ parallel to $I\,J$, and from z draw the line $z\,j$ parallel to $J\,Z$, and the intersection of these two lines will be the point j. The polygon of forces around this joint is from z to g, from g to h, from h to i, from i to j, and from j back to z, the starting point.

The next joint to analyze, in going around the truss, is $D\,E\,J\,I$; $j\,i$, $i\,d$, and $d\,e$ being known, the only remaining force to determine is the stress in the member $E\,J$. The

point e being fixed, draw the line ej parallel to EJ in the frame diagram, and if this line, which completes the diagram, passes through the point j, the diagram is correct and accurately drawn. The stresses around the right-hand heel of the truss are all known, the line ej just drawn having been the only unknown member at this joint.

The polygon of forces around the joint $DEJI$ is from d to e, from e to j, from j to i, and from i to d, the starting point. The polygon of forces around the joint $EFZJ$ is from e to f, from f to z, from z to j, and from j back again to e, completing the stress diagram for the dead or vertical load upon the roof truss.

130. The **wind diagram,** however, remains to be drawn. The student may redraw the frame diagram as shown in Fig. 93. The wind is always considered as acting *normally*, or at *right angles*, to the roof, the amount of its pressure at the different joints of the truss being shown on the frame diagram. As the heels of the trusses are fixed, the reactions act in lines parallel to the wind pressure.

To estimate the magnitude of the reactions R_1 and R_2, consider the left-hand rafter member as a beam, and R_1 and R_2 as the reactions supporting it. The moments due to the wind pressure BC and CD acting about R_1 are:

at BC, $6{,}556 \times 19.5$	$= 127{,}842$ ft.-lb.
at CD, $1{,}950 \times 29.66$	$= \underline{57{,}837}$ ft.-lb.
Total, ...	$185{,}679$ ft.-lb.

The lever arm with which R_2 resists the wind pressure acting at the joints is 29 feet 8 inches, so $185{,}679 \div 29.66 = 6{,}260$ pounds, the amount of the reaction at R_2. The sum of the loads being $4{,}600 + 6{,}556 + 1{,}950 = 13{,}106$ pounds, the reaction at R_1 is $13{,}106 - 6{,}260 = 6{,}846$ pounds.

First, draw the load line ad in the stress diagram, Fig. 94; this line is parallel to the direction of the wind pressure and the reactions. Now lay off to the scale to which the stress diagram is drawn—in this case 2,000 pounds to every $\frac{1}{2}$ inch—the force ab equal to AB in the frame diagram; then lay off bc equal to BC and cd equal to CD. From

d lay off the magnitude of the reaction, R_2, or dz, which determines the point z, and the distance za, according to scale, represents the left-hand reaction R_1. The polygon of external forces is, then, from a to b, from b to c, from c to d, from d to z, and from z back again to a, the starting point. This polygon, as may be readily seen, is a straight line, as in all cases so far analyzed.

Continue the stress diagram Fig. 94, by going around the joint $A\ B\ G\ Z$; from the point b draw the line bg, and from

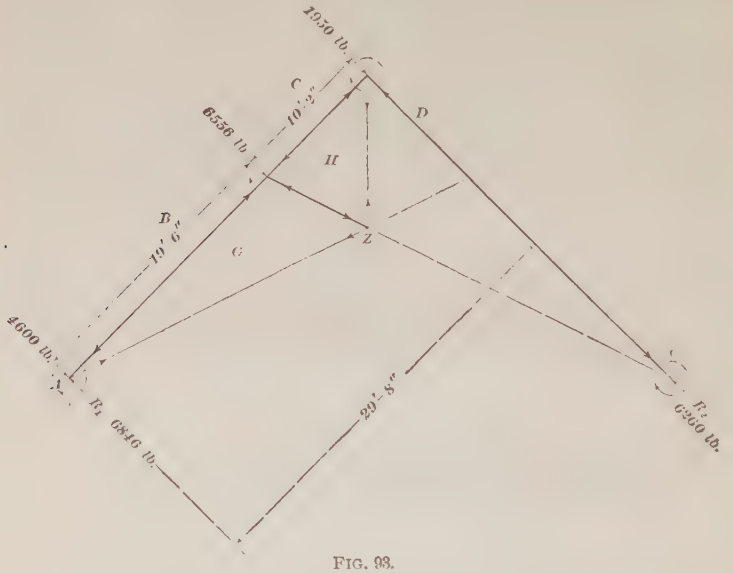
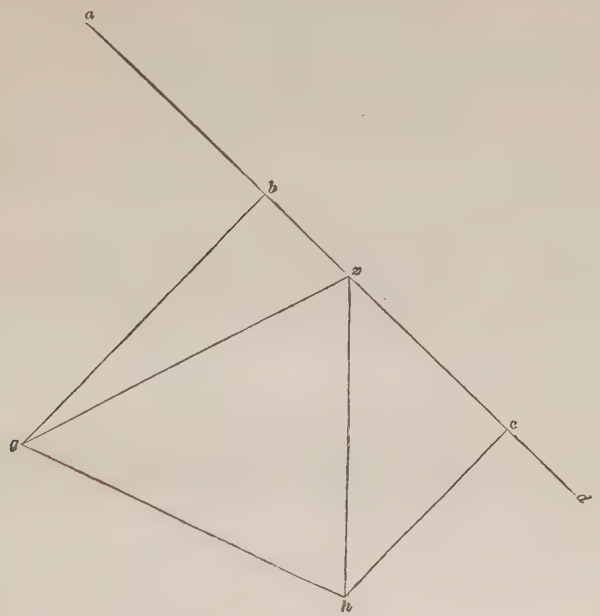


FIG. 93.

the point z draw the line zg parallel to the corresponding members in the frame diagram, the point where the two lines intersect being g. The polygon of forces around this point is from a to b, from b to g, from g to z, and from z back again to a.

The next joint is $B\ C\ H\ G$; bc has been already obtained; then from the point c draw the line ch parallel to CH, and from g draw the line gh parallel to HG, h being the point where these two lines cross. Disregard the two members in the frame diagram shown in dotted lines, which do nothing towards sustaining the wind pressure. Now work around

the joint $H\,C\,D\,Z$; $c\,d$ and $d\,z$ are known; draw from z the line $z\,h$, parallel with $Z\,H$ in the frame diagram; if this closing line of the diagram passes through the point h, the diagram has been drawn accurately. The polygon of forces around the joint $B\,C\,H\,G$ is from b to c, from c to h, from h to g, and g back again to b. The polygon of forces around the joint $C\,D\,Z\,H$ is from c to d, from d to z, from z to h,



Scale: 2000 lb to $\frac{1}{2}$ inch.

FIG. 94.

and from h back again to c. The polygon of forces around the joint $Z\,G\,H$ is from z to g, from g to h, and from h to z, the starting point.

The stress diagram for both the dead and the wind load being complete, to obtain the stress in each member of the truss, it is required to determine, by scale, the stress due to both the dead and wind loads in each member, adding the two together for the maximum load in the member. To determine, for instance, the stress in the strut $H\,G$, measure

the length of the line hg in the stress diagram, Fig. 92, for

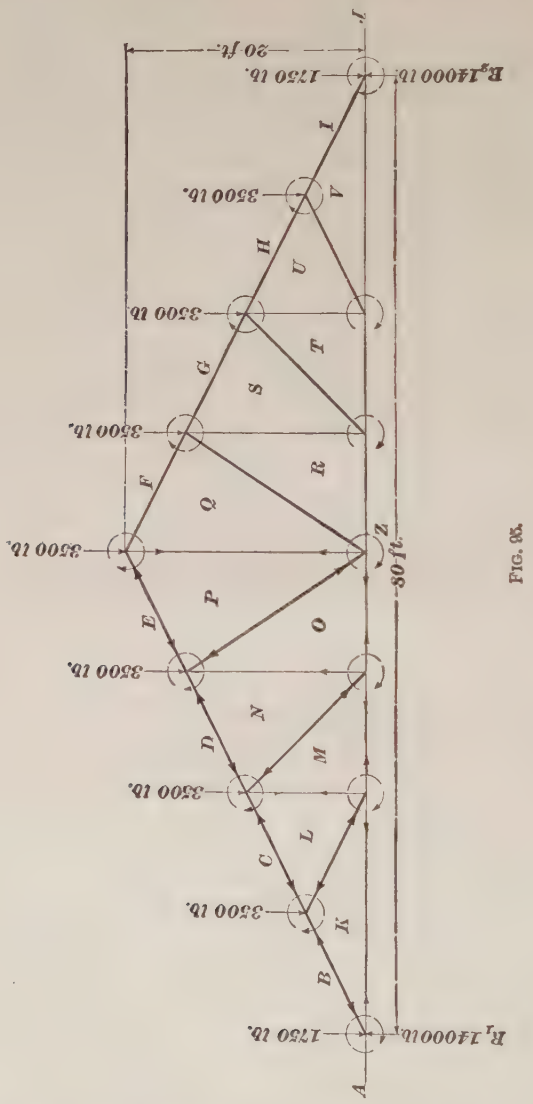


FIG. 93.

the dead load; then measure the same line hg in the stress diagram for the wind, Fig. 94, add the two measurements

together, and determine the maximum stress in the strut hg from the assumed scale of the drawing.

It must be remembered that while the wind acting on one side of a truss does not create stresses in all the members on the opposite side, these members should be proportioned in like manner as the other members, because the wind is quite as likely to blow upon this side of the roof and reverse the conditions.

131. Wooden Truss With an 80-Foot Span.—Fig. 95 is the frame diagram for an 80-foot span wood roof truss.

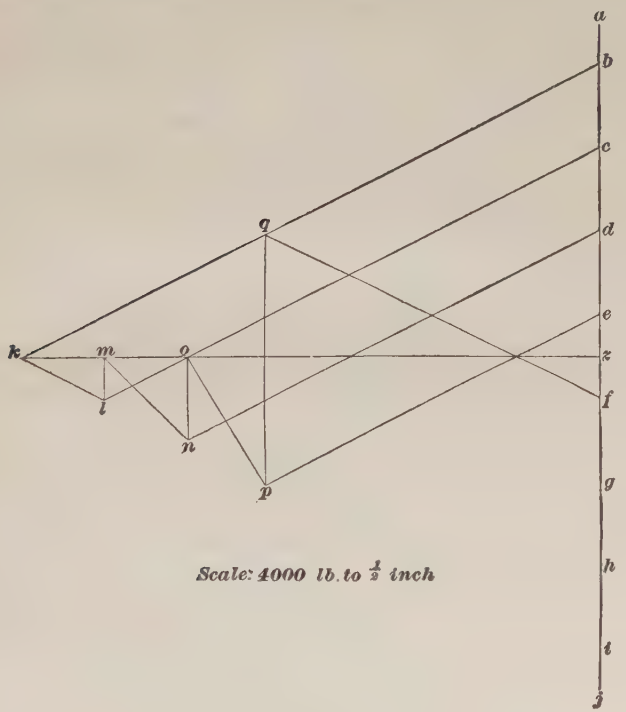


FIG. 96.

It is desired to draw the dead-load diagram and the wind-stress diagram; also to design and properly proportion the roof truss to resist the stresses that the various members may be required to sustain.

Draw the frame diagram shown in Fig. 95, and mark the

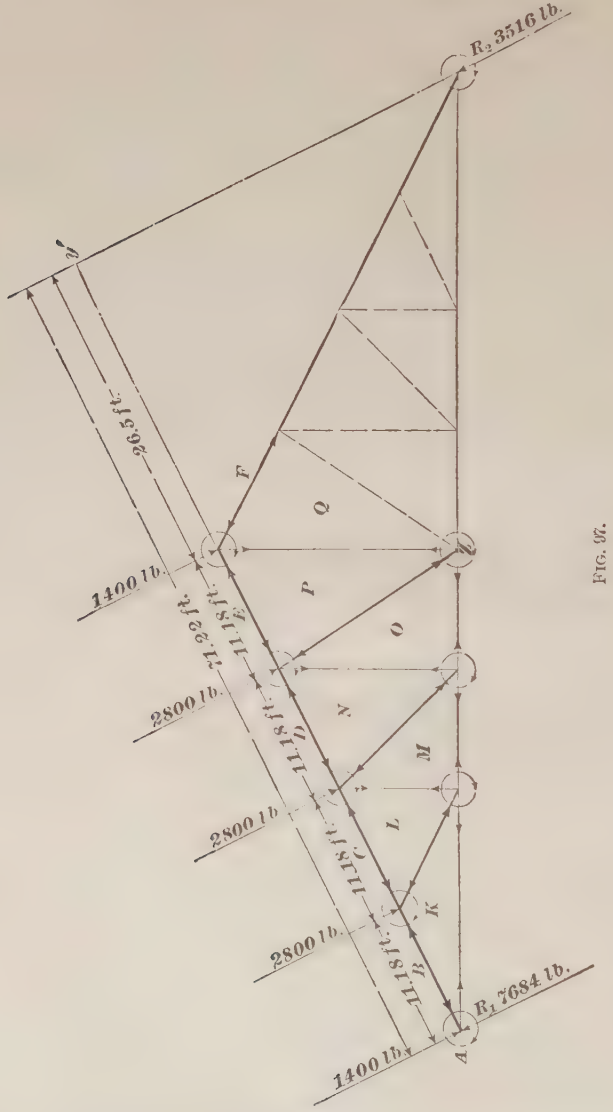


FIG. 97.

dead load coming upon the different panel points, or joints, in the truss. The truss being symmetrically loaded, the

reactions R_1 and R_2 are each equal to half the load upon the truss.

Draw the stress diagram, Fig. 96, for the dead load, say to the scale of 4,000 pounds to ½ inch. Draw the vertical load line $a\,j$, and determine the point z, having previously located upon the line all the loads. Then draw the stress diagram by the methods previously given. Only one-half of the diagram need be drawn, as the stresses obtained on one side of the center of the truss apply to the other side. For instance, $f\,q$ is the same as $p\,e$. Having completed half of the stress diagram for the dead load, the student should redraw the frame diagram as shown in Fig. 97. The direction and amount of the wind pressure at the several panel points, or joints, of the truss, are shown in the frame diagram.

As both ends of the truss are secured against sliding, the reactions act in a direction parallel to the wind pressure. If the left-hand side of the truss be secured, the right-hand side being on rollers, as is sometimes the case with iron or structural steel trusses, to allow for expansion, then the right-hand reaction, instead of being parallel to the direction of the wind, would be vertical. This makes considerable difference in the stress diagram, as will be explained further on.

To determine the magnitude of the reactions R_1 and R_2, let R_1, Fig. 97, be the center around which the moment of R_2 is taken; then the perpendicular distance between the line of action of R_2 and the point R_1 will be 71.22 feet. Extend the left-hand rafter until it cuts the line of action of the force R_2 at the point y'. Regard this extension and the rafter as a beam, and calculate the magnitude of the reactions R_1 and R_2 by the methods given for beams.

The moments about R_1 are as follows:

$$\begin{aligned} 2{,}800 \times 11.18 &= \quad 31{,}304 \text{ ft.-lb.} \\ 2{,}800 \times 22.36 &= \quad 62{,}608 \text{ ft.-lb.} \\ 2{,}800 \times 33.54 &= \quad 93{,}912 \text{ ft.-lb.} \\ 1{,}400 \times 44.72 &= \quad 62{,}608 \text{ ft.-lb.} \\ \text{Total, } \ldots &\quad 250{,}432 \text{ ft.-lb.,} \end{aligned}$$

and $250{,}432 \div 71.22 = 3{,}516$ pounds, the reaction R_2. Having

found R_2, find R_1 by subtracting R_2 from the sum of the loads.

The sum of the loads is $1,400 + 2,800 + 2,800 + 2,800 + 1,400 = 11,200$ pounds. Then $11,200 - 3,516 = 7,684$ pounds, the reaction R_1.

Next, lay out the wind diagram, Fig. 98, by drawing the load line $a\,f$ parallel to the direction of the wind in the frame diagram, Fig. 97. Lay off to the scale (in this case 4,000 pounds to 1 inch) the forces $a\,b$, $b\,c$, $c\,d$, $d\,e$, and $e\,f$ equal to $A\,B$, $B\,C$, $C\,D$, and so on, in the frame diagram. Then

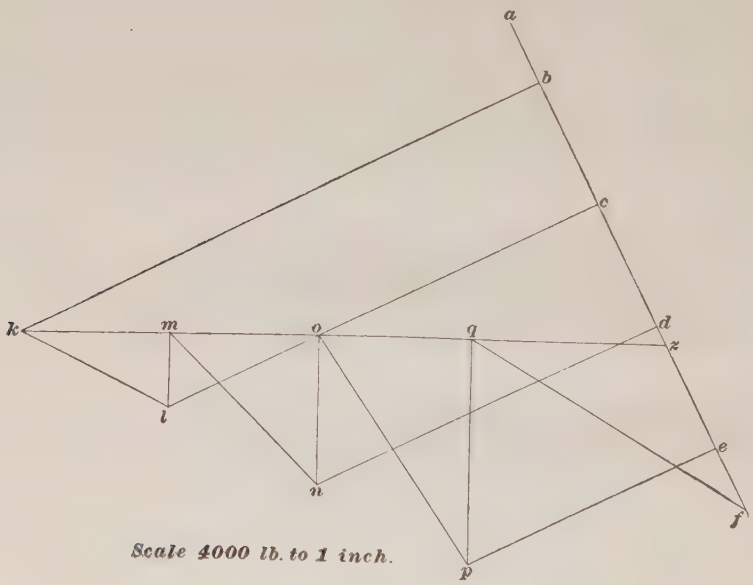


FIG. 98.

from a lay off $a\,z$ equal to the reaction $Z\,A$, or R_1. If the other forces or loads have been laid off accurately, $f\,z$ should, upon measurement, be found equal to the right-hand reaction R_2.

The first joint to analyze is $A\,B\,K\,Z$. Start at b and draw the line $b\,k$ parallel to $B\,K$ in the frame diagram; then from z draw $z\,k$ parallel to $K\,Z$, where the two lines intersect being the point k. Then the polygon of forces around this joint

is from *a* to *b*, from *b* to *k*, from *k* to *z*, and from *z* back again to the starting point *a*.

The next joint to analyze is *B C L K*. From the point *c* draw the line *c l* parallel to *C L* in the frame diagram, and from *k* draw the line *k l* parallel to the member *L K*, the point of intersection being *l*. The polygon of forces around this joint is from *b* to *c*, from *c* to *l*, from *l* to *k*, and from *k* back again to *b*.

To analyze the joint *K L M Z*: *k l* being already known, the next number is *L M*; therefore, from the point *l* draw the line *l m* parallel to *L M* in the frame diagram. As the next member around this joint is *M Z*, to which *m z* in the stress diagram is parallel, the point *m* is located where the line *l m* intersects the line *m z*; this completes this joint, the polygon of forces around it being from *k* to *l*, from *l* to *m*, from *m* to *z*, and from *z* back again to *k*.

To determine the stresses in the members around the joint *C D N M L*, draw from the point *d* the line *d n*, parallel to the member *D N* in the frame diagram; then, from the point *m* draw *m n* parallel to *N M*. The polygon of forces around this joint is from *c* to *d*, from *d* to *n*, from *n* to *m*, from *m* to *l*, and from *l* back again to *c*.

To analyze the forces around the joint *M N O Z*, draw from *n* the line *n o* upwards, parallel to *N O* in the frame diagram; as the next member *O Z* is horizontal, the point *o* must be at the intersection *n o* and *o z*. This completes this joint, and the polygon of forces around it is from *m* to *o*, from *n* to *o*, from *o* to *z*, and from *z* back again to *m*, the starting point.

Now analyze the joint *D E P O N*. From *e* draw the line *e p* parallel to *E P* in the frame diagram, and from *o* draw the line *o p* parallel to *P O*. The intersection of these two lines determines the point *p*, and the polygon of forces around this joint is from *d* to *e*, from *e* to *p*, from *p* to *o*, from *o* to *n*, and from *n* back again to *d*, the starting point.

The analysis of the joint *O P Q Z* is made by drawing the vertical line *p q* from the point *p*; the point where *p q* intersects *q z* is *q*. Then the polygon of forces around this joint is from *o* to *p*, from *p* to *q*, from *q* to *z*, and from *z* back

again to *o*. The members shown in dotted lines do not sustain any stresses from the pressure of the wind, when it blows upon the left-hand side of the truss.

The final joint to consider, thus completing the stress diagram, is *E F Q P*. There is only one unknown force around this joint, and that is the stress in the member *F Q*. A line drawn from *f* in the stress diagram, parallel with the member *F Q*, should pass through the point *q*; if it does not, the diagram has been inaccurately drawn. This is always a test of the accuracy of the stress diagram, and if the last line in this diagram does not close on the proper point, when drawn parallel to the member it represents, there is something so radically wrong as to demand that the stress diagram be redrawn, to determine whether the loads and reactions have been laid out correctly, and whether any of the joints or members in the structure have been passed over.

132. The two diagrams completed, measure the different lines and obtain the stresses in the various members, tabulating the results, as follows:

Member.	Dead-Load Diagram.	Wind-Load Diagram.	Total of Both.	Kind of Stress.
B K	27,000	12,000	39,000	Compressive.
C L	23,500	10,000	33,500	Compressive.
D N	19,500	7,600	27,100	Compressive.
E P	16,000	5,680	21,680	Compressive.
K Z	24,000	13,300	37,300	Tensile.
M Z	21,000	10,000	31,000	Tensile.
O Z	17,500	7,000	24,500	Tensile.
L K	4,000	3,500	7,500	Compressive.
N M	5,000	4,400	9,400	Compressive.
P O	6,500	5,400	11,900	Compressive.
M L	1,600	1,500	3,100	Tensile.
O N	3,500	3,000	6,500	Tensile.
Q P	10,600	4,500	15,100	Tensile.
F Q	16,000	6,600	22,600	Compressive.

The values in the above table represent the stress in round numbers upon the various members in the truss, as obtained from the dead-load and wind diagrams.

The size of the timber and tension rods in the truss may now be calculated.

133. Designing the Members of the Truss.—The first vertical tension member is ML, this member having a total pull upon it of 3,100 pounds. This rod will be made of wrought iron, the tensile strength of which is 52,000 pounds. If a factor of safety of 4 is used, the safe load will be $52{,}000 \div 4 = 13{,}000$ pounds per square inch. A rod $\frac{3}{4}$ inch in diameter has an area at the root of the screw thread of .302 square inch. Then the safe strength of a rod $\frac{3}{4}$ inch in diameter, threaded at the ends, is $.302 \times 13{,}000 = 3{,}926$ pounds. This member being required to support 3,100 pounds only, a wrought-iron rod $\frac{3}{4}$ inch in diameter is amply strong. The tension member ON is subjected to a stress of 6,500 pounds. A rod 1 inch in diameter has an area at the root of the thread of .550 square inch. Hence, its safe strength is $.550 \times 13{,}000 = 7{,}150$ pounds, or slightly in excess of that required. The member QP is subjected to a stress of 15,100 pounds. The area at the root of the thread of a rod $1\frac{1}{2}$ inches in diameter is 1.29 square inches, and the safe strength is $1.29 \times 13{,}000 = 16{,}770$ pounds, which exceeds the strength required.

To calculate the size of timber demanded for the rafter member, note that it is usual in timber trusses to make the rafter members of one size and of one length. This truss requires, then, a large stick of timber, say about 45 feet long, for the rafter members, and obtainable, likely, by special order. As the maximum stress on the rafter, made of one piece of timber throughout, is at BK, we have to calculate its size at this point only. Assume that the truss supports heavy purlins only at the panel points, or joints, of the truss, common rafters being laid up and down the roof, resting on these purlins as shown in Fig. 99. This concentrates all the load on the truss at the panel points, and BK is consequently not to be estimated as a beam, sustaining as it does only the compressive stress as obtained by the diagram.

The total compressive stress in BK is 39,000 pounds, and its length is 11.18 feet = 134 inches, nearly. Using an $8'' \times 8''$ yellow-pine timber, the ultimate compressive strength, parallel to the grain, as given in Table 6, Art. **61,** is 4,400 pounds per square inch. By formula **3,** Art. **70,** the ultimate compressive breaking strength of BK as a column is

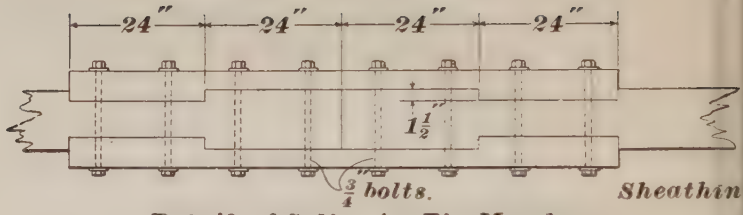
$$S = U - \left(\frac{U \times L}{100\ D}\right) = 4{,}400 - \left(\frac{4{,}400 \times 134}{100 \times 8}\right)$$

$$= 4{,}400 - 737 = 3{,}663 \text{ pounds per square inch.}$$

If, on account of its uncertain nature, a factor of safety of 5 is used for the timber, the safe bearing value of the column is, then, $3{,}663 \div 5 = 733$ pounds per square inch. The area of an $8'' \times 8''$ column is 64 square inches, which multiplied by 733 gives a safe load of 46,912 pounds. While considerably in excess of the 39,000 pounds required, this is the nearest even-sized timber that could be used for this member.

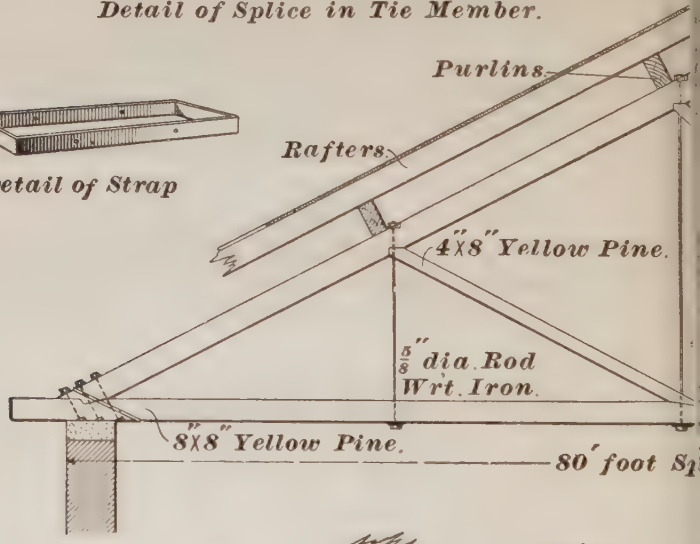
To determine the size of timber required for the tie-member, bear in mind that the greatest pull, or tensile stress, upon this member is at KZ, and amounts to 37,300 pounds. This member, made of yellow-pine timber, must be spliced, say at the center. It is safe to assume that, in making the connection and joints, one-third of this timber will be cut away. The ultimate tensile strength of yellow pine is, according to Table 6, Art. **61,** 8,000 pounds per square inch. This divided by the factor of safety of 5 equals 1,600 pounds, the safe stress that a square inch will sustain.

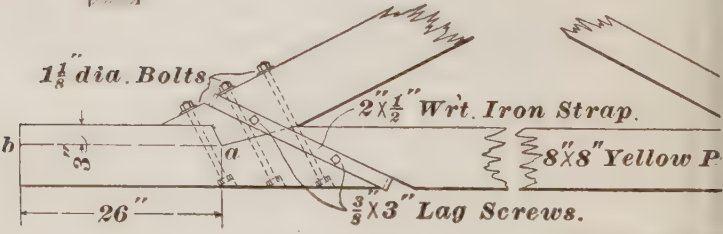
For convenience in construction, a $6'' \times 8''$ timber is selected. The area of this timber is 48 square inches; $\frac{2}{3}$ of this is 32 square inches. Then, $32 \times 1{,}600 = 51{,}200$ pounds. A $6'' \times 8''$ timber is thus amply strong, but upon drawing the truss and laying out the detail of its connections, it may be found necessary to use a timber 8 in. $\times$ 8 in. In determining the size of the member PO, bear in mind that the stress on this member is 11,900 pounds. The member PO in the frame diagram, Fig. 97, measures about 18 feet. The formula for wood columns shows that this member may be made of a $6'' \times 8''$ timber, which is amply strong.



Detail of Splice in Tie Member.

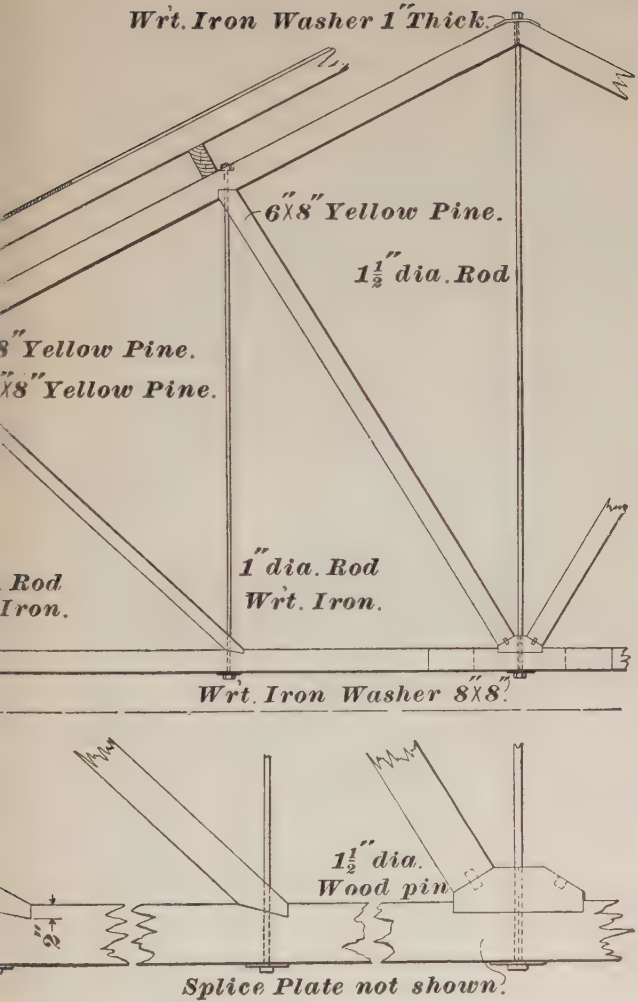
Detail of Strap





Detail of Jo

FIG. 9



Splice Plate not shown.

on Tie Member.

A $4'' \times 8''$ timber would probably carry the load, but the length of the strut being more than 45 times the width of the least side, the next larger stock size of timber is adopted, viz., 6 in. $\times$ 8 in. The timbers, to facilitate making the connections, should all be of one thickness.

In this case, for instance, the face of all the members in the truss are flush, for all are of the same thickness. The other struts, KL and MN, will be found, in like manner, to be amply strong, if made of $4'' \times 8''$ timber. The tie-member is of such a length as to be made up of at least two pieces, spliced at the center. The shear of the wood parallel to the grain determines the strength of the splice, as explained in the example, Art. **62.** The bolts may be used to hold the splice together, but should not be depended upon to withstand any of the pull on the tie-member.

In estimating the strength of the tie-member at the heel, it may be difficult to obtain a sufficient section of wood at the end of the tie to resist the shear parallel to the grain on the line $a\,b$. In this instance, the pull on the member is 37,300 pounds. The area of the shearing section on the line $a\,b$ is $8 \times 26 = 208$ square inches. According to Table 6, Art. **61,** the ultimate shearing strength of yellow pine per square inch, parallel to the grain, is 400 pounds. If a factor of safety as low as 4 is used, the safe strength per square inch will be 100 pounds. Then, $208 \times 100 = 20{,}800$ pounds. Now, as previously stated, the stress is 37,300 pounds, and the remaining stress of 16,500 pounds will have to be taken up by securely bolting the joint and by strapping it, as shown in Fig. 99. It is seldom found that sufficient shear can be obtained in the wood to resist the stress at this point, it being always necessary to depend more or less on the bolts. In a large truss, like that under consideration, the best practice demands that the wrought-iron strap be so proportioned as to be capable of resisting all the stress not borne by the shear of the wood on the line $a\,b$, no reliance whatever being placed on the bolts. The lagscrews simply retain the straps in position. One notch in the tie, to receive the rafter, is always preferable to two, as given in Fig. 100.

Nothing is gained in strength by using two, because one or the other of the joints will open from shrinkage, and, consequently, either the piece *h* or *i* will shear off before any stress is brought upon the other.

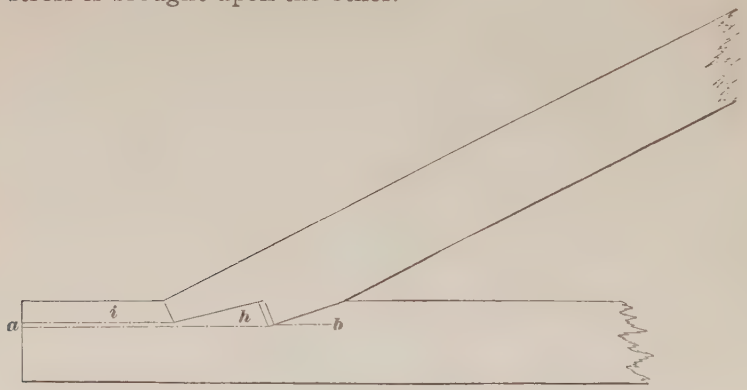


FIG. 100.

Care should be taken that the washers are of sufficient area to prevent them from cutting into the wood. Table 6, giving the allowable compression perpendicular to the grain, is of use in determining the sizes of the washers.

The tension member $\frac{5}{8}$ inch in diameter, Fig. 99, is not needed to resist any stress, but is useful in the truss to hold the strut in position and prevent trouble from any shrinkage that may occur. No further details need be given in the design of wood roof trusses, good judgment and foresight on the part of the designer securing, with the principles and practice already laid down, the desired result, viz., maximum strength in the structure, with a minimum amount of material.

DESIGN FOR A LARGE BUILDING.

134. The combination of general principles and practical rules and formulas heretofore presented, is sufficient to enable the student to solve any problem involved in the design of a large building having floors supported by masonry walls and wooden or cast-iron columns and a roof

carried by timber trusses. A practical example of the application of the preceding rules and formulas is now given in the form of a complete design for a five-story factory building, with basement.

The foundations are of rough stone in cement mortar. The soil on which the building is to rest is of dry clay. The windows are to be 4 feet wide and the floor girders 8 feet from center to center, while the basement columns are to be of cast iron, other columns and posts being of yellow pine. The first-floor girders are of steel, with steel floorbeams placed on 4-foot centers. A 4-inch brick arch is built between the beams and is filled in with concrete, having a cement top finishing coat. The other floors in the building are supported upon yellow-pine trussed beams, the flooring to be of 3-inch yellow-pine plank, splined and grooved with 1-inch finished flooring upon the top.

The live load upon the floor will be 120 pounds. The roof will be supported upon a wood truss making a clear span of the top floor, and the roof covering is slag upon 2-inch yellow-pine planks. The pitch of the roof is 4 inches per foot. The brick walls of the building are laid in lime mortar.

Fig. 101 is a scale drawing of a section and part elevation of the building. Fig. 102, showing details of the various members and connections, is, in fact, a complete working drawing of the entire building. The 4-inch brick arch between the steel beams supports all the load a floor of this character may ever require to carry. The bluestone caps upon which the columns rest are 17 inches thick, or equal in thickness to one-half the width of the side. This is according to the rule previously given in designing foundation piers for columns. It is, however, usual to find capstones made much thinner in ordinary work, but, if good results are desired, adhere to this rule. The cast-iron caps and wall boxes shown in detail in Fig. 103 are known as the Goetz-Mitchell wall box and column cap.

The wall box is especially good as an anchor to bind the wall and the end of the girder together. The cast-iron dowel-pins on the casting, over which the wood girder is

placed, securely hold the girder to the box; the flared sides of the box, when built into the wall, prevent it from being

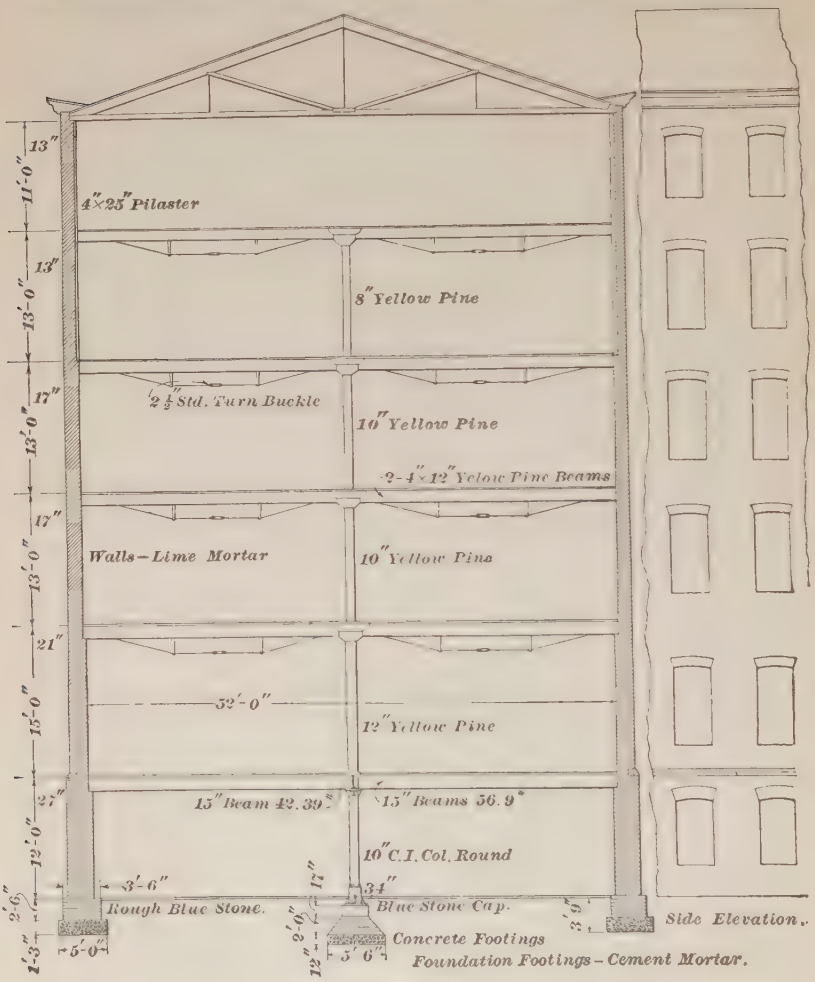
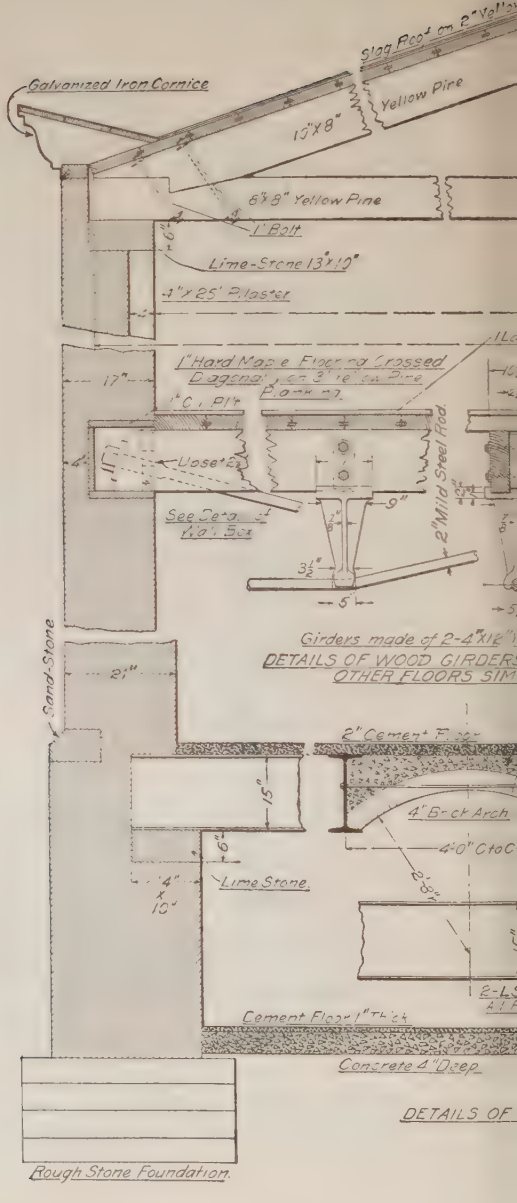
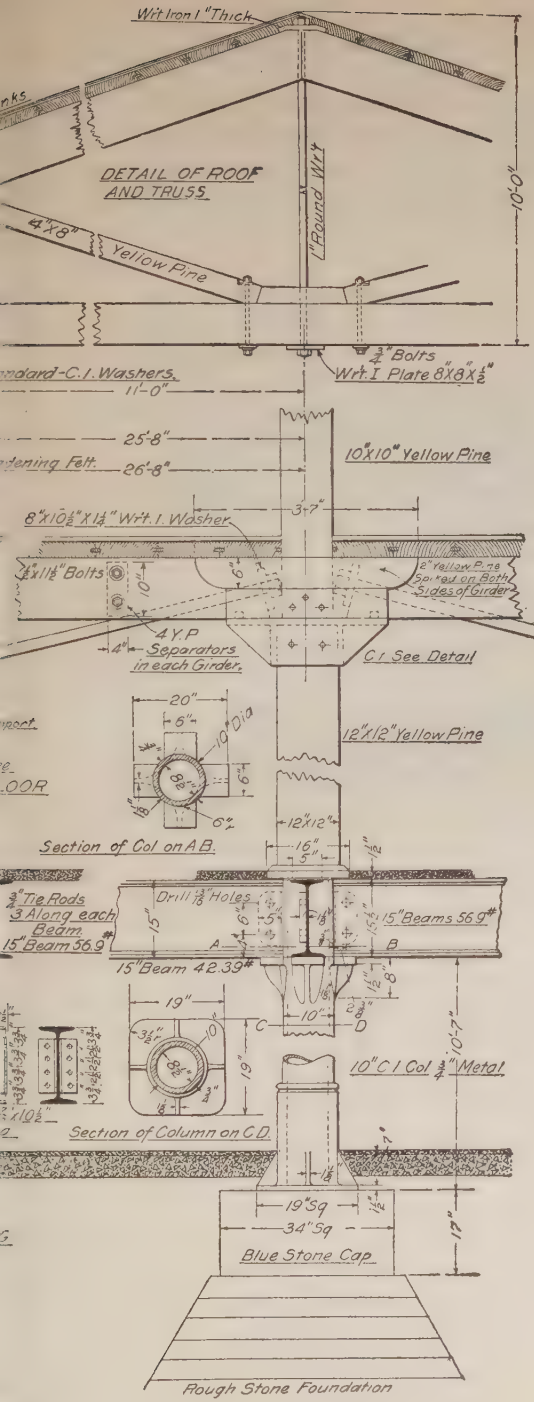


FIG. 101.

pulled out. Vent holes along the edge of the wall box ventilate the end of the wood girder and thus prevent any tendency to dry rot.


Galvanized Iron Cornice
Yellow Pine
10"X8"
8"X8" Yellow Pine
1" Bolt
Lime-Stone 13"X10"
4"X25' Pilaster
17"
2" Mild Steel Rod
9"
3½"
5"
Girders made of 2-4"X12"
DETAILS OF WOOD GIRDERS
OTHER FLOORS SIM
Sand-Stone
21"
15"
4" Brick Arch
4'-0" C to C
6"
4" x 10"
Lime Stone
Concrete 4" Deep
DETAILS OF
Rough Stone Foundation



Wrt Iron 1" Thick
DETAIL OF ROOF AND TRUSS
1" Round Wrt
10'-0"
4"X8"
Yellow Pine
3/4" Bolts
Wrt. I Plate 8"X8"X1/2"
ndard-C.I. Washers.
11'-0"
25'-8"
dening Felt.
26'-8"
10"X10" Yellow Pine
3'-7"
8"X10 1/2"X1 1/4" Wrt. I. Washer
1/2"X11 1/2" Bolts
2" Yellow Pine Spiked on Both Sides of Girder
4 Y.P Separators in each Girder.
C I See Detail
20"
6"
10" Dia
12"X12" Yellow Pine
Section of Col on A B.
12"X12"
16"
5"
3/4" Tie Rods 3 Along each Beam.
15" Beam 56.9#
Drill 13/16" Holes
15" Beams 56.9#
15" Beam 42.39#
A
B
C
D
19"
10"
10'-7"
10" C I Col 3/4" Metal
Section of Column on C.D.
19" Sq
34" Sq
17"
Blue Stone Cap
Rough Stone Foundation

The details shown in Fig. 102 are good practical examples of the application of the principles already set forth; the student is urged to study them carefully and compare their dimensions with the values obtained by the use of the rules.

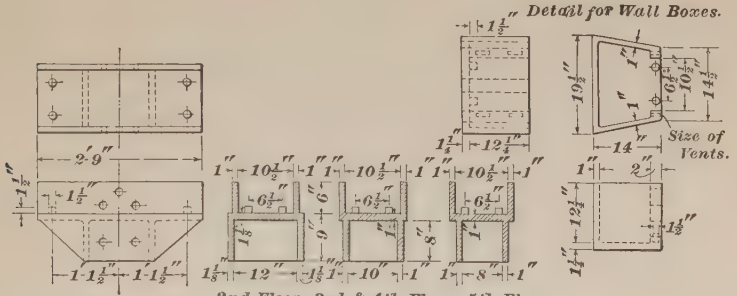


FIG. 103.

He must constantly bear in mind that the engineer often meets conditions to which no published rule or formula applies; these conditions can be successfully met only by the exercise of good judgment in applying the general principles of mechanics, coupled with a thorough and practical knowledge of the properties and uses of available structural materials.

ARCHITECTURAL ENGINEERING.

(CONTINUED.)

PROPERTIES OF SECTIONS.

INTRODUCTORY.

1. The calculations involved in the design of such "built-up" members of a building as steel columns and plate girders—members that are formed by the combination of several of the simple sections produced by the rolling mills—require a knowledge of certain mathematical properties of the simpler sections, together with the methods by which these properties may be calculated. In many cases the exact determination of the required properties is based on complicated mathematical principles; there are, however, numerous formulas and practical methods by means of which the values for all sections used in ordinary practice may be determined, either exactly or with a degree of approximation sufficiently close for all practical purposes.

THE NEUTRAL AXIS.

2. Location of the Neutral Axis.—In Art. **92,** *Architectural Engineering*, § 5, it was stated that the neutral axis—the line separating the fibers in tension from those subjected to compression in a section of a beam—always passes through the center of gravity of the section. It is also evident from what was there stated that the neutral axis is perpendicular to the direction in which the load acts on the beam; therefore, to find the neutral axis of a section

For notice of copyright, see page immediately following the title page.

with reference to a set of loads applied in a given direction, it is only necessary to pass a line through the center of gravity perpendicular to the direction of the load.

A simple approximate method of locating the center of gravity and neutral axis of a section is shown in Fig. 1.

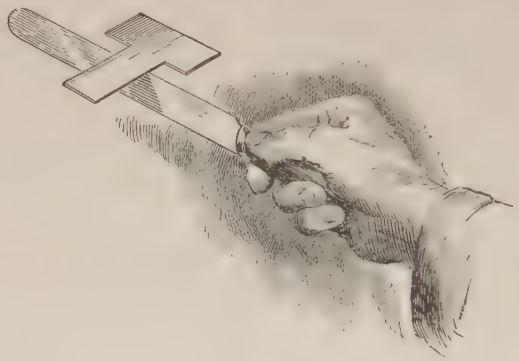


FIG. 1.

Draw the outline of the section, either full size, or to some convenient scale, on a piece of heavy cardboard. Cut the section out and balance it carefully over a knife edge as shown in the figure; the line along which it rests on the edge of the knife is a line passing through the center of gravity, and by locating two such lines in different directions, the center of gravity will be found at their point of intersection.

3. Locating the Neutral Axis by Means of the Principle of Moments.—A convenient method of locating the neutral axis is based on the principle that the moment of any figure, with respect to a given line as an *axis* or *origin of moments*, is equal to the sum of the moments of its separate parts with respect to the same axis. Thus, in Fig. 2, take the line *a b* as the line of origin of moments.

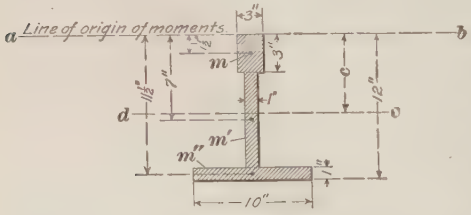


FIG. 2.

Divide the figure into the three rectangles m, m', m''. In accordance with the principles stated in Art. **39**, *Architectural Engineering*, § 5, the center of gravity of each of these rectangles is midway between its edges; the distances of the respective centers from the axis are, therefore, $1\frac{1}{2}$, 7, and $11\frac{1}{2}$ inches. The areas of the figures are respectively $3\times3 = 9$ square inches, $8\times1 = 8$ square inches, and $10\times1 = 10$ square inches. The moments of these areas about the axis $a\,b$ are:

Section m	$9\times 1\frac{1}{2} =$	1 3.5
Section m'	$8\times 7 =$	5 6.0
Section m''	$10\times 11\frac{1}{2} =$	1 1 5.0
Total, . . .		1 8 4.5

The area of the whole section is equal to $9+8+10 = 27$ square inches, the sum of the areas of the rectangles; the distance c of its neutral axis from the line of the origin of moments is, therefore, $184.5 \div 27 = 6.83$ inches, or nearly $6\frac{2}{3}\frac{7}{2}$ inches.

It is not necessary that the line of the origin of moments should coincide with an edge of the figure as in Fig. 2, since any other line parallel with the direction of the required neutral axis gives the same results; in most cases, however, it will be found more convenient to take the axis about which the moments are calculated on one of the extreme edges of the section.

Since the section shown in Fig. 2 is symmetrical with respect to an axis perpendicular to the neutral axis, it is evident that its center of gravity is on their intersection. If, however, there were no axis of symmetry, the center of gravity could have been located by taking a second line perpendicular to $a\,b$ as an origin of moments and finding the neutral axis parallel to it. The intersection of this neutral axis with the one first found is the center of gravity of the section. In accordance with the principles illustrated in the above example, we have the following rule:

Rule.—*To find the neutral axis of any section, first divide it into a number of simple parts, each of whose areas and centers*

of gravity can be readily found; then find the sum of the moments of the areas of each of these parts with respect to an axis parallel to the required neutral axis. Finally, divide this sum by the sum of the areas of the parts of the section, and the result will be the perpendicular distance from the axis of the origin of moments to the required neutral axis.

4. Application of the Rule to a Built-Up Section.— Fig. 3 shows a section of the rafter member of a large roof truss formed of a $\frac{3}{8}'' \times 16''$ web-plate and a $\frac{3}{8}'' \times 12''$ flange plate, the two joined by two $4'' \times 4'' \times \frac{1}{2}''$ angles. What is the distance of the neutral axis of the section from the top edge of the flange plate? By means of the principles previously

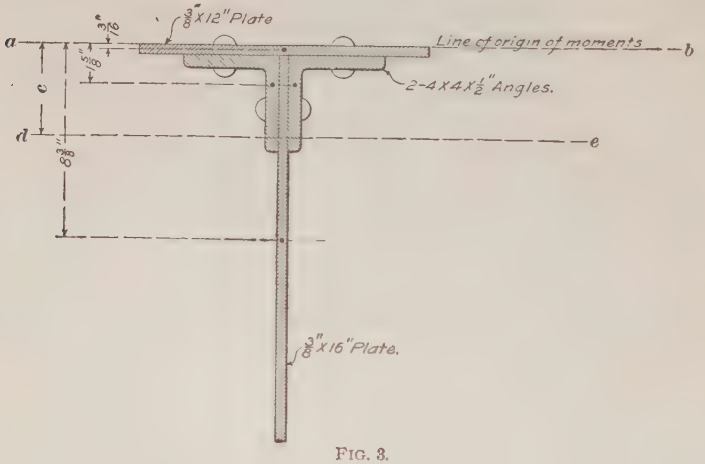


FIG. 3.

given, the centers of gravity of the two rectangular plates are easily located as shown. The centers of gravity of the angles might also be located by applying the rule given in the preceding article; this, however, is unnecessary, since the center of gravity can be obtained directly by referring to the tables of the properties of rolled sections. By referring to the table, "Properties of Angles with Equal Legs," the center of gravity of a $4'' \times 4'' \times \frac{13}{16}''$ angle is found to be 1.29 inches from the back of flange, and of a $4'' \times 4'' \times \frac{5}{16}''$ angle the distance is 1.12 inches; the center of

gravity of a $4'' \times 4'' \times \frac{1}{2}''$ angle is, therefore, nearly $1\frac{1}{4}$ inches from the back of a flange, thus giving us the distance $1\frac{1}{4} + \frac{3}{8} = 1\frac{5}{8}$ inches from the top edge of the flange plate to the axis through the centers of gravity of the angles. From the table, "Areas of Angles," it is also found that the area of the section of a $4'' \times 4'' \times \frac{1}{2}''$ angle is 3.75 square inches.

The area of the section of the flange plate is $\frac{3}{8} \times 12 = 4.5$ square inches, and of the web-plate, $\frac{3}{8} \times 16 = 6$ square inches; the area of the whole section is, therefore, $2 \times 3.75 + 4.5 + 6 = 18.0$ square inches.

The moments of the areas of the separate sections, with respect to the line $a\,b$, are as follows:

Flange plate	$4.5 \times \frac{3}{16} =$	.8 4
Two angles	$2 \times 3.75 \times 1\frac{5}{8} =$	1 2.1 9
Web-plate	$6 \times 8\frac{3}{8} =$	5 0.2 5
Total, ...		6 3.2 8

The distance c from the top edge of the flange plate to the neutral axis $d\,e$ of the section is, therefore, $63.28 \div 18.0 = 3.51$ inches.

5. Graphical Method of Locating the Neutral Axis. Let it be required to determine the position of the neutral axis of the cast-iron beam section shown in Fig. 4. First, divide the depth of the section into any number of parts—as has been done in this case by the dotted lines $w'\,x$, $y\,z$, etc.—whose areas and centers of gravity can readily be found. Then compute the area and locate the center of gravity of each part. In Fig. 4, the area of the top slice is 12 square inches, the area of each of the slices in the web of the section is 3 square inches, and the area of the bottom flange is 28 square inches. Assume some scale whose unit of length represents 1 square inch; for example, in this case, let $\frac{1}{16}$ of an inch represent 1 square inch. Then, commencing at some point a, lay off on the line $a\,l$, lengths $a\,b$, $b\,c$, $c\,f$, etc. which represent the respective areas of the successive parts into which the section of the beam has been divided, beginning at the top part. Thus, with the scale of $\frac{1}{16} = 1$

square inch, the line $a\,b$, which represents an area of 12 square inches, is $\frac{12}{16} = \frac{3}{4}$ inch long; each of the lines $b\,c$,

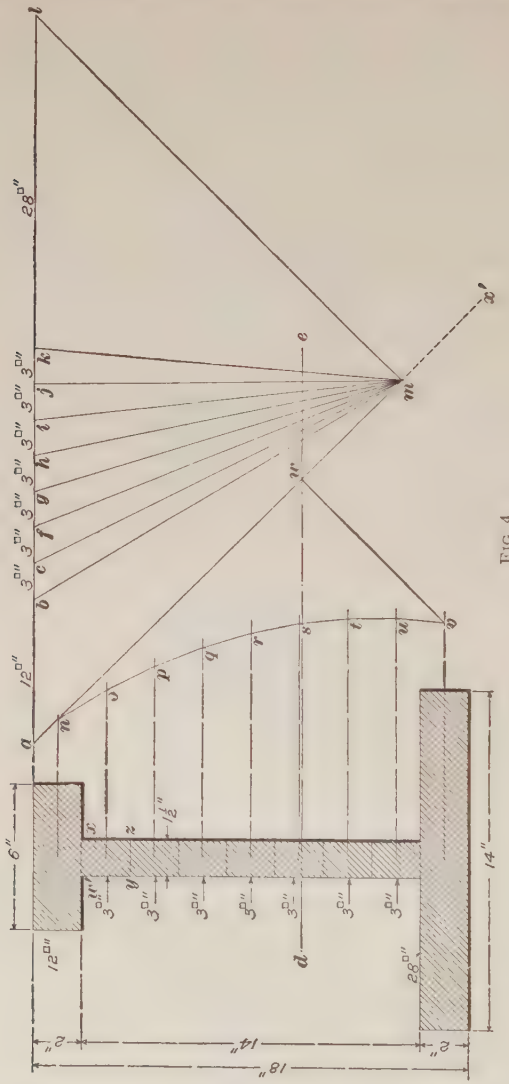


Fig. 4.

$c\,f$, etc. is $\frac{3}{16}$ inch long; and the line $k\,l$ is $\frac{28}{16} = 1\frac{3}{4}$ inches long.

From the points a and l, draw 45° lines, intersecting at the point m. Then from the points b, c, f, etc., draw the lines $b\,m$, $c\,m$, $f\,m$, etc. Through the center of gravity of each of the parts of the section, draw indefinite lines parallel to $a\,l$.

From the point n, where the line through the center of gravity of the top section intersects the line $a\,m$, draw the line $n\,o$ parallel to the line $b\,m$ until it intersects the line passing through the center of gravity of the second slice in the point o; draw the line $o\,p$ parallel to $c\,m$; $p\,q$ parallel to $f\,m$; $q\,r$ parallel to $g\,m$; $r\,s$ parallel to $h\,m$; $s\,t$ parallel to $i\,m$; $t\,u$ parallel to $j\,m$; and $u\,v$ parallel to $k\,m$.

From the point v, which is the point at the intersection of the line $u\,v$ with the line drawn through the center of gravity of the last elementary section, draw the line $v\,w$ parallel to $m\,l$; then its intersection w with the line $a\,m$ is a point on the required neutral axis. If the line $a\,m$ is so short that the line $v\,w$ fails to cut it, it may be extended indefinitely, as shown at $m\,x'$ so as to make it intersect with the line $v\,w$. Having found the point w, draw the horizontal line $d\,e$ through it. This line is the required neutral axis of the figure, and passes through its center of gravity.

This method of determining the position of the neutral axis and center of gravity may be applied to any irregular-shaped section whatever, and in many cases may be found more convenient than the mathematical method.

When, as in Fig. 4, the section is made up of several regular parts whose centers of gravity can be readily located, it is not necessary to subdivide any one of these parts. Thus, the center of gravity of the web of the beam is located on the horizontal line through r, and its area is represented by the distance $b\,k$. We can therefore draw from n a line parallel to $b\,m$ until it intersects the horizontal line through the center of gravity of the web member; then, from this point, draw a line parallel to $k\,m$ until it intersects the horizontal through the lower section at the point v. The point v, as thus located, is identical with the point previously found when the web section was divided into the small parts, and

the line drawn from v parallel to $l\,m$ until it intersects $a\,m$, locates the point w on the neutral axis, as before. If, however, the center of gravity of the web cannot be readily located, it is better to divide it into small parts, as in the first method.

THE MOMENT OF INERTIA.

6. The term **moment of inertia** is a mathematical expression which depends on the distribution of either the material of a body or the area of a surface with respect to a given axis. As applied to the area of a plane figure, the moment of inertia, with respect to an axis lying in the same plane, is numerically equal to the sum of the products obtained by multiplying each of the elementary areas of which the figure is composed by the square of its distance from the given axis.

By **elementary area** is meant an area smaller than any with which we are accustomed to deal in ordinary calculations; it is, therefore, impossible to find an exact expression for the moment of inertia of a figure by the methods of calculation in ordinary use. By means of the Calculus, however, exact formulas have been deduced, by means of which the moments of inertia of many of the simple geometrical forms, with respect to axes through their centers of gravity, have been found. There are also a number of approximate methods by means of which the moment of inertia of an irregular section, to which these formulas do not apply, may be found. Further, the moment of inertia of any section which can be divided into parts, the moments of each of which, with respect to an axis through its center of gravity, can be found, is easily calculated by means of a principle to be given below.

To illustrate the meaning of the term *moment of inertia*, together with a simple approximate method of computing the moment of inertia of a figure, consider the relation of the small **I** section shown in Fig. 5 to the axis $d\,e$ through its center of gravity. Divide the section into a number of

little squares (in this case, each with an area of 1 square inch) and consider the distance of each square from the axis

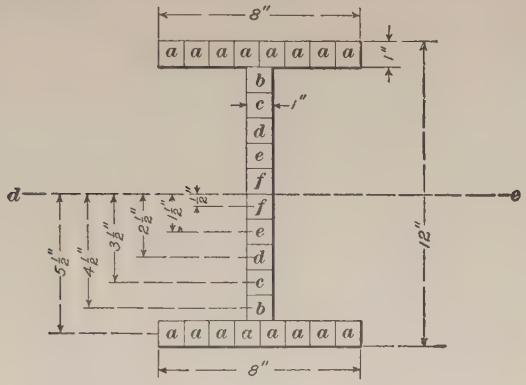


FIG. 5.

to be the distance from the axis to its center of gravity. Then the products of the area of each square, multiplied by the square of its distance from the axis, are as follows:

Squares a, $1 \times (5\frac{1}{2})^2 = \frac{121}{4}$
Squares b, $1 \times (4\frac{1}{2})^2 = \frac{81}{4}$
Squares c, $1 \times (3\frac{1}{2})^2 = \frac{49}{4}$
Squares d, $1 \times (2\frac{1}{2})^2 = \frac{25}{4}$
Squares e, $1 \times (1\frac{1}{2})^2 = \frac{9}{4}$
Squares f, $1 \times (\frac{1}{2})^2 = \frac{1}{4}$

Adding these products for all the squares, we have:

16 squares a, $\frac{121}{4} \times 16 = 484$
2 squares b, $\frac{81}{4} \times 2 = 40\frac{1}{2}$
2 squares c, $\frac{49}{4} \times 2 = 24\frac{1}{2}$
2 squares d, $\frac{25}{4} \times 2 = 12\frac{1}{2}$
2 squares e, $\frac{9}{4} \times 2 = 4\frac{1}{2}$
2 squares f, $\frac{1}{4} \times 2 = \frac{1}{2}$
Total, ... $566\frac{1}{2}$

which is the sum of the products of each of the small areas multiplied by the square of its distance from the axis. This result, however, is only a rough approximation to the moment of inertia, owing to the fact that the assumed areas

are so large. The actual value of the moment of inertia of the section, as will be shown below, is $568\frac{2}{3}$.

7. Rules and Formulas for Moments of Inertia.— In the table "Elements of Usual Sections" are given exact formulas for computing the moment of inertia of such regular figures as are most often met in the design of structures; it also gives approximate formulas for computing this factor for common rolled sections. The tables of properties of rolled sections published by the rolling mills give accurate values of the moment of inertia of all the principal sections used in the construction of buildings, so that it is not generally necessary to make the calculations for these sections; the approximate formulas in the table "Elements of Usual Sections" are, however, sometimes useful in making calculations when the tables published by the rolling mills are not at hand.

Rule.—*To find the moment of inertia, with respect to any axis of any figure whose moment of inertia with respect to a parallel axis through its center of gravity is known, add its moment of inertia with respect to the axis through its center of gravity to the product of its area multiplied by the square of the distance from its center of gravity to the required axis.*

This rule may be expressed by the formula

$$I' = I + a r^2, \qquad (1.)$$

in which I' = the required moment of inertia;
I = moment of inertia of the section, with respect to the axis through its center of gravity and parallel to the given axis;
a = the area of the figure;
r = the distance from its center of gravity to the required axis.

The moment of inertia, with respect to an axis through its center of gravity, of any section which can be divided into a number of parts, the moments of inertia of each of which, with respect to an axis through its center of gravity

parallel to the given axis, is known, is equal to the sum of the moments of inertia of its parts with respect to the given axis. Since the moment of inertia of any figure, with respect to any axis, is given by the formula $I' = I + a r^2$, if we denote the sum of the moments of the separate figures making up a section, with respect to an axis through the center of gravity of that section, by $\Sigma I'$ (in which the Greek letter Σ, read *sigma*, means *sum of*), we have

$$I_s = \Sigma I' = \Sigma(I + a r^2), \qquad (2.)$$

which is a general formula often used to denote the moment of inertia I_s of any built-up section.

8. Graphical Methods of Computing Moments of Inertia.—There are several graphical methods of computing the moment of inertia, one of which is an extension of the graphical method of locating the center of gravity and neutral axis, which was described in Art. **5**, and illustrated by Fig. 4. Thus, let it be required to determine the moment of inertia, with respect to the axis *d e* of the beam section shown in Fig. 4. Using the same scale as that to which the section was drawn, compute or measure the area of the figure enclosed by the lines *n o p q . . . . v w n*; multiply this area by the area of the section—shown graphically by the length of the line *a l*—and the product will be the moment of inertia of the section, with respect to the axis *d e* through its center of gravity. For example, suppose that the section shown in Fig. 4 has been drawn to a scale of $\frac{1}{4}$ inch = 1 inch. Using this scale, and computing the area of the figure enclosed by the lines *n o p q . . . . v w n*, we find it to be 43.36 square inches. The area of the section is 61 square inches; therefore, according to the rule, its moment, with respect to the axis *d e*, is $43.36 \times 61 = 2{,}644.96$.

For finding the moment of inertia, it is necessary to divide the section into a number of parts, for it is evident that the area of the figure enclosed by the lines *n r v w n* is considerably greater than that of the figure *n o p q . . . . v w n*, obtained by dividing the web into the small sections and drawing the lines of the diagram for each.

The area of the figure $n\,o\,p\,q\,.\,.\,.\,.\,v\,w\,n$ may be computed by extending the horizontal lines through $o\,p\,q$, etc., so as to divide it into a series of triangles and trapezoids. The dimensions of these can be readily measured, and their areas can be calculated by means of the principles of mensuration.

This method of computing the moment of inertia will be found convenient in the case of very irregular sections, to which the methods previously given can be applied only with considerable difficulty. The accuracy of the result will in general be greater when the section is drawn to a large scale and divided into a comparatively large number of parts.

EXAMPLE 1.—What is the moment of inertia of the section of a $10'' \times 16''$ beam about an axis through its center of gravity parallel to its shorter side?

SOLUTION.—From the table "Elements of Usual Sections," the formula for the moment of inertia of a solid rectangle is

$$I = \frac{b\,d^3}{12}.$$

Substituting the given dimensions, we have

$$I = \frac{10 \times 16^3}{12} = 3{,}413\tfrac{1}{3}. \quad \text{Ans.}$$

EXAMPLE 2.—Using the approximate formula given in the table "Elements of Usual Sections," compute the moment of inertia of a section of a 10-inch I beam, the area of the section being 10.29 square inches.

SOLUTION.—Substituting in the formula, we have

$$I = \frac{A\,h^2}{6.66} = \frac{10.29 \times 10^2}{6.66} = 154.5. \quad \text{Ans.}$$

EXAMPLE 3.—Compute the moment of inertia, with respect to the axis $d\,e$ through its center of gravity, of the section shown in Fig. 5.

SOLUTION.—This section is made up of 3 rectangles, the moments of inertia of which, with respect to the given axis, can be found by means of formula 1. The moment of inertia of one of the flanges, with respect to an axis through its center of gravity parallel to $d\,e$, is

$$I = \frac{8 \times 1^3}{12} = \tfrac{2}{3}.$$

The area of this figure is $8 \times 1 = 8$ square inches, and the distance of its center of gravity from $d\,e$ is $5\tfrac{1}{2}$ inches; therefore, its moment of

inertia, with respect to de, is $I' = \frac{2}{3} + 8 \times (5\frac{1}{2})^2 = 242\frac{2}{3}$. The axis through the center of gravity of the web section coincides with the axis de, hence the moment of inertia of this section, with respect to de, is

$$I' = \frac{1 \times 10^3}{12} = 83\frac{1}{3}.$$

Then, the moment of inertia I_s of the whole section $= \Sigma I' = 242\frac{2}{3} + 242\frac{2}{3} + 83\frac{1}{3} = 568\frac{2}{3}$. Ans.

Example 4.—What is the moment of inertia, with respect to the axis de, of the column section shown in Fig. 6?

Solution.—The moment of inertia of one of the flange plates, with respect to an axis through its center of gravity, parallel to the axis de, is

$$\frac{12 \times \frac{3}{8}^3}{12} = .05.$$

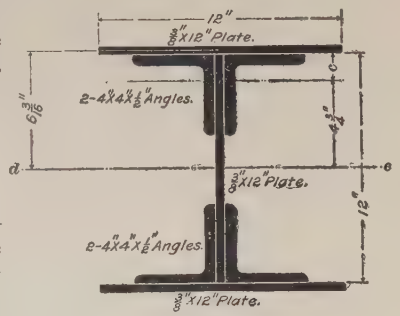


Fig. 6.

The area of the plate is $12 \times \frac{3}{8} = 4.5$ square inches, and the distance of its center of gravity from the axis is $6\frac{3}{16}$ inches. Therefore, the moment of inertia of the plate, with respect to the axis de, is

$$.05 + 4.5 \times (6\tfrac{3}{16})^2 = 172.33.$$

From the table "Areas of Angles," the area of a $4'' \times 4'' \times \frac{1}{2}''$ angle is 3.75 square inches, and the distance of its center of gravity from the back of a flange is approximately 1.25 inches. The distance of the center of gravity of the angle from the axis de, in accordance with the dimensions given in the figure, is $6 - 1\frac{1}{4} = 4\frac{3}{4}$ inches. In accordance with the approximate formula $\frac{A h^2}{10.2}$ given in the table "Elements of Usual Sections," for finding the moment of inertia of an angle with equal legs, the moment of inertia of the $4'' \times 4'' \times \frac{1}{2}''$ angle, with respect to the axis through its center of gravity, is

$$\frac{3.75 \times 4^2}{10.2} = 5.9, \text{ nearly.}$$

The moment of inertia of the angle, with respect to de, is, therefore, $5.9 + 3.75 \times (4\frac{3}{4})^2 = 90.5$. The center of gravity of the web-plate lies on the axis de; therefore, the moment of inertia of the plate, with respect to de, is

$$\frac{\frac{3}{8} \times 12^3}{12} = 54.$$

The moment of inertia of the whole section, with respect to de, is, therefore, $2 \times 172.33 + 4 \times 90.5 + 54 = 760.66$. Ans.

EXAMPLE 5.—What is the moment of inertia, with respect to the axis de, of the column section shown in Fig. 7?

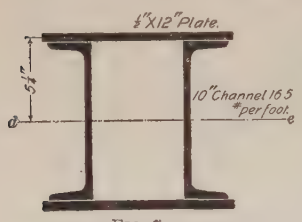


FIG. 7.

SOLUTION.—The moment of inertia of one of the cover-plates, with respect to an axis through its center of gravity, parallel to de, is

$$\frac{12 \times \frac{1}{2}^3}{12} = .125.$$

The area of the plate is $12 \times \frac{1}{2} = 6$ square inches, and the distance of its center of gravity from de is $5\frac{1}{4}$ inches, therefore, its moment of inertia, with respect to de, is $I' = .125 + 6 \times (5\frac{1}{4})^2 = 165.5$. From Table 12, Art. **107**, *Architectural Engineering*, § 5, the area of a 10-inch 16½-pound channel is 4.84 square inches, and its moment of inertia, with respect to an axis through its center of gravity, corresponding in this case with the axis de, is 71.09; therefore, the moment of inertia of the *whole* section, with respect to de, is $2 \times 165.5 + 2 \times 71.09 = 473.18$. Ans.

RESISTING MOMENT.

9. In Art. **99**, *Architectural Engineering*, § 5, it was stated that the moment of resistance of a section is equal to the product of the greatest unit stress in any part of the section multiplied by a factor called the *section modulus*. In calculating the strength of beams, it is, therefore, important to be able to determine the value of the section modulus; this can be readily done for any section whose moment of inertia is known by means of the formula

$$K = \frac{I}{c}, \qquad (3.)$$

where K = the section modulus;

I = the moment of inertia with respect to the neutral axis;

and c = the distance from the neutral axis to the farthest edge of the section.

10. Relation Between Bending Moment and Moment of Resistance.—In order that the forces acting on a beam may be in equilibrium, we know that the moment of the external forces, with respect to a given axis, must be equal to the moments of the stresses with respect to the same axis.

The axis about which these moments are assumed to act is the neutral axis of a section, and the relation is expressed by saying *that the resisting moment must equal the bending moment when the beam is in equilibrium.*

Letting S denote the greatest stress in any fiber of a beam subjected to a bending stress, and M, the bending moment in inch-pounds, the relation between the bending moment and the resisting moment is

$$M = SK = S\frac{I}{c}. \qquad (4.)$$

Values of the section modulus K for the various simple sections most often used for beams are given in column 2 of the table "Elements of Usual Sections." These values are obtained by dividing the values of I given in column 1 by the distance c from the neutral axis to the most distant fiber of the section.

RADIUS OF GYRATION.

11. In computing the strength of columns, frequent use is made of a property of a section which is numerically equal to the square root of the quotient of its moment of inertia, with respect to an axis through its center of gravity, divided by its area. This property is called the **radius of gyration** of the section, with respect to the given axis. It is usually expressed by the letter R, and its value, with respect to a given axis, for any section whose area A and moment of inertia I, with respect to the same axis, are known, is given by the formula

$$R = \sqrt{\frac{I}{A}}. \qquad (5.)$$

The form in which the radius of gyration appears in most formulas for calculating the strength of columns is its square; hence, it is convenient to express the above relation by the formula

$$R^2 = \frac{I}{A}, \qquad (6.)$$

which gives directly the value to be substituted in the column formulas. Formulas for computing the least radius of gyration and its square for the sections most often used

in the design of structures, are given in columns 4 and 5 of the table "Elements of Usual Sections." The tables of the properties of rolled sections also give accurate values of R and R^2 for the sections used in the examples given in the following pages.

EXAMPLE.—Compute the square of the radius of gyration, with respect to the axis de, of the column section shown in Fig. 6.

SOLUTION.—By referring to example 4, Art. 8, we find the moment of inertia of the section to be $I = 760.66$, and the area of the section is $3 \times 4.5 + 4 \times 3.75 = 28.50$ square inches. Substituting these values in formula 6, we have

$$R^2 = \frac{760.66}{28.50} = 26.69. \quad \text{Ans.}$$

STEEL COLUMNS.

TYPES OF COLUMNS.

12. Columns of rolled-steel shapes riveted together are now largely used in the construction of buildings, and, especially in tall building construction, are rapidly superseding cast-iron columns. On account of their unreliability under sudden stress, cast-iron columns are no longer used in bridge work. In buildings, although the loads are generally quiescent, and the liability to sudden shock is more remote, the columns seldom receive their loads as favorably as in bridges, because in most cases they are subjected to considerable eccentricity in loading; that is, the loads upon one side of a column are heavier than those on the other, thus tending to produce bending stress and materially decreasing its ultimate strength to direct compression.

It may readily be seen that the placing of columns centrally, one over another, necessitates the application of loads to their sides; and, unless the loads are equal and on opposite sides, the effect is to increase the stress upon the side where the greatest load occurs.

The method of securing the ends of columns, or struts, also exercises an important influence on their resistance to bending, and, consequently, on their ability to resist compression

stresses. Columns may therefore be classified, according to the arrangement of their ends, into the four following forms: (*a*) columns with *round* ends; (*b*) columns with *hinged* ends; (*c*) columns with *flat* ends; and (*d*) columns with *fixed* ends. Fig. 8 shows the typical form of each of these classes.

Round-ended columns are struts which have only central points or lines of contact, such as balls or pins resting upon flat plates. The centers of the balls or pins should lie in a line through the center of gravity of a section of the strut.

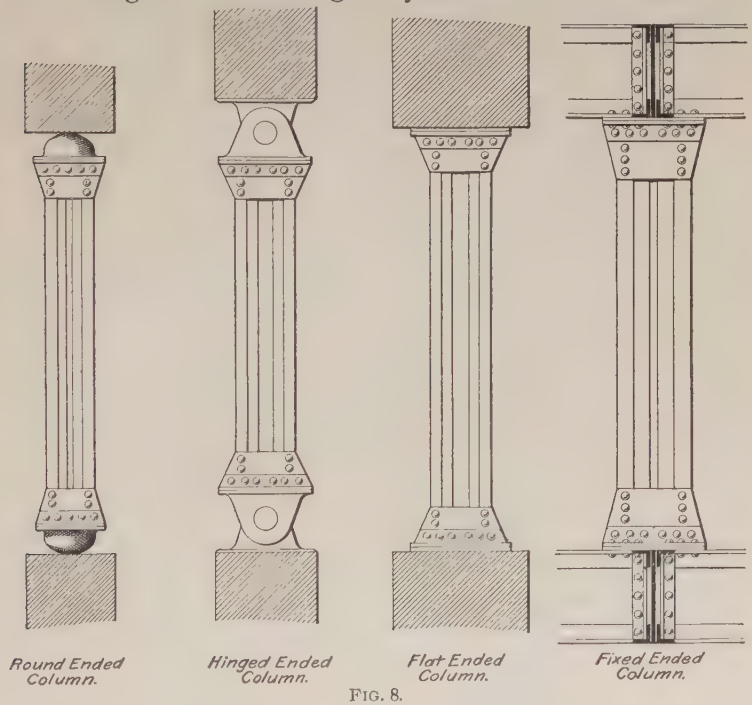


FIG. 8.

Columns with **hinged ends** are struts which have both ends properly fitted with either pins or ball-and-socket joints; the center of the end joints should lie on the central axis of the column.

Flat-ended columns are struts which have flat ends normal to the central axis of the strut, but not rigidly secured to adjoining parts of the structure.

Columns with **fixed ends** are rigidly secured at both ends to the contiguous parts of the structure in such a manner, that the attachment would not be severed if the member was subjected to the ultimate load.

The round-ended column is seldom used, and in this course will be disregarded entirely.

The column with hinged ends is mostly used in pin-connected trusses, that is, trusses in which the several members at a joint are connected by means of a pin or single bolt so

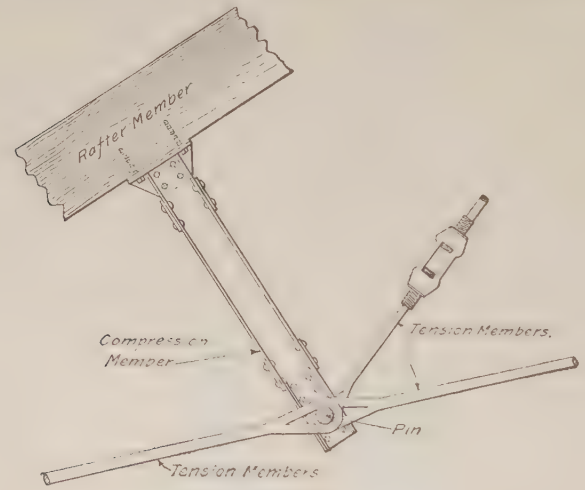


FIG. 9.

as to be free to turn, as on a hinge, instead of being firmly riveted, bolted, or spiked together. Fig. 9 shows one of the lower joints of a pin-connected roof truss.

FORMULAS FOR STEEL COLUMNS.

13. Columns With Hinged Ends.—The strength of wrought-iron and steel columns with hinged ends, may be found from the formula

$$S = \frac{U}{1 + \frac{L^2}{18{,}000\ R^2}}, \qquad (7.)$$

in which S = the ultimate breaking strength of the column in pounds per square inch of section;
U = the ultimate compressive strength of the material in pounds per square inch;
L = the length of the columns in inches;
R = the least radius of gyration of the section.

This formula may be expressed as a rule as follows:

Rule.—*To find the ultimate strength per square inch of sectional area of a wrought-iron or steel column with hinged ends, divide the ultimate compressive strength per square inch of the material composing the column by 1 plus the quotient obtained by dividing the square of the length of the column, in inches, by 18,000 times the square of the least radius of gyration of the section of the column.*

The *safe bearing strength* of the column is obtained by multiplying its sectional area by the ultimate strength per square inch, as determined by formula **7**, and dividing this product by the factor of safety demanded by the given conditions.

EXAMPLE.—Using a factor of safety of 4, compute the safe bearing strength of a strut of a pin-connected truss; the length of the strut being 20 feet from center to center of pins, and its section made up of four $6'' \times 6'' \times \frac{3}{8}''$ angles connected as shown in Fig. 10.

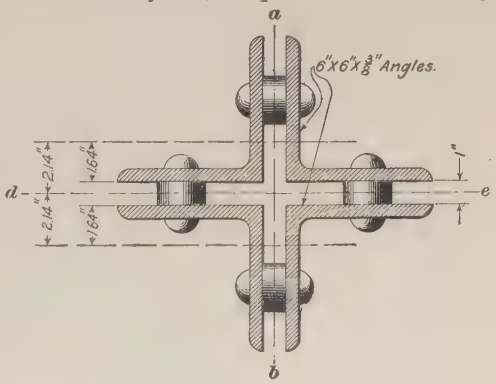


FIG. 10.

SOLUTION.—Since the relation of the section to each of the axes $a\,b$ and $d\,e$ is the same, its moment of inertia and radius of gyration is the same about either axis, and it is therefore necessary to compute these factors for only one of these axes. From the table "Areas of Angles," the area of the section of a $6'' \times 6'' \times \frac{3}{8}''$ angle is 4.36 square inches, and from the table "Properties of Angles," the distance of its center of gravity

from the back of either leg is 1.64 inches, nearly. By the formula given in the table "Elements of Usual Sections," we find the moment of inertia of one of the angles, with respect to an axis through its center of gravity parallel with the given axis, to be

$$I = \frac{A h^2}{10.2} = \frac{4.36 \times 6^2}{10.2} = 15.39.$$

From Fig. 10 the distance of the axis through the center of gravity of the angle from the axis $d e$ of the section is 2.14 inches; therefore, substituting the known values in formula **1**, the moment of inertia of a single angle, with respect to the axis $d e$, is

$$I' = I + a r^2 = 15.39 + 4.36 \times 2.14^2 = 35.35.$$

The moment of inertia of the whole section is, therefore,

$$I_s = \Sigma I' = 4 \times 35.35 = 141.40.$$

The total area of the section is $4.36 \times 4 = 17.44$ square inches; therefore, the square of its radius of gyration, from formula **6**, is

$$R^2 = \frac{I}{A} = \frac{141.4}{17.44} = 8.1, \text{ nearly.}$$

In Table 6, Art. **61**, *Architectural Engineering*, § 5, the ultimate compressive strength U of structural steel is given as 52,000 pounds per square inch. In accordance with the statement of the problem the length of the strut is $L = 20$ feet $= 240$ inches. Substituting in formula **7**, we find the ultimate strength, per square inch of section of the strut, to be

$$S = \frac{52{,}000}{1 + \frac{240^2}{18{,}000 \times 8.1}} = 37{,}270 \text{ pounds.}$$

The safe bearing strength of the strut is, therefore,

$$\frac{37{,}270 \times 17.44}{4} = 162{,}500 \text{ pounds, nearly. Ans.}$$

14. Columns With Flat or Square Ends.—The columns used for supporting the tiers of floorbeams in an office building may properly be considered as columns with flat or square ends, and their strength may be calculated by the following formula for flat or square ended columns:

$$S = \frac{U}{1 + \frac{L^2}{24{,}000\, R^2}}, \qquad (8.)$$

in which the symbols S, U, L, and R have the same meaning as in formula **7**.

EXAMPLE.—The moment of inertia, with respect to the axis $Y Y$ of the steel **Z**-bar column shown in Fig. 11, is 287.92, and, with respect to

the axis XX, 337.17. The total area of the section is 21.36 square inches. What is the safe bearing strength with flat ends, if the length of the column is 20 feet and a factor of safety of 3 is used?

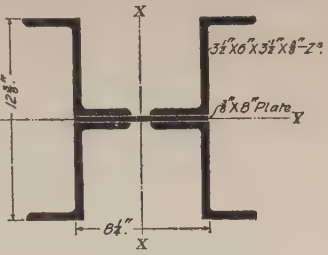


FIG. 11.

SOLUTION.—Using the least moment of inertia, which is that with respect to the axis YY, the square of the least radius of gyration is found to be

$$R^2 = \frac{287.92}{21.36} = 13.48, \text{ nearly,}$$

therefore, the ultimate strength per square inch of section is

$$S = \frac{52{,}000}{1 + \frac{240^2}{24{,}000 \times 13.48}} = 44{,}100 \text{ pounds.}$$

The safe bearing strength is, therefore,

$$\frac{44{,}100 \times 21.36}{3} = 314{,}000 \text{ pounds, nearly.} \quad \text{Ans.}$$

15. Columns With Fixed Ends.—Letting S, U, L, and R represent the same quantities as in the two preceding formulas, the strength of columns with fixed ends may be calculated by the formula

$$S = \frac{U}{1 + \frac{L^2}{36{,}000\, R^2}}. \qquad \textbf{(9.)}$$

EXAMPLE.—A section of the compression member of a large structural-steel roof truss is shown in Fig. 12. The ends of the member are firmly riveted to the adjacent members of the truss, and the length of the member is 15 feet; what is its safe bearing strength with a factor of safety of 4?

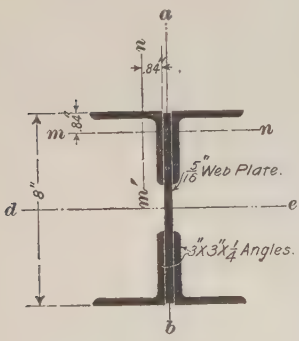


FIG. 12.

SOLUTION.—From the table "Areas of Angles," the area of a $3'' \times 3'' \times \frac{1}{4}''$ angle is 1.44 square inches, and from the table "Properties of Angles," the distance of its center of gravity from the back of a flange is .84 inch; also from the latter table, the moment of inertia of the angle, with respect to an axis through its center of gravity, is found to be 1.24. In accordance with the dimensions

given in the figure, the distance from the axis $d\,e$ to the centers of gravity of the angles is $4 - .84 = 3.16$ inches. The moment of inertia of one of the angles, with respect to the axis $d\,e$, is, therefore,

$$I' = 1.24 + 1.44 \times 3.16^2 = 15.62.$$

The moment of inertia of the web-plate, with respect to $d\,e$, is

$$I' = \frac{\frac{5}{16} \times 8^3}{12} = 13.33.$$

The moment of inertia of the whole section, with respect to $d\,e$, is

$$I_s = 4 \times 15.62 + 13.33 = 75.81.$$

The total area of the section is $4 \times 1.44 + 8 \times \frac{5}{16} = 8.26$ square inches; the square of its radius of gyration, with respect to $d\,e$, is, therefore,

$$R^2 = \frac{75.81}{8.26} = 9.18.$$

Referring now to the axis $a\,b$, the distance from the axis to the center of gravity of one of the angles is $.84 + \frac{5}{32} = .84 + .156 = .996$, say 1 inch.

The moment of inertia of the angle, with respect to $a\,b$, is, therefore,

$$I' = 1.24 + 1.44 \times 1^2 = 2.68.$$

The moment of inertia of the web-plate, with respect to $a\,b$, is

$$\frac{8 \times \frac{5}{16}^3}{12} = .02.$$

Taking the sum of the moments of inertia of the several parts, the moment of inertia of the whole section, with respect to $a\,b$, is found to be $4 \times 2.68 + .02 = 10.74$; dividing this by the area of the section, the square of the radius of gyration, with respect to this axis, is

$$R^2 = \frac{10.74}{8.26} = 1.3, \text{ nearly,}$$

which, being much less than the value of R^2 when referred to the axis $d\,e$, is the value to use in computing the strength of the member.

Substituting now in formula **9**, the ultimate strength per square inch of the section is

$$S = \frac{52{,}000}{1 + \frac{180^2}{36{,}000 \times 1.3}} = 30{,}700 \text{ pounds,}$$

and the safe bearing strength of the member is

$$\frac{30{,}700 \times 8.26}{4} = 63{,}400 \text{ pounds.} \quad \text{Ans.}$$

16. Columns With Eccentric Loads.—Columns designed by the foregoing formulas are reasonably safe

against crushing, bending, or buckling unless subjected to considerable bending stress, due to eccentric loading.

In modern buildings nearly all columns are subjected to more or less eccentric loading. In fact, it would be a difficult matter to find a column loaded with a perfectly concentric load. It is, however, good practice to disregard the eccentric loading generally encountered in ordinary building construction, since the preceding formulas make due allowance for the slight eccentricity caused by the lack of symmetry in the arrangement of the brackets and the application of the loads with regard to the central axis of the column. For example, the beam connections may all be upon one side of the column, and of the form shown in Fig. 13, or the beams attached to one side of the column may be more heavily loaded than those attached to the other side. Some of these conditions are usually unavoidable and tend to produce the undesirable effect of eccentric loading.

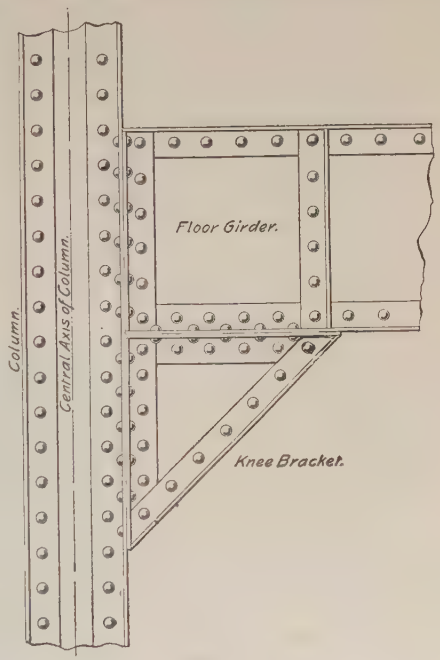


FIG. 13.

Should the eccentricity of the load be considerable, and liable to produce dangerous transverse or bending stresses, it would materially decrease the ability of the column to withstand direct compressive stresses. The bending or transverse stresses should in such a case be calculated, and an additional amount of material should be added to the section of the column in order to resist them.

A column which is a fair example of eccentric loading is shown in Fig. 14. The bending moment or transverse stress is equal to the product of the load by the lever arm through which it acts, in this case 2 feet. Hence, the bending moment upon the column, due to the eccentric load, is $10,000 \times 2 = 20,000$ foot-pounds, or 240,000 inch-pounds. The problem now is to determine the amount of material required and its disposition in order to provide sufficient resistance to this transverse or bending stress; its solution, however, depends on the principles involved in the design of beams and girders, principles

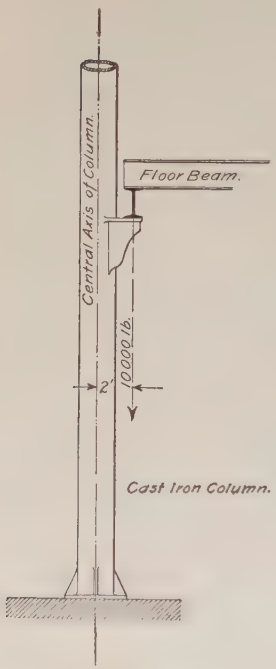


FIG. 14.

that will be more fully discussed in connection with the following articles.

FIG. 15.

The failure of a structural-steel column may occur by reason of the buckling between the rivets of the plates or structural shapes, of which it is constructed; the homogeneousness of the section will thus be affected, as may be seen by reference to Fig. 15.

17. Factors of Safety.—The ability of a column to resist the transverse or bending stresses due to eccentric loading decreases as its length increases, and this ability is less for columns with round or hinged ends than for those with flat or fixed ends. For these reasons it is good practice to assume a minimum factor of safety for the shortest columns or struts and increase the factor with the increase in length, making the rate of increase greater for round or hinged than for flat or fixed ends.

Assuming a minimum factor of safety of 3 for very short struts, the factors prescribed by good practice for longer struts with flat or fixed ends are given by the formula

$$F = 3 + .01 \frac{l}{R}; \qquad \textbf{(10.)}$$

and for struts with round or hinged ends,

$$F = 3 + .015 \frac{l}{R}. \qquad \textbf{(11.)}$$

In both of these formulas F is the required factor of safety; l, the length of the strut in inches; and R, the least radius of gyration of the section.

EXAMPLE.—What is the least factor of safety that should be used with (*a*) a column with flat ends, and (*b*) a column with hinged ends, the length of the column being 20 feet, and its least radius of gyration 2.5?

SOLUTION.—(*a*) Applying formula **10**, we have

$$F = 3 + .01 \times \frac{240}{2.5} = 3.96, \text{ say } 4. \quad \text{Ans.}$$

(*b*) Applying formula **11**,

$$F = 3 + .015 \times \frac{240}{2.5} = 4.44. \quad \text{Ans.}$$

Table 1 gives values of the factor of safety obtained by substituting different values of $\frac{l}{R}$ in formulas **10** and **11.**

TABLE 1.

FACTORS OF SAFETY.

$\frac{l}{R}$	Fixed and Flat Ends.	Hinged and Round Ends.	$\frac{l}{R}$	Fixed and Flat Ends.	Hinged and Round Ends.	$\frac{l}{R}$	Fixed and Flat Ends.	Hinged and Round Ends.
20	3.2	3.30	110	4.1	4.65	200	5.0	6.00
30	3.3	3.45	120	4.2	4.80	210	5.1	6.15
40	3.4	3.60	130	4.3	4.95	220	5.2	6.30
50	3.5	3.75	140	4.4	5.10	230	5.3	6.45
60	3.6	3.90	150	4.5	5.25	240	5.4	6.60
70	3.7	4.05	160	4.6	5.40	250	5.5	6.75
80	3.8	4.20	170	4.7	5.55	260	5.6	6.90
90	3.9	4.35	180	4.8	5.70	270	5.7	7.05
100	4.0	4.50	190	4.9	5.85	280	5.8	7.20

The table gives values of the factor of safety for columns whose length is as great as 280 times their least radius of gyration; it is not, however, good practice to use a column in which the value of $\frac{l}{R}$ is greater than 150. Another condition to be observed is that the length of the column should not exceed 45 times its least dimension.

FORMS OF STEEL COLUMNS.

18. Conditions Which Affect the Choice of a Type of Column.—There are at present numerous forms in which rolled-steel shapes are combined to make up columns for structural purposes. The type of column to be used in a building is sometimes prescribed by the owner, or the design of the building is furnished by the architect, thus leaving the engineer little latitude in his choice of design. There are, however, a number of conditions demanded by considerations, partly practical and partly theoretical, which should be carefully studied and compared before selecting

the type of column for a particular purpose. In many cases it will be found that these considerations impose conditions that conflict with each other; and in order to make such a compromise as will meet this difficulty in the most satisfactory manner, there is demanded of the engineer a most careful exercise of judgment guided by practical experience. Some of the most important points to be considered in choosing the type of column to be used, in any case, are the following:

1. The cost and availability of the material.

2. The amount of labor required and the facility with which it may be performed in both shop and field.

3. The distribution of material in the column so as to give the maximum strength with the least weight.

4. The facility with which connections may be made between the column and the members which it supports.

5. The application of the connections in such a manner that they will transfer the compressive stresses directly to the axis of the column.

6. The facility with which the thickness of the metal in the different parts composing the column can be reduced in order to meet the reduced loads of the upper floors. It is not desirable to make the columns supporting the upper floors of the building very small, since the beams and girders supporting the upper floors are usually of the same dimensions as those for the lower floors, and consequently require connections as heavy and secure; it is almost impossible to make such connections to small columns, consequently, in order to reduce the weight of the column for the lighter load which it will carry, it is better to reduce the thickness of the material used and keep the section the same.

7. The facility with which fireproofing may be attached to the section. Columns of circular sections may be fireproofed more compactly than rectangular columns. Architects in some instances have utilized the space lost by the use of rectangular columns for the purpose of conduits in which to run pipe lines, etc. In one instance, for example,

circular holes were cut in the bed and cap plates between the columns of several stories, to allow for the passage of pipes inside of the column. Such practice cannot be condemned too severely. It is bad practice to run pipe lines or electric wires inside of, or alongside of, the fireproofing of columns; separate conduits, provided with suitable covers, which will allow of inspection, should be provided in the walls.

In regard to the cost and availability of the material, such shapes should be selected as are easily rolled and placed on the market at a reasonable price. **I** beams, channels, angles, and **Z** bars, together with plates, are the most common commercial sections; these are manufactured at nearly every structural-steel mill, and may be obtained promptly on large orders at any locality where a skeleton-constructed building is likely to be erected. Patented sections do not fulfil this condition, and should therefore be avoided in most cases. The consideration of prompt delivery is an important one, and greatly influences the cost and facility with which the modern building is erected.

The consideration of the facility with which the labor in both shop and field can be performed is one that should receive careful attention. In the shop the complexity of the column section, and the number of pieces of which it is composed, greatly influence the element of labor. If there are numerous small pieces, such as brackets or splice plates, each of which requires cutting, bending, and fitting together, with frequent handling, the cost of labor may be proportionately very great. The number of rivets also greatly influences the cost of a column; not only should there be as few rivets as is consistent with strength, but the construction should permit the rivets to be readily driven by machine, so as to avoid hand riveting, which is slow and expensive and now generally admitted to be inferior to machine work.

The same general remarks apply to the labor in the field; the connections should be as simple as possible, and with as few rivets as is consistent with the strength required; they should be easy of access, so that the work upon them may be executed conveniently and rapidly.

In connection with the question of the distribution of the material in the column so as to develop the maximum amount of strength with the least weight of material, it is necessary to consider the facility with which economical connections may be made between the columns and floor-beams, and the directness with which the connections transfer the stresses to the central axis of the column. The relation between these conditions may be illustrated and analyzed by comparing the sections shown in Fig. 16, where

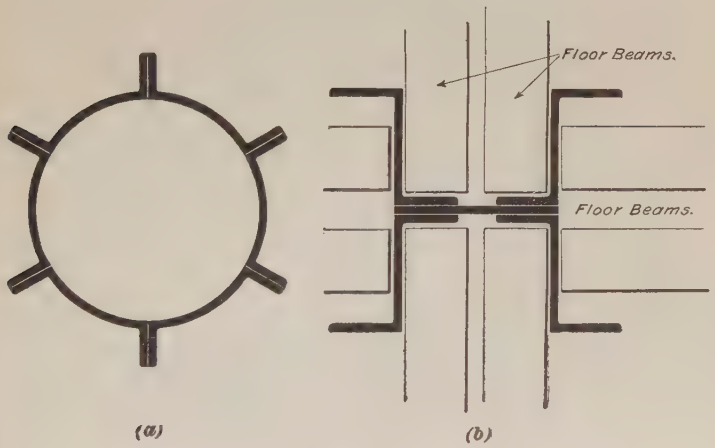


FIG. 16.

(*a*) represents a section of a Phœnix column, while (*b*) is a section of a **Z**-bar column. The apparent advantage of the Phœnix column is that the material composing it is placed where it will be the most efficient, thus fulfilling the third condition of the above list in a satisfactory manner; the radius of gyration of the column is also practically the same upon any of its diameters, and the metal is placed at the greatest possible distance from its center.

On the other hand, by referring to the section of the **Z**-bar column, it is seen that a considerable portion of the material is concentrated on the axis, a condition that gives it a relatively small radius of gyration and demands the use of a greater weight of material for a given load. This column,

however, offers greater advantages for the proper connection of the floorbeams than does the other, and, owing to its open construction, the beams transmit their loads almost directly to the central axis of the column, thus avoiding the disadvantages of eccentric loading.

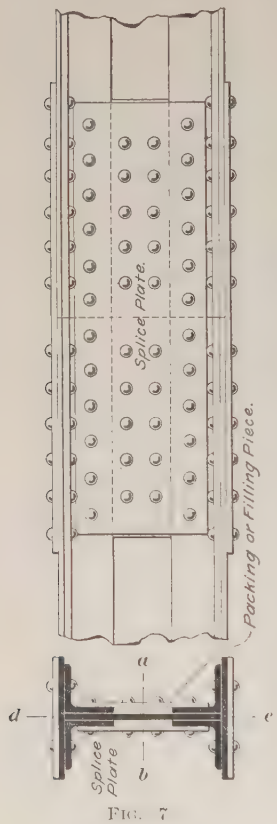


FIG. 7

Thus it is seen that the section having the best theoretical distribution of material is not always the best to use, on account of these several practical considerations; in fact, the section shown in Fig. 17 is probably as much used for structural-steel columns as any other, although it is not an economical section, for the reason that its radius of gyration on the axis *de* is very small in proportion to the weight of material used, and, since the least radius of gyration is always used in calculating the strength of a structural column, all the material which goes to form the greater radius of gyration adds nothing to the theoretical strength.

The great advantage of this section is that it is composed of the cheapest rolled sections, which are put together with a minimum of labor. It is also one of the best forms for attaching the beams and girders, and, as will be seen by a study of the details of the splice shown in the figure, for making connections between two adjacent columns.

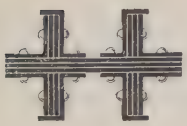
19. Sections of Columns Frequently Used in Skeleton Building Construction.—The following list shows the general forms of the principal types of steel-column sections now in use:

Larimer column, 1 row of rivets (patented).

Z-bar column without cover-plates, 2 rows of rivets.

Z-bar column with one cover-plate, 6 rows of rivets.

Z-bar column with two cover-plates, 6 rows of rivets.



Z-bar column, additional section obtained by the use of angles and plates, 8 rows of rivets.

Z-bar column, rectangular section, 6 rows of rivets.

Channel column with plates or lattice, 4 rows of rivets.

Box column of plates and angles, 8 rows of rivets.

Latticed angle column, 8 rows of rivets.

A column much used by the Pennsylvania Railroad, composed of angles and plates, 10 rows of rivets.

Keystone octagonal column, 4 rows of rivets (patented).

Four-section Phœnix column, 4 rows of rivets (patent expired).

Eight-section Phœnix column, 8 rows of rivets (patent expired).

Grey column, 4 rows of rivets (patented).

DETAILING OF STRUCTURAL COLUMNS AND CONNECTIONS.

20. Rivets and Rivet Spacing.—Before discussing the design of structural-steel columns and their connections, we will consider the best practice in proportioning and spacing the rivets which connect the several rolled sections of which the completed column is composed.

Where brackets are riveted to the column to support floor-beams, a sufficient number of rivets should be used to take the entire shear due to the reaction at the end of the beam; also where a plate girder, which forms the principal member of a floor system, is riveted directly to the column, sufficient rivets should be provided so that their combined shearing strength is equal to the end reaction on the girder. If, as shown in Fig. 13, there is a bracket or knee brace in connection with the girder, the calculation may be based on the combined shearing strength of the rivets in both bracket and girder.

EXAMPLE.—If the reaction at the end of a plate girder is 60,000 pounds, and $\frac{7}{8}$-inch rivets, each with an allowable shearing strength of 6,500 pounds, are used, how many rivets will be required to support the end of the girder?

SOLUTION.— 60,000 lb. ÷ 6,500 lb. = 9.2, say 10 rivets. Ans.

It is usual to space the rivets closer at the joints and foot of a column than in the body; for example, where $\frac{3}{4}$-inch rivets are used, it is customary to space them on 3-inch centers at the joints and bottom and from $4\frac{1}{2}$ to 6 inch centers throughout the remainder of the column, and where $\frac{7}{8}$-inch rivets are used, they should be spaced about 4 inches at the joints and about 6 inches throughout the length of the column.

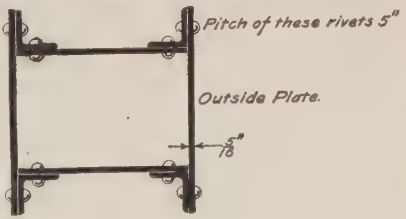


FIG. 18.

In a compression member the pitch of the rivets in inches should never exceed the thickness in 16ths of an inch of the thinnest outside plate. For example, in Fig. 18, where the outside

plate is $\frac{5}{16}$ inch thick, the greatest allowable pitch of the rivets would be 5 inches; under no conditions, however, should the pitch of the rivets in a compression member be greater than 6 inches, center to center, except where the rivets are staggered, in which case the pitch should not be more than 6 inches in a staggered line.

The diameter of rivets to be used in built-up columns, is determined by the thickness of the metal in the parts to be joined. Where the aggregate thickness of the metal between the heads of the rivet is not more than $1\frac{1}{4}$ inches, a $\frac{3}{4}$-inch rivet may be used; if the aggregate thickness of the several plates is more than $1\frac{1}{4}$ inches and less than 3 inches, it would be advisable to use $\frac{7}{8}$-inch rivets; and if the aggregate thickness of the metal is 3 inches or more, 1-inch rivets should be used.

As far as practicable the rivets should be of the same size throughout the column, as this saves annoyance in both shop and field. The size of the rivets to be used in the column splices, girder and beam connections, knee braces, and brackets is determined by the size of the rivets in the column.

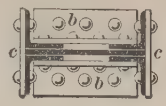
In conjunction with the above rules the student should exercise his judgment in the matter of details, and should study the requirements of each condition to be fulfilled.

21. Column Splices and Connections.—The excellence of the detail design of structural-steel columns is largely governed by the experience and judgment of the designer, and the care with which he studies the local conditions which will be met with in nearly every new piece of work.

The several points of excellence to be attained in the design of structural columns have already been discussed, and it now remains for the student to consider a few first-class details of column splices and connections.

As the strength of a building depends almost entirely upon the strength of the connections to the columns, great care should be taken in their design. Rigid column

connections for the floorbeams and girders at the several floors, and efficient splice connections between the sections of the columns on the different floors are of the utmost importance, and should be given the most careful attention. The ideal system of column construction would be to make the various columns on the several floors of one continuous set of sections running from foundation to roof; it is evident, however, that this is impossible in high modern building construction, therefore the next best thing to do is to make the spliced connections as rigid as possible.



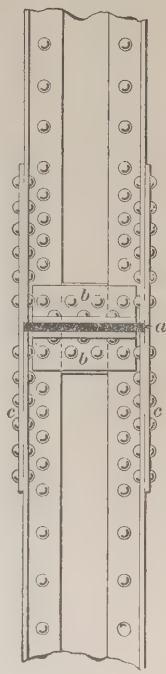
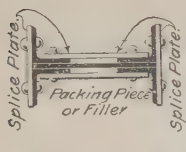


FIG. 19.



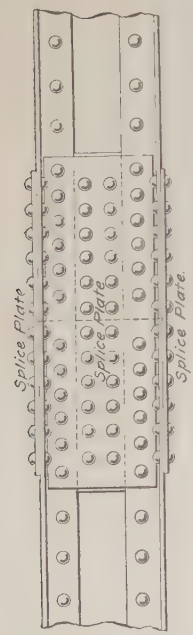


FIG. 20.

Columns may be spliced as shown in Fig. 19. The bedplate *a* separates the two columns, and through it, by means of the angles *b*, *b*, the columns are riveted securely together; they are also additionally secured by means of the two splice plates *c*, *c*.

The abutting ends of structural columns should be milled or planed so as to secure a square and firm bearing and obviate the danger of their being thrown out of line when erected.

Another very good method of connecting two columns is by the use of splice plates on all sides, thus doing away with the bedplates or packing pieces between the ends; this construction is shown in Fig. 20. The bedplate is sometimes extended beyond the column and forms a rest or support for the floorbeams or girders. If

this is done, it is advisable to further support the bedplate by a bracket directly under the bearing of the beams, as shown in Fig. 21.

If, in the same floor, the beams or girders are of different

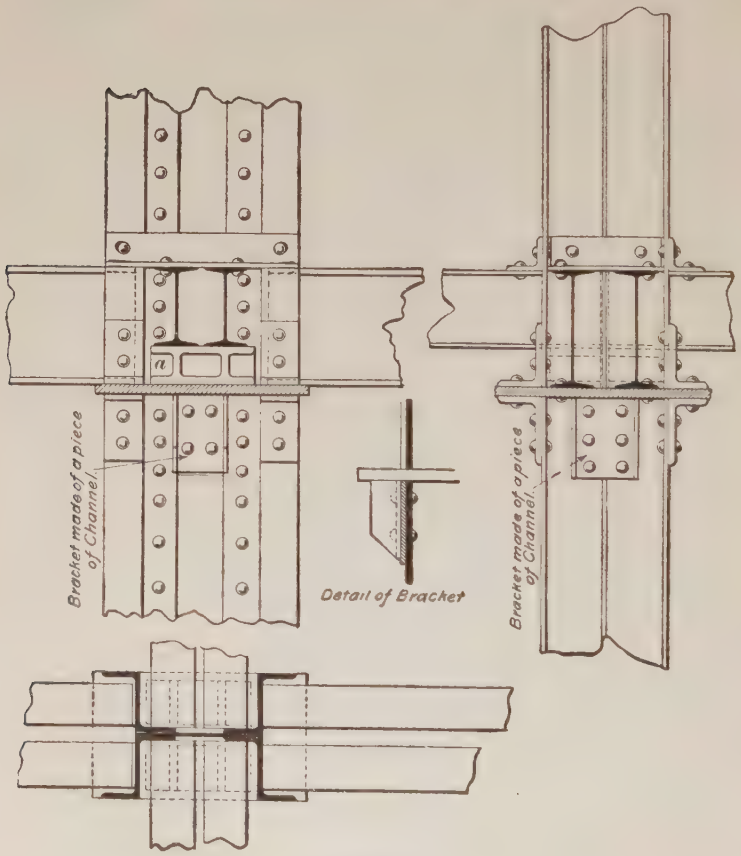


FIG. 21.

depths and are at the same level with regard to the top flange, the shallow beams may be supported by introducing cast-iron blocks beneath them, as shown at *a*, Fig. 21.

Where the floor girders are of the built-up plate-girder type, there is no difficulty in securing a rigid connection

to the column. The connection to be recommended where the architectural features of the interior arrangement of the building do not interfere is shown in Fig. 22. This connection is particularly efficient when the building is high and narrow, and in danger of being acted on by heavy wind pressure.

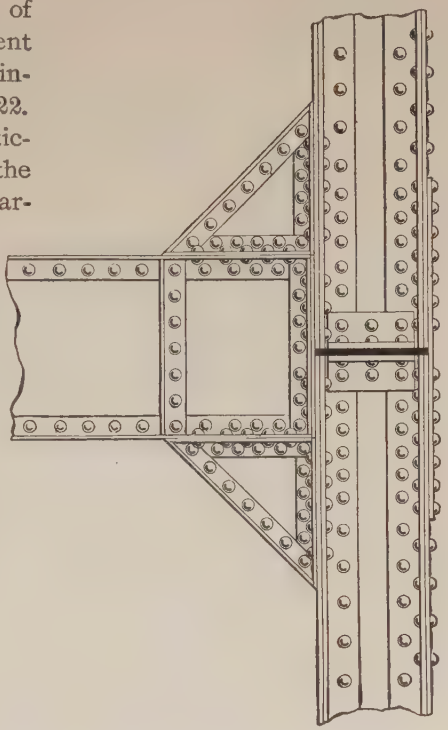


FIG. 22.

Other forms of connections, where plate girders are the principal supporting members of a floor system, are shown in Figs. 23 and 24. In Fig. 23, it will be seen that a filler is introduced at *c* because the upper column is smaller than the lower and it becomes necessary to pack between the splice plates. It is, however, preferable to pack equally on both sides of the

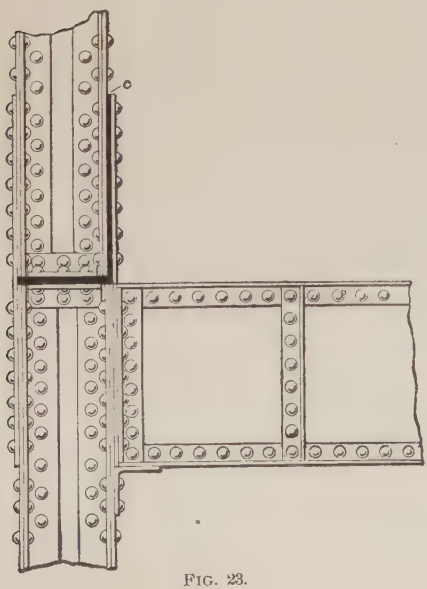


FIG. 23.

upper column whenever practicable, as this insures the central axis of the one column being over the central axis of the other.

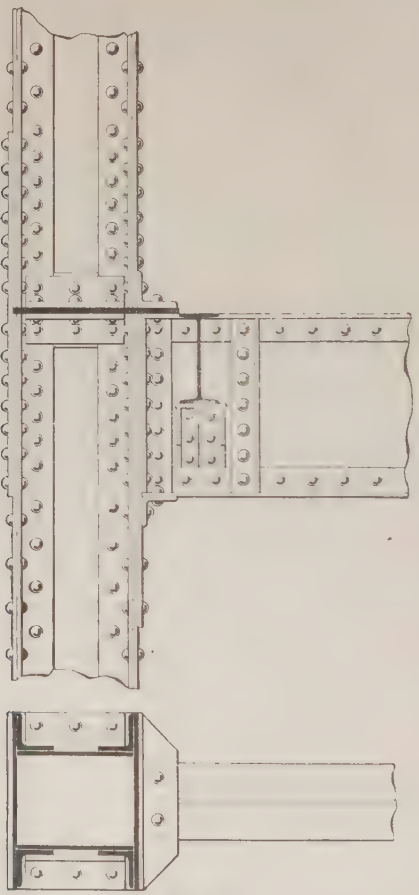


FIG. 24.

22. Examples of Good Column Design.—In Figs. 25 and 26 are shown complete detail working drawings of the first-floor columns of a building designed according to the best modern practice.

Fig. 25 is a channel column. The floorbeams are rigidly secured to the column at a considerable distance below the

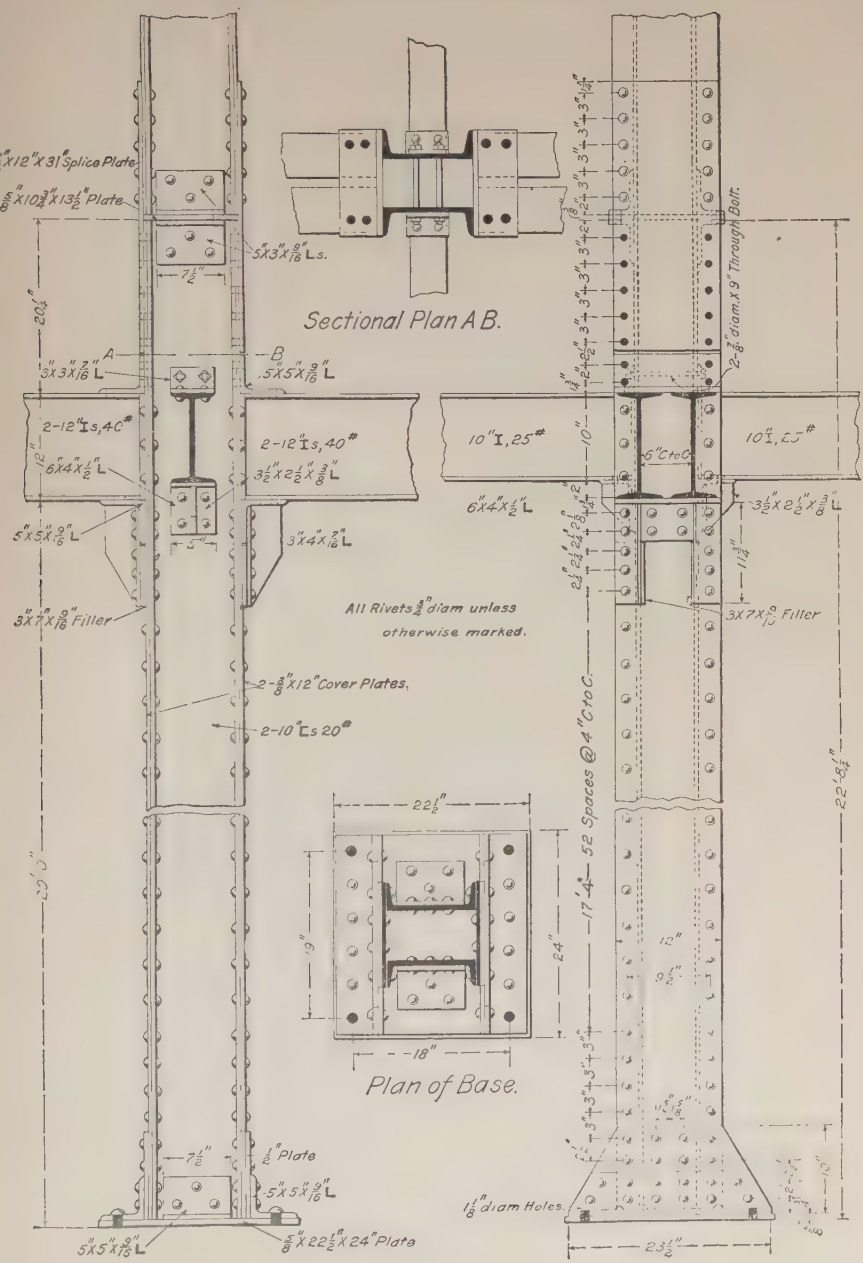
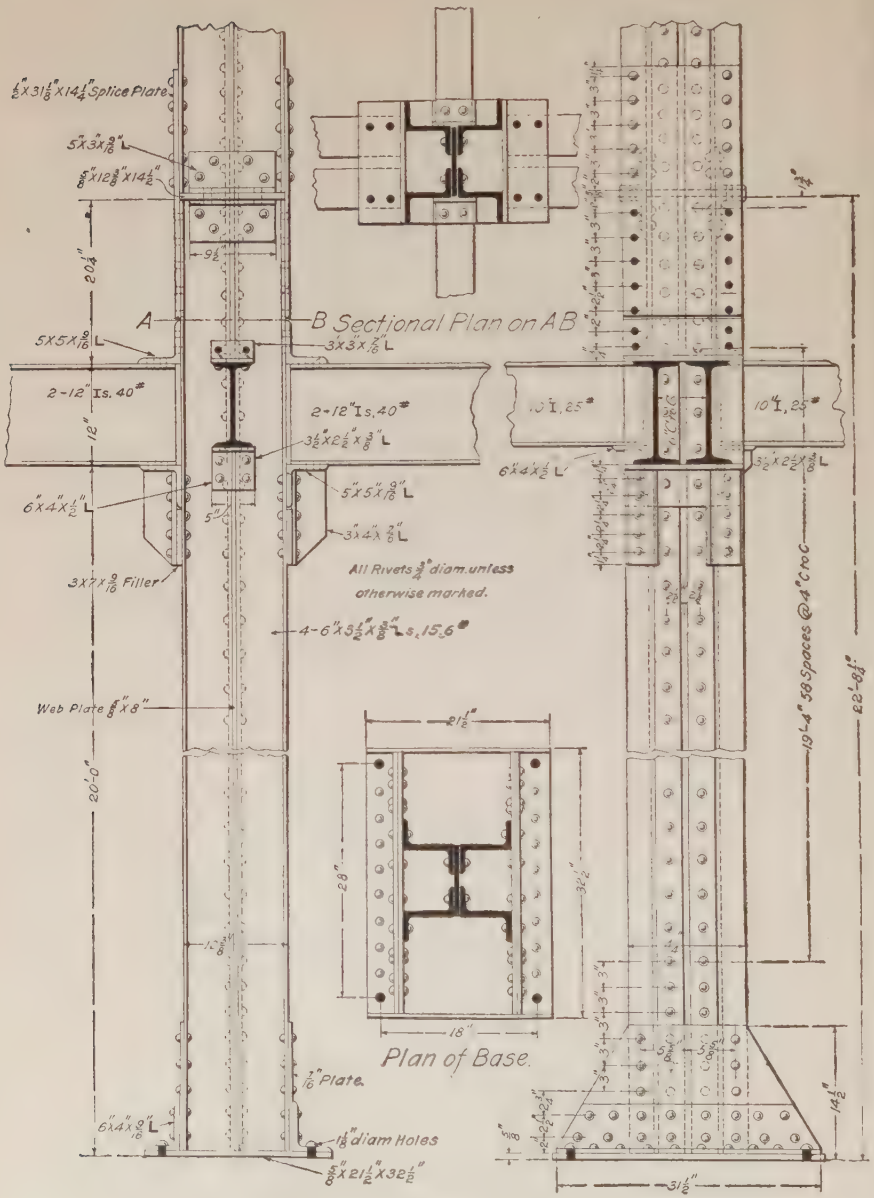

Sectional Plan A B.
Plan of Base.
All Rivets 3/4" diam unless otherwise marked.
2-12" Is, 40#
10" I, 25#
6" C to C
2-7/8" diam. x 9" Through Bolt.
3X7X9/16 Filler
2-3/8"X12" Cover Plates.
2-10" Cs 20#
1/2" Plate
5"X5"X9/16" L
5"X5"X9/16" L
5/8"X22 1/2"X24" Plate
1 1/8" diam Holes.
52 Spaces @ 4" C to C.
17'-4"
22'-8 1/4"
23 1/2"
20 1/4"
20'-0"
22 1/2"
24"
18"
19"
12"
9 1/2"
7 1/2"
5"
11 3/4"

FIG. 25.



½"x31⅛"x14¼" Splice Plate
5"x3"x9/16" L
⅝"x12⅜"x14½"
9½"
20¼"
A
B Sectional Plan on AB
5x5x9/16 L
3"x3"x7/16" L
2-12" Is. 40#
2-12" Is. 40#
12"
3½"x2½"x⅜" L
6"x4"x½" L
5"x5"x9/16" L
5"
3"x4"x7/16" L
3x7x9/16 Filler
All Rivets ¾" diam. unless otherwise marked.
4-6"x3½"x⅜" Ls. 15.6#
Web Plate ⅝"x8"
20'-0"
21½"
28"
32½"
18"
Plan of Base.
7/16" Plate.
6"x4"x9/16" L
1⅛" diam Holes
⅝"x21½"x32½"
10" I, 25#
10" I, 25#
6"x4"x½" L
3½"x2½"x⅜" L
19'-4" 58 Spaces @ 4" C to C
22'-8¼"
14½"
31½"

FIG. 26.

splice, which has been designed with a bedplate between the two sections. The splice could have been made equally as well, and would possibly have been more efficient, by the use of side-splice plates as previously explained.

In Fig. 26 is shown a **Z**-bar column, the principal points in the detail of which are similar to those in the channel-bar column.

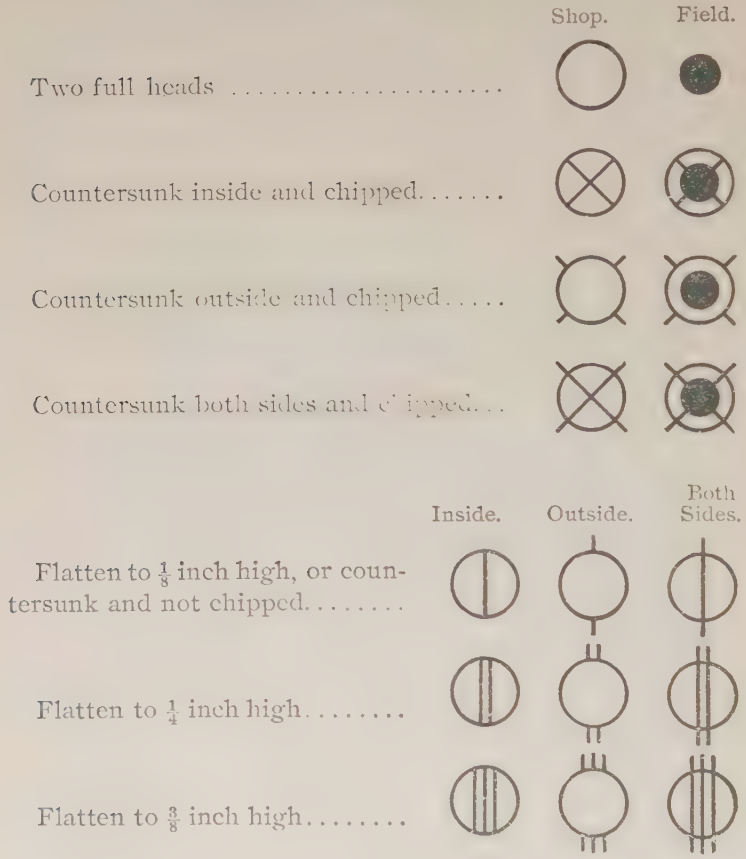
In each of the above examples, the shear on the rivets supporting the floorbeam brackets has been calculated to safely sustain the load carried upon the ends of the floorbeams.

All the connections and splices to a column should be riveted together with hot rivets; this insures more rigidity against wind pressure than can be obtained by bolted connections.

23. Rivet Signs.—In making drawings of structural-steel work, the form of rivet to be used should be designated by some system of conventional signs. The accompanying list shows Osborn's code of conventional signs for rivets, which has been adopted by many of the leading bridge companies and consulting engineers, and by many of the railway companies.

The basis of this system consists of the open circle to represent a shop rivet and the blackened circle to represent a field rivet, the diagonal cross to indicate a countersunk head and the vertical stroke to indicate a flattened head. The position of the diagonal lines with reference to the circle (inside, outside, or both) indicates whether the rivet head is countersunk into the inside, outside, or both sides of the material. Similarly, the number and position of the vertical strokes indicate the height and position of the flattened head. Any combination of shop or field rivets with full, countersunk, or flattened heads, may be readily indicated by the proper combination of these signs. The diagonal cross indicates not only that the rivet head is to be countersunk, but that it is also to be chipped off even with the surrounding material; if the rivet is to be countersunk, but not chipped, the countersunk sign may be combined with the sign to flatten to $\frac{1}{8}$ of an inch.

Conventional Signs for Rivets.



In the case of simple flat joints the *front side*, or the side which is seen, is considered as the *outside*, while the *rear side*, or the side which is hidden, is considered as the *inside*.

24. Conventional Signs for Rolled Shapes.—When designating the rolled structural shapes, such as angles, tees, channels, and **I** beams, they should be represented by their conventional signs. For instance, a $6'' \times 6'' \times \frac{1}{2}''$ angle, weighing 20 pounds per foot, would be designated upon the

drawing as 1–6″ × 6″ × ½″–20 ♯ L, or, to be more explicit, it could be written 1–6″ × 6″ × ½″ L–20 pounds per foot. If two angles, or a pair of angles of this size, are used, they could be designated as 2–6″ × 6″ × ½″–20 ♯ L's. It is not usual, however, to give the weight of an angle, when its thickness is given, consequently, a 6″ × 6″ × ½″ angle, weighing 20 pounds per foot, would generally be expressed on a drawing as 1–6″ × 6″ × ½″ L. A 12-inch channel, weighing 40 pounds per foot, would be marked on the drawing 1–12″–40 ♯ C. Sometimes the length of the rolled section is marked on the drawing, together with its size and weight, as 2–6″ × 6″ × ½″ × 10′ 0″ L's.

The above remarks, together with a careful study of the working drawings given in Figs. 25 and 26, are sufficient to give the student such information as will enable him to make a businesslike shop drawing of any structure or structural member made up of rolled-steel sections.

STRENGTH OF RIVETS AND PINS.

25. Rivets and pins are the elements by which the different sections and members of a steel structure are bound or tied together. Pins are also sometimes used in the better class of timber construction, in which case, however, the tension members are usually made of steel or wrought iron. Such a pin connection was shown in Fig. 9.

Where plates or rolled shapes are joined by either pins or rivets, as in Fig. 27, there is more or less friction between the several parts, which acts to prevent them from being pulled apart. This is especially true when rivets are driven close against the plates while hot; in cooling, they contract between the heads and bind the plates tightly together.

The friction between plates bound together by rivets and pins, however, is a very uncertain quantity and should not be considered in calculating the strength of the joint.

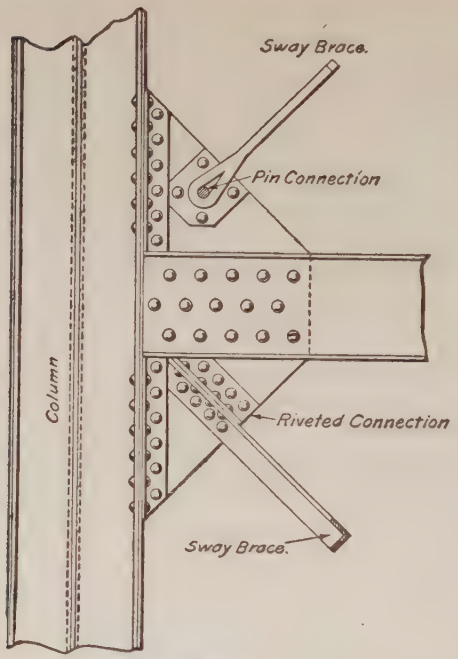


FIG. 27.

26. Methods of Failure.—Riveted joints may fail either by the shearing of the rivet or the crippling of the plates

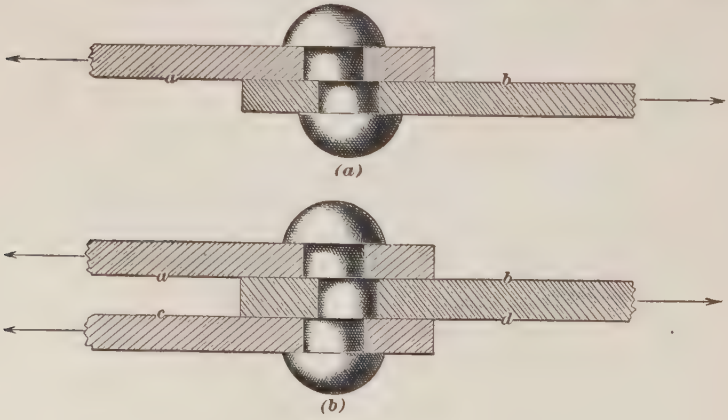


FIG. 28.

which they connect. When a rivet shears, the tendency is to cut straight through it across its section, as shown at (*a*) and (*b*), Fig. 28.

Where there are only two plates connected, as shown at (*a*), the tendency is to cut the rivet on the single plane *a b*. A rivet in this position is said to be in *single shear*.

At (*b*), the tendency is to cut through the rivet on both the planes *a b* and *c d*; under these conditions the rivet is in *double shear*, and it is evident that, since the rivet will shear across at two places, it will be twice as strong as where the tendency is to shear through only one section.

When the diameter of the rivet is large in proportion to the thickness of the plate, the joint or connection is liable to fail by the crippling of the plate as shown in Fig. 29. This occurs when the resistance of the plate to crushing or crimping is less than the resistance of the rivet to shear.

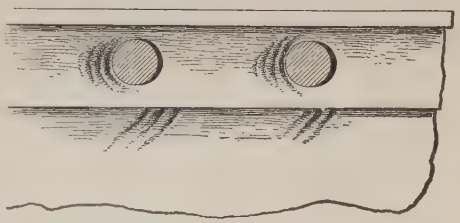


FIG. 29.

27. Bearing Value of Rivets.—The condition of the plates in the joint shown at (*a*), in Fig. 30, is called *ordinary bearing*, while the plate *m* connected as shown at (*b*) is said to be in *web bearing*. This distinction is important, because the bearing value of a web-plate is greater than that of an outside plate, the value for web bearing being about $\frac{1}{8}$ greater than for ordinary bearing.

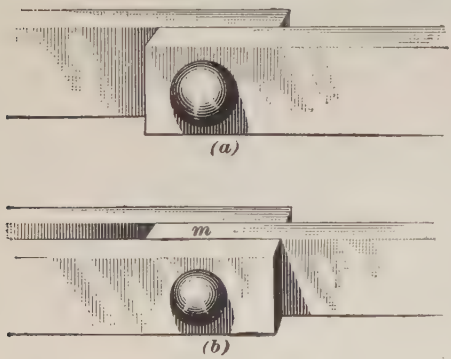


FIG. 30.

As the tensile strength of iron and steel used in the manufacture of rivets, pins, and plates for structural work is more

easily determined by tests, and, therefore, better known than either its compressive or shearing strength, it is customary to use this as a basis from which to calculate the bearing value of plates and the shearing strength of rivets and pins.

Good practice assumes that the compressive strength of steel or high-test iron is about $\frac{13}{15}$ of its tensile strength; that is, if the safe tensile strength of the material per square inch of section is 15,000 pounds, the safe compressive strength may be taken as $\frac{13}{15}$ of 15,000 = 13,000 pounds.

The shearing strength is considered to be $\frac{5}{6}$ of the compressive strength. For example, if, as above, the tensile strength is 15,000 pounds and the compressive strength 13,000 pounds, the shearing strength becomes $\frac{5}{6}$ of 13,000 = 10,833 pounds per square inch of section.

In order to determine the bearing value of the plates around a rivet hole, it is necessary to consider the bearing area of the rivet on the plate. This is always assumed to be the product of the diameter of the rivet multiplied by the thickness of the plate. Owing, however, to the support which the material around the hole receives from the rest of the plate, it will safely sustain a pressure greater than the compressive strength of the material when not so supported. The safe bearing strength of the rivet on the plate, for ordinary bearing, is, therefore, assumed to be $1\frac{1}{2}$ times its compressive strength, and for web bearing the safe bearing strength is assumed as double the compressive strength of the material.

In deducting the rivet holes, to ascertain the net section of a riveted plate, the diameter of the hole is taken as $\frac{1}{8}$ inch larger than the diameter of the rivet.

EXAMPLE 1.—Two pieces of structural steel are joined by rivets, as shown in Fig. 31. If the tensile strength of the steel is 60,000 pounds per square inch, and a factor of safety of 4 is used, what is the safe strength of this joint?

SOLUTION.—The safe tensile strength of the steel is 60,000 ÷ 4 = 15,000 pounds per square inch. The width of the pieces connected is $2\frac{1}{2}$ inches, from which is to be deducted 1 inch for the rivet hole, leaving a net width of $1\frac{1}{2}$ inches, which, multiplied by the thickness of

the plate, gives a net area of $1\frac{1}{2} \times \frac{3}{8} = .5625$ square inch. Then, $.5625 \times 15{,}000 = 8{,}437$ pounds, the strength of the plate.

Now, to determine whether the strength of the rivets is equal to the net section of the plate: Taking the compressive value of the plate as $\frac{13}{15}$ of 15,000 pounds, or 13,000 pounds per square inch, and the rivets being in ordinary bearing, the safe bearing strength is $13{,}000 \times 1\frac{1}{2} = 19{,}500$ pounds per square inch of bearing area. The bearing area is $\frac{7}{8} \times \frac{3}{8} = .328$ square inch, therefore, the safe bearing strength for one rivet is $19{,}500 \times .328 = 6{,}396$ pounds, and for the two it is $2 \times 6{,}396 = 12{,}792$ pounds.

The next point to consider is the resistance of the rivets to shear: The shearing strength of the steel is $\frac{5}{6}$ of $13{,}000 = 10{,}833$ pounds per square inch. The area of a $\frac{7}{8}$-inch rivet is .601 square inch, which,

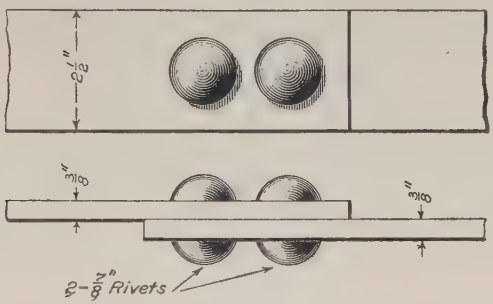


FIG. 31.

multiplied by 10,833, gives 6,510 pounds, the shearing strength of 1 rivet. The total resistance to shear of the rivets in the joint is, therefore, $6{,}510 \times 2 = 13{,}020$ pounds.

The safe resistance of the three elements entering into the strength of the joint is, therefore, as follows: Resistance of net section of the plate = 8,437 pounds; bearing value of the plate = 12,792 pounds; shearing strength of the two rivets = 13,020 pounds; from which it is easily seen that the strength of the joint is that of the net section of the plate, 8,437 pounds. Ans.

Since the bearing value of the plate and the shearing strength of the rivets are considerably in excess of this amount, it appears that the rivets are large for the joint, and it is probable that $\frac{3}{4}$-inch diameter rivets would give better results.

EXAMPLE 2.—One of the tension members in a structure is connected as shown in Fig. 32. The tension bars are made of structural steel with a safe tensile strength of 15,000 pounds per square inch. (*a*) What is the bearing value of the bar *c*? (*b*) What is the bearing value of the two bars *a*?

Solution.—(*a*) The safe compressive strength of the material is $\frac{13}{15}$ of 15,000 = 13,000 pounds. Then the bearing value of the bar *c*, which may be considered as a web, is $2 \times 13{,}000 = 26{,}000$ pounds per square inch. The bearing area of the pin in the bar *c* is $4 \times 1 = 4$ square inches; therefore, the bearing strength of the bar is $26{,}000 \times 4 = 104{,}000$ pounds. Ans.

(*b*) As the piece *a* is in ordinary bearing, its bearing value is 13,000

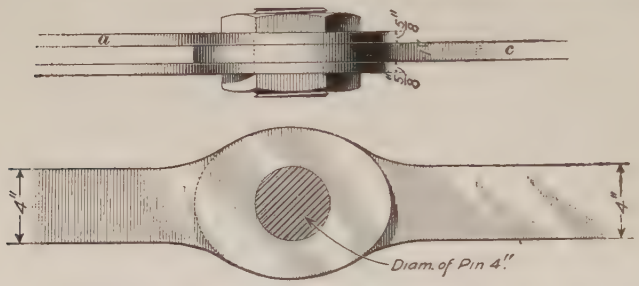


Fig. 32.

$\times 1\frac{1}{2} = 19{,}500$ pounds per square inch. The bearing area of the two bars is $2 \times 4 \times \frac{5}{8} = 5$ square inches; their combined bearing strength is, therefore, $19{,}500 \times 5 = 97{,}500$ pounds. Ans.

Example 3.—Determine the safe strength of the riveted joint shown

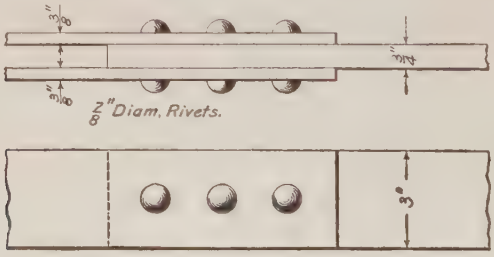


Fig. 33.

in Fig. 33, in which the plates and rivets each have a safe tensile strength of 16,000 pounds per square inch.

Solution.—The safe tensile strength of the material being 16,000 pounds, the safe compressive strength is $\frac{13}{15}$ of 16,000 pounds, or 13,867 pounds, and the shearing strength of the rivets is $\frac{5}{6}$ of 13,867, or 11,556 pounds per square inch of section. The area of the section of a $\frac{7}{8}$-inch diameter rivet is .601 square inch; therefore, the total shearing strength of the three rivets, each of which is in double shear, is $2 \times .601 \times 11{,}556 \times 3 = 41{,}670$ pounds.

The two outside plates are in ordinary bearing, and their bearing value is $1\frac{1}{2} \times 13{,}866 = 20{,}800$ pounds per square inch. There are three

rivet holes in each plate, each with a bearing area of $\frac{3}{8} \times \frac{7}{8} = .328$ square inch; the total bearing strength of the two plates is, therefore, $20{,}800 \times .328 \times 3 \times 2 = 40{,}934$ pounds.

The bearing value of the central or web-bearing plate at one rivet hole is $2 \times 13{,}867$ pounds $= 27{,}734$ pounds per square inch, and the bearing area is $\frac{3}{4} \times \frac{7}{8} = .656$ square inch; the total bearing strength for the three rivets is, therefore, $27{,}734 \times .656 \times 3 = 54{,}580$ pounds.

The safe tensile strength of the central plate is equal to its net section multiplied by 16,000, the safe unit tensile strength of the material. The net width of the plate is 3 in. $-$ 1 in. $=$ 2 inches, and its net area, $2 \times \frac{3}{4} = 1.5$ square inches; therefore, the safe strength is $1.5 \times 16{,}000 = 24{,}000$ pounds. The strength of the two outside plates, calculated in the same manner, is found to be 24,000 pounds also; therefore, it is evident that, since the strength of the net section of the plate is much less than either the strength of the rivets or the bearing value of the plates, it determines the strength of the joint, which is, therefore, 24,000 pounds. Ans.

28. Table of Bearing Values of Rivets.—In order to avoid the necessity of calculating the shearing value of the rivets and the bearing value of the riveted plates and rolled

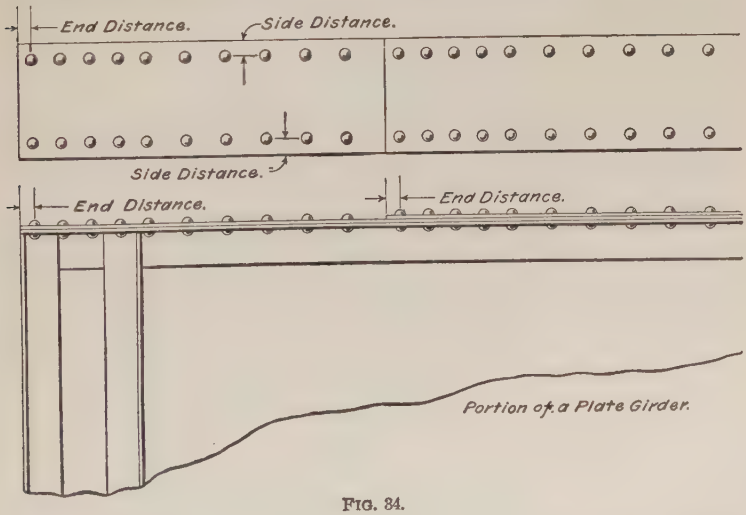


FIG. 34.

sections, the table "Values of Rivets" has been prepared. It will be noticed that the areas and shearing values for both double and single shear are given for rivets from $\frac{1}{2}$ inch to $\frac{7}{8}$ inch in diameter.

The least distance that rivets may be placed from the end or side of a plate is given in the third and fourth vertical columns from the left-hand side of the table. By referring to Fig. 34, the terms "end distance" and "side distance" used in the table will be readily understood.

The minimum allowable distance from the end and side of plate, as given in the table, is readily ascertained by the following: The end distance equals the thickness of the plate plus one-half the diameter of the rivet plus one-half inch, while the side distance equals one-half the thickness of the plate plus one-half the diameter of the rivet plus one-eighth inch.

The values under "allowable stress per square inch for high-test iron or structural steel," refer to the tensile strength of the material. If an allowable stress of 15,000 pounds is desired, use the value of the plates and rivets given under this head; should a still safer connection be desired, and an allowable stress of 10,000 pounds per square inch be considered advisable, use the values given in the column headed 10,000.

If the requirements of the structure are such that values other than those given in the table are desired, the required values may be obtained by finding the sum or the difference of two of the given columns. For example, a stress of 16,000 pounds per square inch is to be allowed, and the ordinary bearing value of a $\frac{1}{4}$-inch plate at a $\frac{5}{8}$-inch rivet hole is required; by adding the value given for an allowable stress of 1,000 pounds to that given for 15,000 pounds, the required bearing value may be determined; in this case it is $203.13 + 3,046 = 3,249.13$, which is the allowable bearing value of a $\frac{1}{4}$-inch plate in ordinary bearing around a $\frac{5}{8}$-inch diameter rivet, where the safe tensile stress of the materials is taken at 16,000 pounds per square inch.

29. Pins Subjected to Bending Stresses.—Pins, when used to connect the several members of a structure, besides being subjected to shearing in the same manner as rivets, may be required to resist heavy bending stresses; they may then be regarded as solid cylindrical beams and calculated

to resist the greatest bending moment that may come upon them.

Take, for example, the pin in Fig. 35 which connects the three tension bars *a*, *a*, and *c*; the pull of the two bars *a*, *a*,

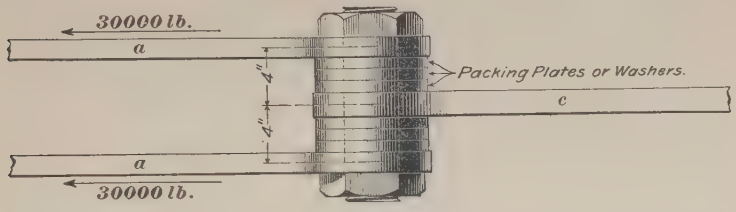


FIG. 35.

both acting in the same direction, is transmitted to the bar *c* by means of the pin. The stress upon each bar *a* is 30,000 pounds, consequently the stress upon the bar *c* must be 60,000 pounds. Assuming a maximum unit stress of 15,000 pounds per square inch, it is desired to find what diameter of pin is required to resist the bending moment produced by the stresses exerted upon it.

In calculating the bending moment on a pin, the forces acting upon it through the several members are considered as being applied at the center of the bearings. In Fig. 35, the distance between the centers of the bearings of the members is 4 inches, and by referring to the diagram, Fig. 36,

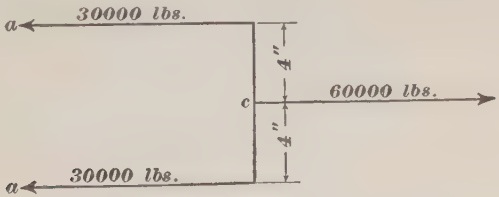


FIG. 36.

it is seen that the greatest bending moment is at *c*, and is equal to $30,000 \times 4 = 120,000$ inch-pounds.

Having found the greatest bending moment on the pin, it is necessary to determine its diameter, in order that its resisting moment may equal the moment of the bending stresses.

In the table "Elements of Usual Sections" we find the section modulus of a circular section to be

$$K = \frac{A\,D}{8} = \frac{.7854\,D^2 \times D}{8} = .0982\,D^3;$$

the allowable unit stress on the material is $S = 15{,}000$ pounds per square inch, and the bending moment is $M = 120{,}000$ inch-pounds; substituting these values in formula **4,** Art. **10,** we have $M = SK$, or $120{,}000 = 15{,}000 \times .0982\,D^3$, from which we get

$$D^3 = \frac{120{,}000}{15{,}000 \times .0982} = 81.46.$$

The diameter of the pin is, therefore, $D = \sqrt[3]{81.46} = 4.335 = 4\frac{3}{8}$ inches, nearly.

30. Table of Resisting Moments of Pins.—To avoid the necessity of calculating the resisting moment of pins, the table "Resisting Moments of Pins" will be found convenient. This table gives the resisting moments of pins from 1 to 12 inches in diameter, calculated for allowable fiber stresses of 15,000, 20,000, 22,000, and 25,000 pounds per square inch.

31. Resultant Moment of Several Stresses.—When

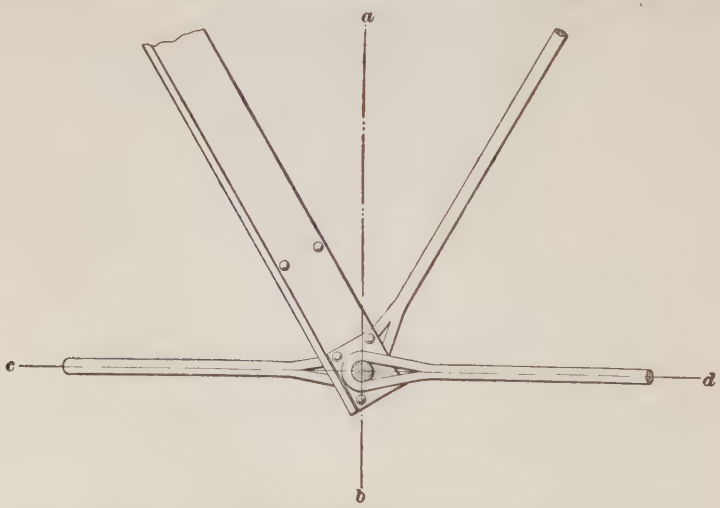


FIG. 37.

a pin is used at a joint at which several members, extending

in different directions, meet, as shown in Fig. 37, it is necessary to combine the stresses so as to find the resultant that gives the greatest bending moment. This is conveniently done by first resolving the stresses on each of the different members into vertical and horizontal components, and calculating the bending moments produced in each of these

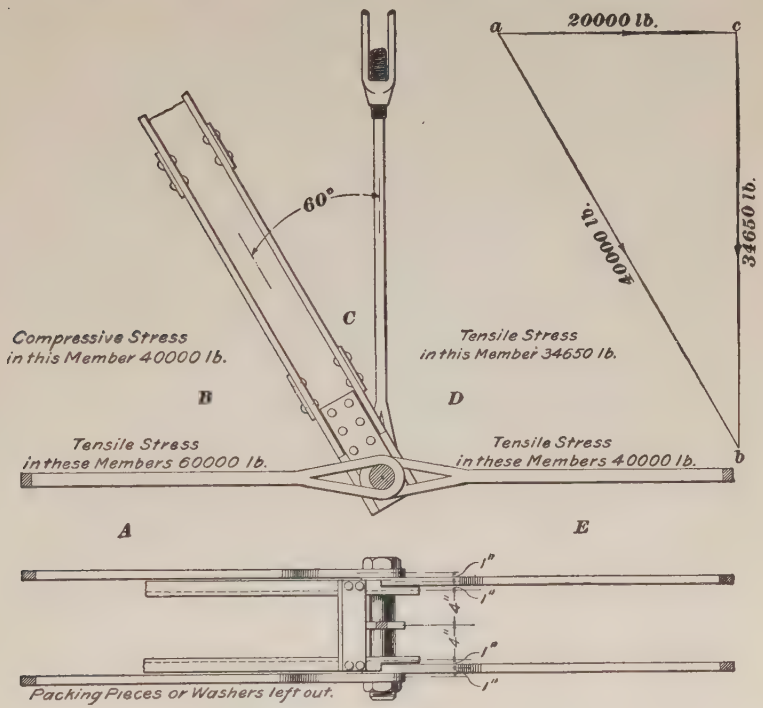


FIG. 38.

directions by all the corresponding components. The maximum bending moment is then given by the resultant of these two bending moments.

The details of the method for finding the maximum bending stress on a pin, will be made clear by a study of the following illustrative examples:

Fig. 38 shows one of the lower joints of a roof truss. At

this joint there are four sets of members, two of which act in a horizontal, while one acts in a vertical direction; since they already act in the directions of the required components, these forces need not be resolved. There is, however, one inclined member in which there is a compressive stress of 40,000 pounds, which stress must be resolved into its vertical and horizontal components. Draw the line $a\,b$ parallel to the strut and of such a length as to represent the magnitude of the stress. From a, draw the horizontal line $a\,c$ intersecting the vertical line at the point c. The direction of the forces around the triangle is shown by the arrows. Upon measuring the line $a\,c$, the horizontal component of the stress in $a\,b$ is found to be 20,000 pounds, while the vertical component of the stress is found to be 34,650 pounds.

Having determined these components, a diagram showing all the horizontal stresses acting upon the pin and tending to bend it, and also another showing all the vertical forces, should be drawn as illustrated at (a) and (b), Fig. 39, the distance from center to center of the members being taken from the detail plan of the joint, Fig. 38. It must always be remembered that, in accordance with the principles of equilibrium, the sum of the resultants of the forces acting upon the pin in any one direction must equal the sum of all the resultants acting in the opposite direction; otherwise, the pin would move in the direction of the greater sum, and the

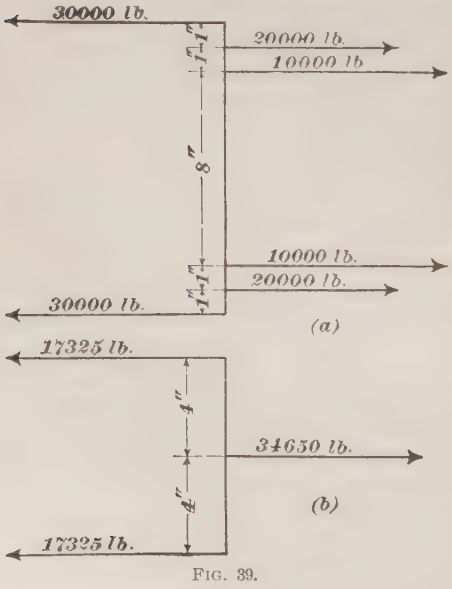


FIG. 39.

structure would fall. Thus, from Fig. 38, it is readily seen that the vertical component of the stress in BC acts in an opposite direction to the stress in the member CD, while the horizontal component acts in opposition to the stresses in the member AB, and in the same direction as the stress in DE. This makes the algebraic sum of all the components in either the horizontal or vertical direction equal to zero, and fulfils the condition of equilibrium.

The resultant of the vertical and horizontal bending moments may also be calculated by the rule for finding the length of the hypotenuse of a right-angled triangle; for example, in this case the lengths of the sides are represented by the horizontal bending moment of 40,000 inch-pounds, and the vertical bending moment of 69,300 inch-pounds; the resultant bending moment is, therefore, $\sqrt{40{,}000^2 + 69{,}300^2} = 80{,}015$ inch-pounds.

In order to determine the required size of pin for this joint, assume a safe fiber stress of 15,000 pounds per square inch, then, by referring to the table "Resisting Moments of Pins," it is seen that a pin $3\frac{7}{8}$ inches in diameter, under a fiber stress of 15,000 pounds per square inch, has a resisting moment of 85,700 inch-pounds, which is very nearly the value required by the conditions.

The student must remember that in all cases the pin should be examined for both shear and bending stresses.

EXAMPLES FOR PRACTICE.

1. What are the safe strengths of $\frac{7}{8}$, $\frac{3}{4}$, and $\frac{5}{8}$ inch rivets, in double shear, and also in single shear, assuming that the safe tensile strength of the material used in their manufacture is 15,000 pounds per square inch of section?

	Diameter of Rivet. Inch.	Double Shear. Pounds.	Single Shear. Pounds.
Ans.	$\frac{7}{8}$	13,022	6,511
	$\frac{3}{4}$	9,577	4,788
	$\frac{5}{8}$	6,652	3,326

2. What pulling force will two pieces of $\frac{3}{8}'' \times 2\frac{1}{2}''$ bar safely resist, providing they are connected at the ends by two $\frac{3}{4}$-inch diameter rivets, as shown in Fig. 40? The safe tensile strength of the material in rivet and bar is 15,000 pounds.

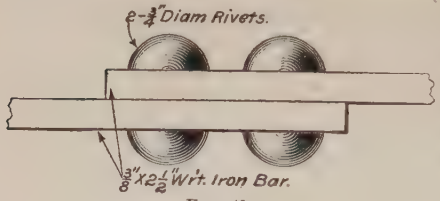


FIG. 40.

Ans. 9,140 lb.

3. What is the safe resisting moment of a pin 5 inches in diameter, if the safe fiber strength of the material is 20,000 pounds?

Ans. 245,400.

4. In Fig. 41 is shown a pin connection, the pull on the tension bar *a* being 140,000 pounds. If the safe shearing strength of the material in the pin is 10,000 pounds per square inch, and the safe fiber stress in

FIG. 41.

bending is 15,000 pounds per square inch, (*a*) what size of pin will be required to resist the shear? (*b*) what size will be required to resist the bending?

Ans. { (*a*) 3 in. in diameter.
(*b*) $4\frac{1}{8}$ in. in diameter.

5. It is necessary to construct the connection of a tension member as shown in Fig. 42. What is the safe load that this member will

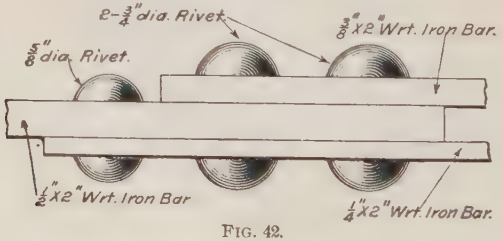


FIG. 42.

carry, if the safe tensile strength of the material in both the rivets and bars is 18,000 pounds per square inch?

Ans. 11,250 lb.

PLATE GIRDERS.

GENERAL CONSTRUCTION.

32. A **plate girder** is a beam built up of a number of plates and angles securely riveted together.

The names given to the different parts of a plate girder may be understood by referring to Fig. 43, in which *a* is the **flange plate,** of which there may be one or more on each flange, depending upon the strength required. The flange plates are the principal elements for resisting the bending stresses in the girder. The **flange angles** *b*, *b* are the means of connecting the flange plates and the **web-plate** *c*. When the load on the girder is small, the flange plates may be omitted, in which case the flange angles are the members which chiefly act to resist the bending stresses.

FIG. 43.

33. Stiffness.—On account of the construction of a plate girder, there is very little stiffness in the web-plate, consequently, there is always a strong tendency for it to fail by buckling and twisting under the load imposed upon the girder. This tendency to buckle is greatest at the supports or abutments of the girder and at points where concentrated loads are applied. Because of this buckling tendency it becomes necessary to reinforce the girder by riveting at stated intervals **stiffeners** generally made of angles.

The most common and cheapest form of stiffener is shown at (*a*), Fig. 44. This is simply a straight piece of angle riveted to the web-plate and flange angles. The space between the stiffeners and web-plate, due to the thickness of the flange angles, is filled with a piece of bar iron or plate, as shown at *d*; this is called a **filler** or **packing piece.**

A more expensive form of stiffener for plate girders is shown at (*b*), Fig. 44. The angle is swaged out, to allow it to fit over the flange angles, and is riveted directly to the web-plate, thus doing away with the filler or packing piece. This construction does not require as much material as that shown at (*a*), but, unless there are a large number of girders of the same dimensions to be built, in which case dies, in connection with a power or hydraulic press, may be used for swaging the ends, the labor required is so much greater as to make the girder more expensive.

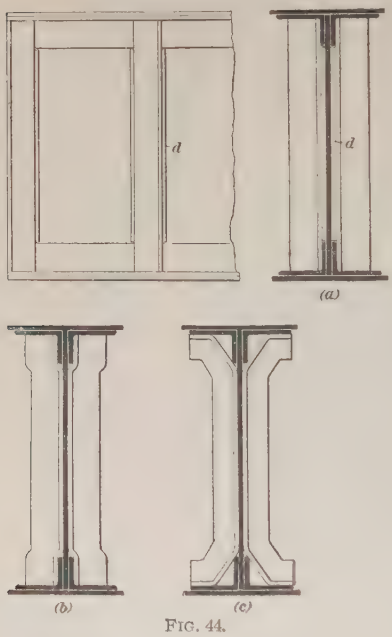


FIG. 44.

The stiffeners shown at (*c*) are sometimes used, but are subject to the same general criticism in regard to cost of manufacture as those shown at (*b*). Stiffeners of this shape possess a possible advantage in the fact that they stiffen the flanges considerably more than either of the other two styles.

34. Usual Forms of Sections.—The four principal sections used in plate-girder construction are shown in Fig. 45.

A simple plate girder with a web-plate and two flange angles, but with no flange plate, is shown at (*a*). This section is used for short spans or light loads. At (*b*) is shown a similar girder with one flange plate. This girder is used to support heavier loads and to clear longer spans; while the girder at (*c*), which may have two or more flange plates at each flange, may, if the conditions require, be made as

heavy as is necessary in order to carry great loads over long spans. In fact, the strength of a girder of this character may be increased almost indefinitely by adding flange

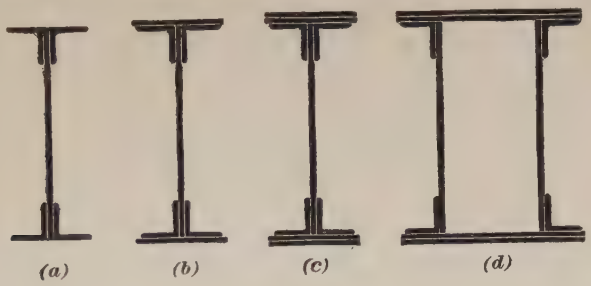


FIG. 45.

plates. The section (*d*), Fig. 45, is a plate girder of box section. It is stiffer laterally than the forms shown at (*a*), (*b*), and (*c*), but the difficulty of reaching the interior for painting and inspecting, and the excessive amount of labor required in its construction, are such serious objections that it is much less used than the sections shown at (*a*), (*b*), and (*c*). On account of their open construction, the latter are especially good forms to use in buildings where the objection in regard to lateral stiffness does not hold good, as when the girder is used in the position in which it is usually found, being generally prevented from deflecting laterally by the floorbeams; any lack of stiffness in comparison with the box girder is more than compensated by the simplicity of construction and easy access on all sides for painting and inspecting.

PRINCIPLES OF DESIGN.

STRESSES.

35. The external forces, loads, and reactions produce the same kind of stresses in a plate girder as in an ordinary beam, but, on account of its special construction, the distribution of these stresses in the girder is assumed to be somewhat different from that in a beam made of a single piece.

In the girder, the shear is generally assumed to be borne wholly by the web-plate, while the bending moment is assumed to be resisted by the stresses in the flange members. The method of calculating the magnitude of the shear and bending moment is the same as that for beams, already discussed in *Architectural Engineering*, § 5; owing, however, to the different assumption in regard to the distribution of these forces, a different method of calculation is used in determining the relations between them and the stresses in the girder.

36. Shearing Stresses in Web-Plate.—In discussing the methods of calculating the dimensions of a plate girder for a given purpose, we will first consider the shear, which is the principal factor that determines the thickness of the web-plate and the number and size of stiffeners required. In *Architectural Engineering*, § 5, it was shown that the greatest shear in a beam occurs at the point of support at which the reaction is greatest, and that the magnitude of the shear is equal to the reaction at that point; consequently, in a simple plate girder, the greatest shear occurs at a point of support, and is equal in amount to the reaction at that point.

WEB-PLATES AND STIFFENERS.

37. Depth of Girder.—Having calculated the shear, the depth of the girder is assumed in accordance with practical rules which fix the relation between the depth and span. In accordance with the best practice, the depth should not be less than $\frac{1}{15}$ of the span, though some authorities consider $\frac{1}{20}$ as being ample. The latter proportion, however, gives an exceedingly shallow girder, and cannot be recommended except where the loads are very light and the span short, or where it is absolutely necessary that an extremely shallow girder be used, on account of decorative features, or lack of space in regard to headroom, in which case the girder should be so proportioned that when fully loaded its deflection will not be excessive.

38. Thickness of Web-Plate.—Knowing the depth of the girder, and the shear at the points of support, the thickness of the web-plate is proportioned so as to give it sufficient area to resist the maximum shear. It is always necessary to stiffen the plates over the supports, as shown in Fig. 47; these stiffeners are riveted to the plate and transfer the shearing stress from it to the supports.

A considerable portion of the plate is cut away by the holes for the rivets by which it is fastened to the stiffeners; hence, the least strength of the plate is along the line of the rivet holes. It can readily be seen, by referring to Fig. 46, which shows the end of a plate with the holes punched for riveting to the stiffener, that the net or efficient depth of the plate is equal to the actual depth minus the sum of the diameters of the rivet holes.

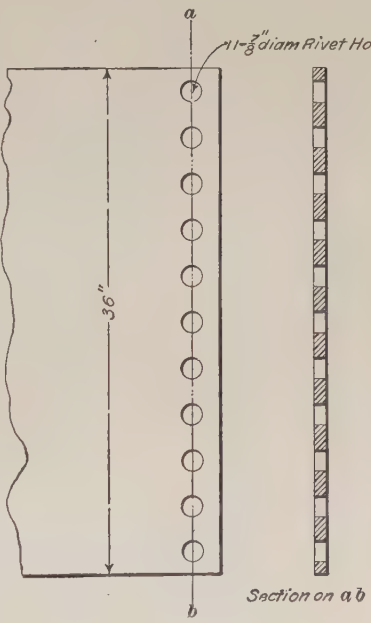


FIG. 46.

The following rule may be used for calculating the thickness of a web-plate so that it will have sufficient strength to resist the shearing stress:

Rule.—*From the total depth of the web-plate, deduct the sum of the diameters of the rivet holes, which will give the net or efficient depth of the web-plate; multiply the net depth by the safe resistance of the material to shear, and divide the maximum shear in pounds by the product; the quotient will be the required thickness of the metal in the web of the girder.*

The above rule may be expressed by the formula

$$T = \frac{R}{D \times S}, \qquad (12.)$$

in which T = thickness of the web-plate;
R = greatest reaction or maximum shear;
S = safe shearing resistance of the material per square inch;
D = net depth of the web-plate after all the rivet holes have been deducted.

The safe resistance of the material to shear is of course governed by the factor of safety required in the girder. For example, the ultimate shearing strength of structural steel being 52,000 pounds per square inch, if a factor of safety of 4 is required, the safe resistance of the metal will be $52{,}000 \div 4 = 13{,}000$ pounds, while if a factor of safety of 5 is desired, the safe strength will be $52{,}000 \div 5 = 10{,}400$ pounds.

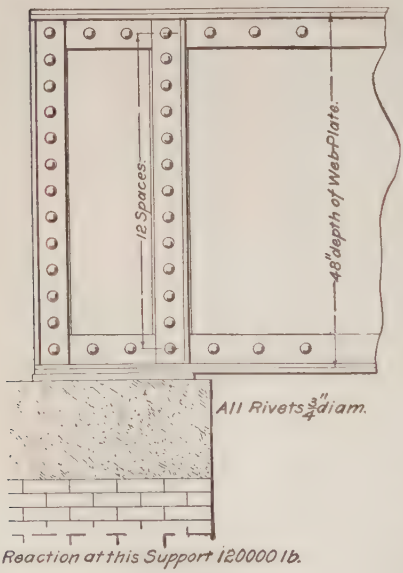


FIG. 47.

In deducting the metal for the rivet holes in order to ascertain the net depth of the web-plate, the holes should always be considered as being $\frac{1}{8}$ inch larger than the nominal diameter of the rivet; this allowance is made because the holes are always made $\frac{1}{16}$ inch larger in diameter than the rivet so that the rivet may be inserted easily, and another $\frac{1}{16}$ inch should also be allowed in the diameter of the hole to compensate for any injury that the metal immediately around it may suffer from the punch.

It will often be found that the calculated thickness of the web-plate is less than is allowable for practical reasons.

The thinnest plate that should be used for any case is $\frac{5}{16}$ inch.

EXAMPLE.—Fig. 47 shows the end of a plate girder in which the greatest reaction is 120,000 pounds. The girder is made of structural steel, the safe fiber stress of which is assumed to be 11,000 pounds per square inch for shear. What should be the thickness of the web-plate?

SOLUTION.—The width of the plate is 48 inches, and there are 13 holes punched for $\frac{3}{4}$-inch rivets. The deduction to be made for each rivet hole is $\frac{1}{8}$ inch $+ \frac{3}{4}$ inch $= \frac{7}{8}$ inch, therefore, the net depth of the plate is $48 - 13 \times \frac{7}{8} = 48 - 11\frac{3}{8} = 36\frac{5}{8}$ inches. Applying formula **12**, the thickness of the plate is

$$T = \frac{120{,}000}{36\frac{5}{8} \times 11{,}000} = .297 \text{ inch.}$$

In no case, however, should the web-plate of a girder be less than $\frac{5}{16}$ inch in thickness. Hence, as .297 is less than $\frac{5}{16}$, the thickness of the web-plate in this girder should be $\frac{5}{16}$ inch. Ans.

39. Buckling of Web-Plate and Distribution of Stiffeners.—The shearing stresses in a web-plate, in addition to their tendency to shear the plate, as above described, are liable to cause it to fail by buckling; therefore, in order to properly resist the vertical shearing stresses and prevent them from buckling the web-plate before its full shearing strength is realized, it is necessary to provide the stiffeners, which have been previously described.

It was shown in *Architectural Engineering*, § 5, that the shearing stresses in a simple beam are always greatest at the points of support, and diminish towards the center of the beam until a point is reached between which and the support the sum of the loads is equal to the reaction; at such a point the shear is said to change sign. The natural inference from the above fact is that the stiffeners should be more numerous at the points of greatest vertical shear, decreasing in number as the shear decreases. Theoretically, this would be a correct method of locating the stiffeners, but practically they are spaced at equal distances along the length of the girder, except at the points of support, where several are placed near each other in order to give the end of the girder more nearly the character of a column and enable

it to successfully resist the great vertical shear, due to the reaction at this point. In no case should the stiffeners at the end of a plate girder be omitted, even if the conditions make the intermediate ones unnecessary.

It is also good practice to place stiffeners directly under any concentrated load that may be placed upon the girder. These stiffeners are necessary not only to stiffen the web-plate at the point of application of the load, and thus prevent buckling, but also, through the medium of the rivets, to assist in distributing the load on the web-plate and other members of the girder.

The end stiffeners of a plate girder may be considered as columns subjected to a compressive stress equal to the reaction, and calculated by the rules and formulas already given for columns. For safety the stress upon the end stiffeners should never exceed 15,000 pounds per square inch of section.

Practice in regard to the placing of stiffeners on plate girders varies considerably, being more a matter of judgment and experience than of calculation. Some engineers determine the resistance of the web-plate to buckling by the formula

$$B = \frac{11{,}000}{1 + \frac{d^2}{3{,}000\, t^2}}, \qquad \textbf{(13.)}$$

in which B = safe resistance of the web to the buckling, in pounds per square inch;

d = depth of the web-plate in inches;

t = thickness of the web-plate in inches.

If the value of B given by this formula is less than the unit shearing stress, the girder should be stiffened.

EXAMPLE.—The allowable shearing stress upon the web of a plate girder is 11,000 pounds per square inch. The stiffeners at the end supports are riveted by nine $\frac{7}{8}$-inch rivets to the web-plate, which is 36 inches wide. The end reaction on the girder is 100,000 pounds. (*a*) What should be the thickness of the web-plate? (*b*) Will it be sufficiently strong without the addition of stiffeners?

SOLUTION.—(*a*) Since $\frac{7}{8}$-inch rivets are used, the allowance to be made for one rivet hole is $\frac{7}{8} + \frac{1}{8} = 1$ inch, and the effective width of the

plate along the line of rivets is $36 - 9 \times 1 = 27$ inches. Applying formula **12**, the thickness of the plate is found to be

$$T = \frac{100{,}000}{27 \times 11{,}000} = .33 \text{ inch.}$$

The thickness of the nearest standard-size plate above this is $\frac{8}{8}$ inch, which will be the thickness used. Ans.

(*b*) By formula **13**, the safe unit resistance of the plate to buckling is

$$B = \frac{11{,}000}{1 + \frac{36^2}{3{,}000 \times \frac{8}{8}^2}} = 2{,}700 \text{ pounds per square inch;}$$

which, since it is much less than the unit shearing stress on the web-plate, shows that stiffeners are required. Ans.

40. Practical Rule for Spacing Stiffeners.—It is not the general practice to make the above calculations to determine whether stiffeners are required; according to the best engineering practice, stiffeners should be provided, unless the thickness of the web-plate is at least $\frac{1}{60}$ of the clear distance between the vertical legs of the flange angles.

EXAMPLE.—Assume the girder in the previous problem to be provided with $6'' \times 6''$ flange angles; the depth and thickness of the plate, as previously shown, are 36 inches and $\frac{8}{8}$ inch, respectively. According to the above rule, does this girder require stiffeners?

SOLUTION.—The unsupported depth of the plate between the flange angles is $36 - 2 \times 6 = 24$ inches; $\frac{1}{60}$ of 24 inches $= .48$, say, $\frac{1}{2}$ of an inch. As the thickness of the web-plate is only $\frac{8}{8}$ of an inch, the girder must be provided with stiffeners. Ans.

Another rule, which gives nearly the same result, is as follows:

Rule.—*Provide stiffeners whenever the thickness of the web-plate is less than* $\frac{1}{60}$ *of its total depth.*

This rule is modified by some authorities so as to allow a thickness of $\frac{1}{80}$ of the total depth of the plate as being amply safe without stiffeners. The more conservative rule, however, which requires the thickness to be at least $\frac{1}{60}$ of the unsupported depth of the web or the distance between the flange angles, is the one to be recommended, and will be used in this section.

The spacing and size of stiffeners to be used on a plate

girder is almost entirely a matter of experience and judgment. As a general rule, it may be said that stiffeners should be provided at the ends of all plate girders over the supports or abutments, and they should be so proportioned that they will take care of the entire reaction at these points. The stiffeners between the abutments or supports should be of such a size that they will best suit the general requirements of the design of the girder. The practice in spacing intermediate stiffeners is to make the distance between their center lines equal to the depth of the girder, thus dividing the girder into equal square panels. Under no conditions, however, should stiffeners be placed more than 5 feet apart from center to center of line of rivets.

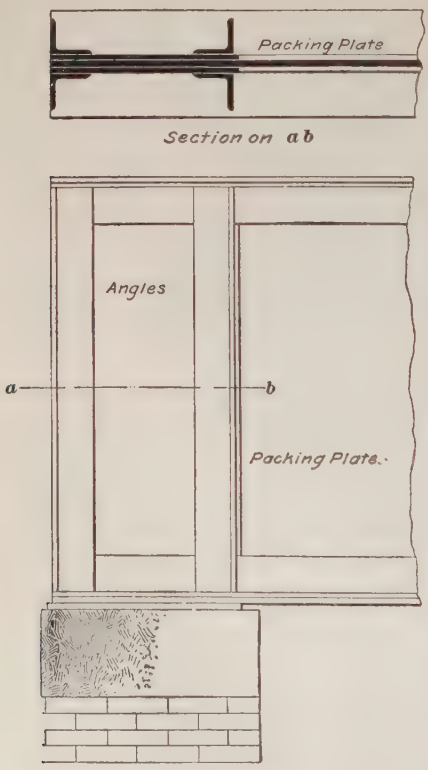


FIG. 48.

Having proportioned the stiffeners at the abutments to take the entire reaction, it is good practice, when possible, to make the intermediate stiffeners of the same size as the end ones. In general, the angles used for stiffeners should not be less than 3 in. × 3 in. × $\frac{5}{16}$ in., though on shallow girders, with extremely light loads, it might be economical to use angles as light as $2\frac{1}{2}$ in. × $2\frac{1}{2}$ in. × $\frac{5}{16}$ in. Sizes smaller than this should certainly never be used as stiffeners.

Stiffeners should always extend over the vertical legs of the flange angles; they should always be either swaged out

to fit over the flange angles, or be provided with a filling piece as illustrated in Fig. 44.

EXAMPLE.—The end reaction on a plate girder is 300,000 pounds. If a compressive fiber stress of 13,000 pounds per square inch is allowed on the stiffeners, and 4 stiffeners are used, as shown in Fig. 48, what should be the size of the angles?

SOLUTION.—The reaction or greatest shear being 300,000 pounds, and the allowable stress 13,000 pounds per square inch, the area of stiffeners required must be $300{,}000 \div 13{,}000 = 23$ square inches; this sectional area divided among 4 angles, gives $23 \div 4 = 5.75$ square inches as the area required for each angle.

By referring to the list of angles with even legs, in the table "Areas of Angles," it is seen that a $5'' \times 5'' \times \frac{5}{8}''$ angle has a sectional area of 5.86, while, in the list of areas of the uneven angles, a $5'' \times 4'' \times \frac{11}{16}''$ is shown to have an area of 5.72 square inches; therefore, either of these angles may be used. Ans.

FLANGES.

41. The flanges of a riveted girder include all the metal at the top and bottom of the girder, and are sometimes called the top and bottom chords, though this term is more frequently applied to lattice or open girders, such as are more often used for railroad and highway bridges.

In building construction, it is customary to include in the flange the two flange angles, the flange plates, and $\frac{1}{6}$ of the web-plate included between the flange angles. The building ordinances of some of the large cities in the United States, however, disregard this last item, and will not allow any portion of the web-plate to be included as part of the flange. In this section the web-plate will not be considered in calculating the flange area.

If, because of economic considerations, $\frac{1}{6}$ of the web-plate must be included as part of the flange, it must be remembered that the plate should never be spliced near the center, when the girder is uniformly or symmetrically loaded, or directly under the point of greatest bending moment, when the load upon the girder is unsymmetrically placed. Especial care must also be taken to insure that any splice made upon the length of such a web-plate is so designed as to furnish

the greatest possible percentage of strength of the solid plate included within $\frac{1}{6}$ of the depth of the web.

The best practice dictates that where flange plates are used, the sectional area of the flange angles should equal the sectional area of the flange plates. This, however, is not possible in heavy work, where the best that can be done is to use the heaviest sections obtainable for the flange angles.

42. Flange Stresses.—In a simple girder the top flange is subjected to compression, and the bottom flange to tension. Nevertheless, it is customary in practice to make the two flanges equal and composed of the same size of rolled plates and angles.

In proportioning the flanges of a plate girder, the lower flange is calculated for tension; the areas of the rivet holes cut out of the flanges are deducted from the total area, so as to give the net or actual area of the flange at the point of least strength.

The stresses in the flanges are assumed to be produced wholly by the bending moment on the girder, and the moments of these stresses are assumed to be equal to the moments of the external forces.

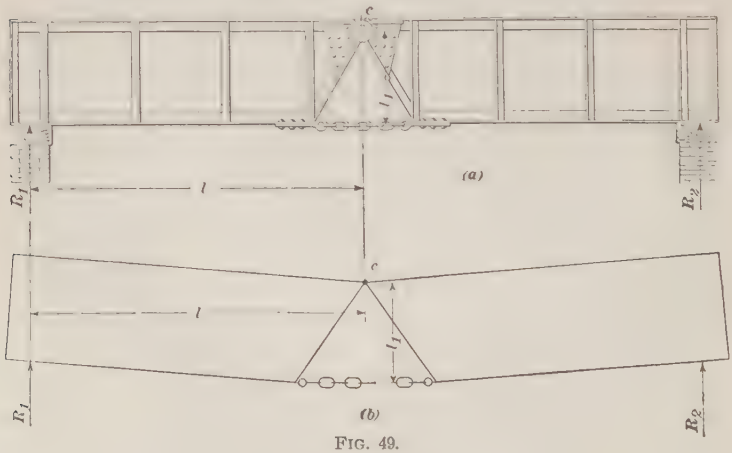


FIG. 49.

The principles on which the flange stresses of a plate girder are calculated will be made clear by reference to

Fig. 49, which shows a girder in two sections joined by a hinge pin c at the upper flange, and a chain at the lower flange. The resultant moment of the loads and reactions tends to produce rotation about the center c, which, however, is taken at a point in the upper flange instead of on the neutral axis, as was done in the case of the beam composed of a single section; in reality, owing to the fact that the web is entirely neglected in calculating the resistance of the girder to the bending stresses, there is no neutral axis, in the sense in which that term was used in connection with ordinary beams.

The stress in the chain, which represents the lower chord or flange of the girder, resists the tendency to rotation about the center of moments c, a lever arm l_1, which is the perpendicular distance from the chain to the point c. It is evident, then, that the strength of the girder depends upon two factors, the tensile strength of the lower chord, and its distance from the center of the hinge c, the latter of which represents the depth of the girder.

If now the center of moments is taken on the center line of the chain, directly under the point c, it is evident that the resultant moment of the external forces, with respect to this center, is the same as when the center was taken at c; it is also evident that the force in the beam whose moment, with respect to this center, balances the resultant moment of the external forces, is the compression on the pin c. Since the moment and the lever arm of the compressive stress on the pin are respectively equal to the moment and the lever arm of the tensile stress in the chain, it follows that these two stresses are equal; in other words, the compressive stress in the top flange of the girder is equal to the tensile stress in the bottom flange.

43. Proportioning the Flanges.—Having determined the principles upon which the bending strength of a plate girder is calculated, it remains to show a method for proportioning the metal in the flanges. The usual process is as follows:

First calculate the maximum bending moment upon the girder; this may be done by the principles and rules given in *Architectural Engineering*, § 5. In calculating the bending moment on a plate girder, however, it is customary to express the moment in foot-pounds, the depth of a girder being generally given in feet and not in inches, as in solid beams of shallow depth. If, however, the depth of the girder is expressed in inches, the bending moment must be calculated in inch-pounds.

Having found the maximum bending moment on the girder, it is necessary to assume an allowable fiber stress for the material of which the flanges are composed. The following rule may then be used to calculate the sectional area of either flange.

Rule.—*Divide the bending moment on the girder in foot-pounds by the product obtained by multiplying the depth of the girder in feet by the safe fiber stress.*

The safe fiber stress for a given case is obtained by dividing the ultimate fiber stress per square inch of the material by the factor of safety required in the girder.

The rule may be expressed by the formula

$$A = \frac{M}{D \times S}; \qquad (14.)$$

in which A = net area of one flange in square inches;
D = depth of the girder in feet;
S = safe fiber stress per square inch of the material;
M = bending moment on the girder in foot-pounds.

Example.—The depth of a plate girder is 6 feet, the span is 80 feet, and the load upon the girder is 3,000 pounds per lineal foot. (*a*) What will be the required net flange area for structural steel, if a factor of safety of 4 is used? (*b*) Of what size rolled sections should the flange be composed?

Solution.—The span being 80 feet and the load 3,000 pounds per lineal foot, the entire load on the girder will be $80 \times 3{,}000 = 240{,}000$ pounds.

Substituting in the formula $M = \frac{W L}{8}$ for the bending moment on a simple beam (see Table 10, Art. **97**, *Architectural Engineering*, § 5), we have

$$M = \frac{240{,}000 \times 80}{8} = 2{,}400{,}000 \text{ foot-pounds.}$$

The net area of the flange, from formula **14**, is

$$A = \frac{2{,}400{,}000}{6 \times 15{,}000} = 26.7 \text{ square inches.} \quad \text{Ans.}$$

(*b*) In order to determine the size of the flange plates and angles, it is useful to assume some particular size of angle and plates, and make a detail sketch of the flange, as shown in Fig. 50, marking on it the size of the respective plates and angles that have been assumed. The rivets should also be shown, so that the metal cut out of the rivet holes may be deducted from the sectional area of the flange in order to determine that area.

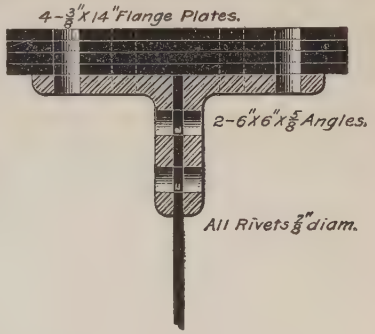


FIG. 50.

It is assumed in the section under consideration that there are two rows of rivets through the vertical legs of the angles, each pair of these rivets being placed in the same vertical plane in consequence of which the amount to be deducted from the net section is double the area cut out for one rivet. The rivets in the two rows through the horizontal legs of each angle are staggered, and consequently only one rivet hole in each horizontal leg affects the area of the flange section.

According to the table "Areas of Angles," the area of a $6'' \times 6'' \times \frac{5}{8}''$ angle is 7.11 square inches, therefore, the total area of the metal in the flange is

2–$6'' \times 6'' \times \frac{5}{8}''$ angles,	$7.11 \times 2 =$	14.22 sq. in.
4–$14'' \times \frac{3}{8}''$ plates,	$4 \times 14 \times \frac{3}{8} =$	21.00 sq. in.
Total, ...		35.22 sq. in.

From the total area of the flange it is necessary to deduct the metal cut out for the rivet holes. As $\frac{7}{8}$-inch rivets are used, the rivet holes are considered to be $\frac{1}{8}$ of an inch larger, or 1 inch in diameter.

There are four 1-inch holes through $\frac{5}{8}$ inch of metal to be deducted from the vertical legs of the angles, and in the plates and the horizontal

legs of the angles there are two 1-inch holes through $2\frac{1}{8}$ inches of metal. The areas to be deducted for the rivet holes are, therefore,

4–1-inch holes through $\frac{5}{8}$ in. of metal	$= 2\frac{1}{2}$ sq. in.
2–1-inch holes through $2\frac{1}{8}$ in. of metal	$= 4\frac{1}{4}$ sq. in.
Total, ...	$6\frac{3}{4}$ sq. in.

and the net area of the flange is $35.22 - 6.75 = 28.47$ square inches. Since the calculations showed that the net area required in this flange is 26.6 square inches, it is evident that the assumed flange is amply strong. Ans.

44. Lengths of Flange Plates.—Since the bending moment in a simple beam varies along the entire length of the beam, the location of the maximum bending moment depending on the distribution of the load, it would seem that, in order to design an economical girder, the area of the flange should vary with the bending moment. Where flange plates are used, this condition may be partially fulfilled by the use of plates of different lengths, each extending only as far as may be required in order to provide the flange section demanded by the bending moment.

Reference to Fig. 51 will make this construction more

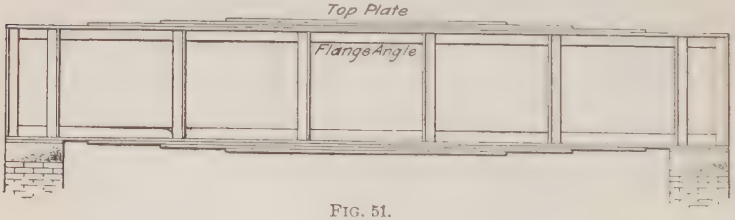


FIG. 51.

clear. In this figure, it is seen that the topmost plate of the top flange is the shortest, and extends over a small portion of the girder only, each successive plate under this one being longer than the one above it. The third plate from the top is the longest and extends nearly the full length of the girder, while the angles extend from end to end.

Where the beam is uniformly loaded, the following method may be used to obtain the theoretical length of each of the flange plates:

Commencing with the outside plate of the flange, find the

sum of all the net areas in square inches of the plates to and including the plate in question. Thus, in Fig. 52, if it be required to obtain the length of the third plate from the outside, find the sum of the areas of the first, second, and third plates. If the length of the second plate is required, then the sum of the areas of the first and second plates is to be taken. Divide the area so obtained by the net area of the whole flange in square inches, and multiply the square root of this quotient by the length of the girder in feet; the product will be the theoretical length of the plate in feet.

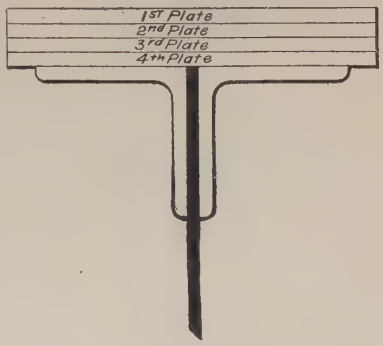


FIG. 52.

Having obtained the theoretical length of the plate, it is necessary to add from 12 to 16 inches to each end, in order that the plate in question may be carried sufficiently past the point of bending moment which governs the area of the flange at its ends to be securely riveted to the plates and angles making up the flange from there on to the abutment.

The method for determining the length of flange plates where the beam is uniformly loaded may be expressed by the formula

$$l = L\sqrt{\frac{a}{A}}, \qquad \textbf{(15.)}$$

in which l = theoretical length in feet of the plate in question;

L = the length of the girder in feet;

a = net area of all the plates to and including the plate in question, beginning with the outside plate;

A = total net area of the entire flange.

EXAMPLE.—In Fig. 53 is shown a section through the flange of a plate girder the span of which is 60 feet. What is the theoretical length of each of the three flange plates?

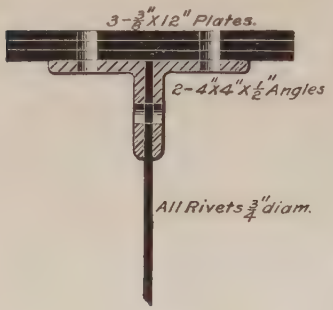


FIG. 53.

SOLUTION.—Area of a $4'' \times 4'' \times \frac{1}{2}''$ angle according to the table "Areas of Angles," is 3.75 square inches. The area of each plate is $\frac{3}{8} \times 12 = 4.5$ square inches. The diameter to be deducted for the rivet holes is $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ inch.

The area cut out by a $\frac{7}{8}$-inch hole through a $\frac{3}{8}$-inch plate is $.875 \times .375 = .328$ square inch. Then, as there are two rivet holes in each plate, its net area is 4.5 sq. in. $-$ (.328 sq. in. $\times 2$) $= 3.844$ square inches.

The net area of the angles is (3.75 sq. in. $\times 2$) $-$ (.4375 sq. in. $\times 4$) $= 5.75$ square inches.

The net area of the flange section is, therefore,

3 plates 3.844×3	=	11.532 sq. in.
2 angles	=	5.75 sq. in.
Total, . .		17.282 sq. in.

Now calculate the length of the outside plate. Substituting in the formula, we have

$$l = 60\sqrt{\frac{3.844}{17.282}} = 28.29 \text{ feet.}$$

By substituting the proper values, the theoretical length of the second or middle plate is

$$l = 60\sqrt{\frac{7.688}{17.282}} = 40.0 \text{ feet.}$$

The length of the third or last plate in the flange, that is, the one next to the flange angles, is next to be calculated, though some engineers prefer to run this the entire length of the girder, as it stiffens the girder laterally and assists in preventing any tendency towards side deflection. The theoretical length of this plate is

$$l = 60\sqrt{\frac{11.532}{17.282}} = 49.12 \text{ feet.}$$

45. Graphical Method of Determining Length of Flange Plates.—The graphical method for determining the theoretical length of flange plates in built-up girders is

more convenient than the analytical method previously given. In order to explain this method, we will assume a section through the flange of a plate girder, and find the lengths of the several flange plates.

Fig. 54 shows a section through the flange of a girder, built up of four $\frac{1}{2}'' \times 14''$ flange plates, the span of the girder being 90 feet. It will be noticed that there are two rows of rivets in the flange, and two rows in the vertical leg of the angles, but as the latter are staggered, there will be but one rivet hole to be deducted from the vertical leg of each angle.

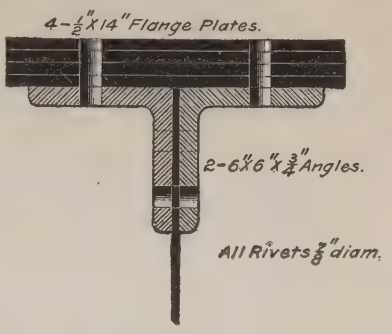


FIG. 54.

The sectional area of a $6'' \times 6'' \times \frac{3}{4}''$ angle is found by the table "Areas of Angles" to be 8.44 square inches; from this deduct $1\frac{1}{2}$ square inches, the area cut out by the two rivet holes, making the net area of each flange angle 6.94 square inches.

The sectional area of a $\frac{1}{2}'' \times 14''$ flange plate is 7 square inches, from which there is to be deducted 1 square inch for the sectional area cut out by the two rivet holes. Hence, the net area of one flange plate is 7 sq. in. − 1 sq. in. = 6 square inches.

The net area of the entire flange will, therefore, be

2–$6'' \times 6'' \times \frac{3}{4}''$ angles	=	13.88 sq. in.
4–$\frac{1}{2}'' \times 14''$ flange plates	=	24.00 sq. in.
Total, . . .		37.88 sq. in.

We will now proceed with the diagram, see Fig. 55. Since the load is uniformly distributed, the flange plates extend equally on each side of the center; consequently, the diagram for only one-half of the girder will be drawn. Draw, to any scale, a horizontal line *a b*, Fig. 55, equal to one-half of the span; divide this line into any number of

equal parts (in the figure, twelve parts have been used). Upwards from the points of division, draw indefinite

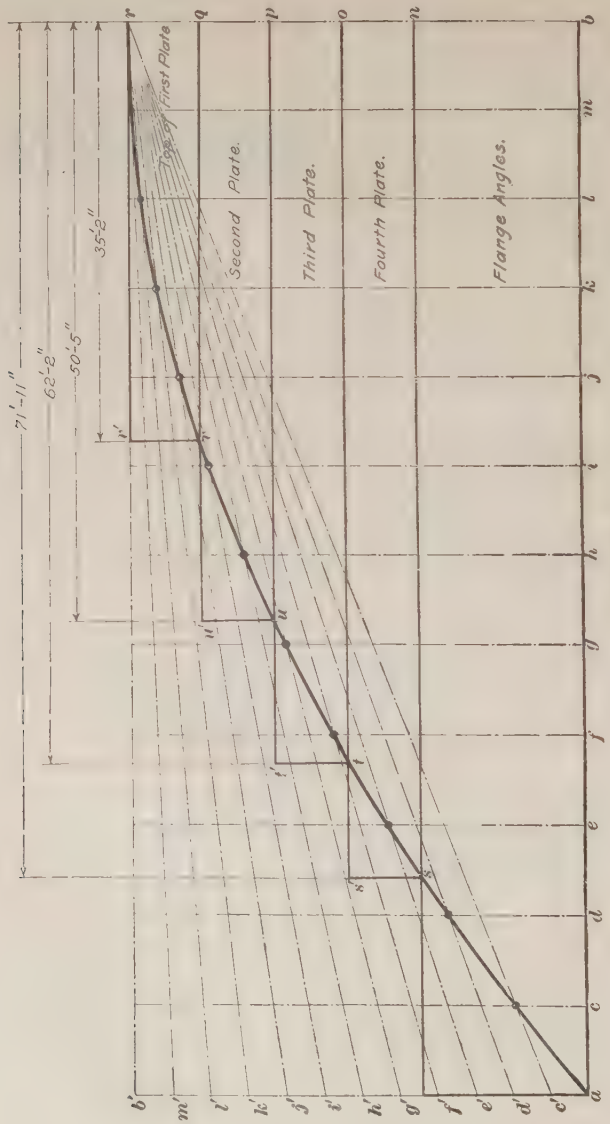


FIG. 55.

perpendicular lines. On the perpendicular from b, lay off to some scale a distance which represents the entire net section of the flange, thus locating the point r. For example, the net area of the flange in this case is 37.88 square inches; letting $\frac{1}{16}$ inch represent 1 square inch, the distance $b\,r$ must be $37.88 \times \frac{1}{16} = 2.37$ inches, nearly.

Lay off to the same scale on the line $b\,r$ a distance $b\,n$, which represents 13.88 square inches, the net area of the two flange angles; also the distances $n\,o$, $o\,p$, $p\,q$, and $q\,r$, each representing 6 square inches, the net area of each of the flange plates. From the point r, draw a horizontal line cutting the vertical line erected at a, thus locating the point b'. Divide the vertical line $a\,b'$ into the same number of equal parts as the line $a\,b$, thus locating the points c', d', e', f', etc.; and from these points, draw the lines $c'\,r$, $d'\,r$, etc. Draw the curve $a\,s\,v\,r$ through the points where the vertical lines from c, d, e, etc. intersect the corresponding lines $c'\,r$, $d'\,r$, $e'\,r$, etc. Now from the points q, p, o, and n, draw horizontal lines as shown, cutting the curve in the points s, t, u, and v, and from each of these points of intersection, draw a perpendicular, extending it until it intersects the horizontal line next above. The rectangles $v'\,r\,q\,v$, $u'\,q\,p\,u$, $t'\,p\,o\,t$, etc., thus formed, represent the flange plates and angles.

Now in order to obtain the theoretical length of any flange plate, measure the length of the corresponding rectangle by the scale to which the half span was laid out on the line $a\,b$; this length multiplied by 2 gives the length of the plate in question. For example, if it is desired to obtain the length of the first, or top, plate, with the scale to which the half span was laid out, measure the length of the line $v'\,r$; as only one-half of the diagram is drawn, this gives one-half of the length of the top, or first, plate, and by doubling this the entire theoretical length of the plate in question is obtained. The length of the other plates may be determined in like manner. It is to be borne in mind that to the theoretical length given by the diagram it is necessary to add a length of about 1 foot at each end of the

plate. From the diagram, the theoretical lengths of the flange plates of the girder shown in Fig. 54 are found to be 35 ft. 2 in., 50 ft. 5 in., 62 ft. 2 in., and 71 ft. 11 in., respectively.

The student will find upon checking these lengths by formula **15** that they are approximately correct. Fig. 56,

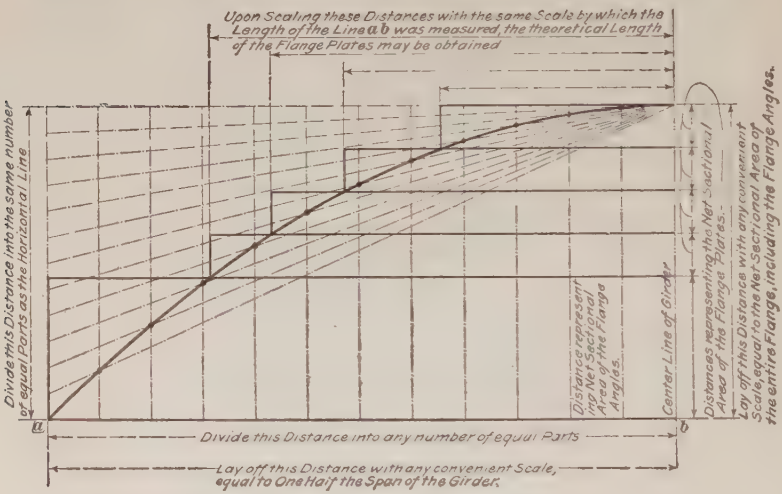

Upon Scaling these Distances with the same Scale by which the Length of the Line a b was measured, the theoretical Length of the Flange Plates may be obtained
Divide this Distance into the same number of equal Parts as the Horizontal Line
Distance representing the Net Sectional Area of the Flange Angles.
Center Line of Girder
Distances representing the Net Sectional Area of the Flange Plates.
Lay off this Distance with any convenient Scale, equal to the Net Sectional Area of the entire Flange, including the Flange Angles.
a
b
Divide this Distance into any number of equal Parts
Lay off this Distance with any convenient Scale, equal to One Half the Span of the Girder.

FIG. 56.

in which all the different steps are indicated, is presented in order that the student may always have a guide for laying out this diagram.

46. Application of the Graphical Method to Girders With Concentrated Loads.—The graphical method for determining the theoretical length of flange plates when the girder is loaded with concentrated loads, which is similar to that given for a uniformly loaded girder, will be illustrated by constructing a diagram for the lengths of the four flange plates required for a girder with a span of 80 feet and a depth of 6 feet, carrying a concentrated load of 185,000 pounds at 30 feet from one end.

The bending moment on the girder may be calculated by

the method given in *Architectural Engineering*, § 5, or by the formula

$$M = W \times \frac{a\,b}{L}, \qquad \textbf{(16.)}$$

in which M = bending moment;
W = load on the girder;
L = span;
a = distance that the load is located from one abutment;
b = distance it is located from the other.

In Fig. 57 it is seen that the load is located at the point f 30 feet from R_1, and 50 feet from R_2. Substituting these values in formula **16,** the bending moment is found to be

$$M = 185{,}000 \times \frac{30 \times 50}{80} = 3{,}468{,}750 \text{ foot-pounds.}$$

From this bending moment the required net flange area, assuming a safe unit stress of 15,000 pounds per square inch, is found to be, approximately, 38 square inches, which is provided by the use of four $\frac{1}{2}'' \times 14''$ plates and two $6'' \times 6'' \times \frac{3}{4}''$ angles. The flange therefore has the section shown in Fig. 54.

To construct the diagram, which is shown in Fig. 57, draw the base line $c\,d$ to any scale equal to the span of the girder in feet. (In this case, owing to the fact that the load is not symmetrically placed, the center line of the girder will not divide the lengths of the plates in halves, and it will be necessary to draw the entire diagram.) Locate the point of application of the concentrated load at 30 feet from R_1, and draw the perpendicular line $e\,f$. On this line lay off to any scale a distance which represents the bending moment. For example, in this case, a scale has been used on which 50,000 foot-pounds is represented by $\frac{1}{32}$ of an inch; the distance to be laid off is, therefore, $\frac{3\,4\,6\,8\,7\,5\,0}{5\,0\,0\,0\,0} \times \frac{1}{32} = 2.168$ inches. This locates the point g, which is then connected by straight lines to the points c and d. Draw vertical lines from the points c and d, until they meet a horizontal line through the point g, at q and r.

In the flange section, the angles have a combined net area of 13.88 square inches, and each flange plate a net area of

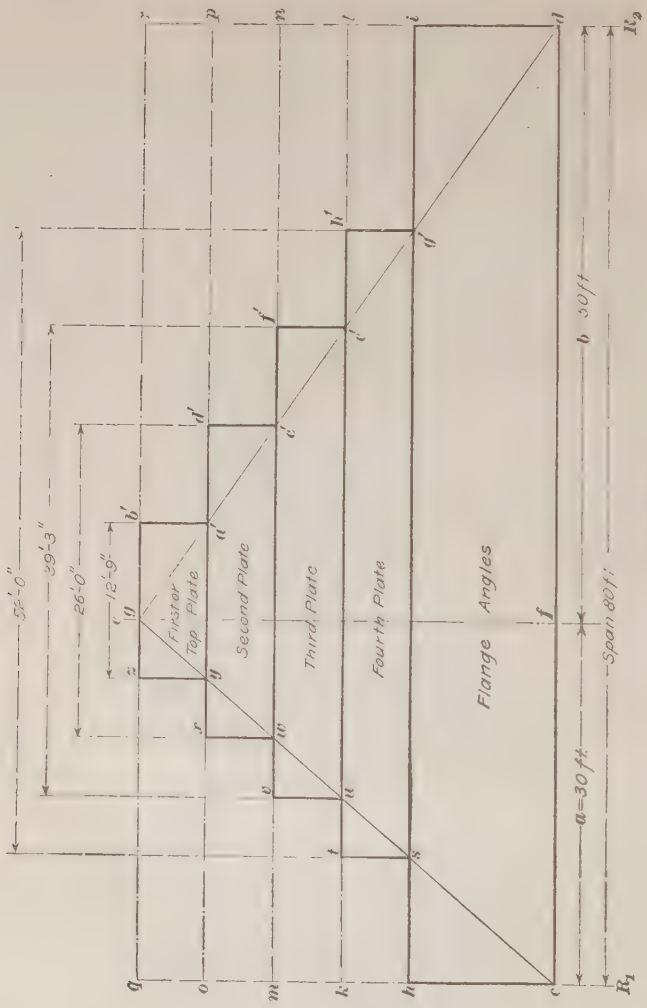


Fig. 56.

6 square inches. The combined net sectional area of the flange is 37.88 square inches.

Place the zero mark of any convenient scale at the point

f, and slant the scale until the marking on the scale which represents 37.88 square inches, the net area of the flange, falls upon the horizontal line $q\,r$. Thus, in this case, a $\frac{1}{16}$-inch scale has been used, and the zero mark is placed at f, while the mark on the scale representing the division 37.88 is placed on the horizontal line $q\,r$. With the scale still in this position, begin at the zero mark and lay off a distance 13.88 to represent the net sectional area of the two angles; then lay off four distances, each equal to 6, to represent the net sectional area of each of the flange plates.

Through the points thus found, draw the horizontal lines $h\,i$, $k\,l$, $m\,n$, and $o\,p$. At the points $s\,u\,w\,y$, etc., where these horizontal lines cut the oblique lines $g\,c$ and $g\,d$, draw the perpendicular lines $s\,t$, $u\,v$, $w\,x$, $y\,z$, $a'\,b'$, etc., extending each until it meets the next horizontal line above. Then the rectangles enclosed by the horizontal and vertical lines, shown heavy in the diagram, represent the cover-plates and flange angles. By measuring the length of these rectangles with the scale to which the span was laid off on the line $c\,d$, the theoretical length of the plates may be determined.

In this case the length of the angles is equal to the span, 80 feet, as marked on the diagram. The length of the first, or top, plate measures 12 ft. 9 in., of the second plate 26 ft. 0 in., of the third 39 ft. 3 in., and of the fourth, or last, 52 ft. 0 in. It may be that when the student lays out the diagram for himself, he will obtain results which will vary slightly from those given. However, a variation of a fractional part of an inch in the length of a flange plate on a girder need not be considered.

47. Graphical Diagram for Several Concentrated Loads.—In order to illustrate this method still further, a more complicated problem, in which there are three concentrated loads, will now be presented.

Assume that a girder having a span of 80 feet with a depth of 6 feet is loaded as shown in Fig. 58. What flange area is required, provided a safe unit fiber stress of 16,000

pounds per square inch is used, and of what should the flange be constructed, also what will be the length of the several flange plates?

The reactions at R_1 and R_2 are found to be 142,500 and 117,500 pounds, respectively. It will first be necessary to

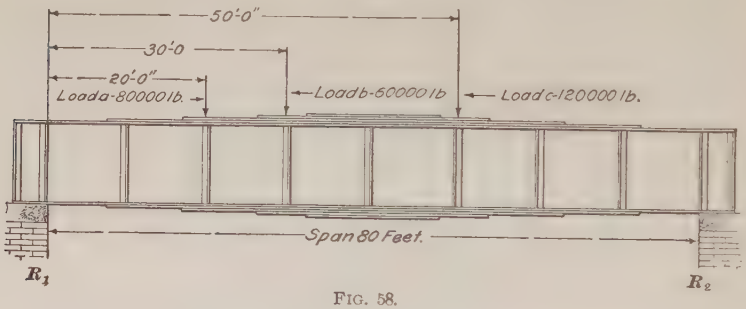


FIG. 58.

calculate the bending moment in foot-pounds at the points *a*, *b*, and *c*, where the loads are concentrated.

The bending moment at *a* is $142,500 \times 20 = 2,850,000$ foot-pounds; at *b* the bending moment is $(142,500 \times 30) - (80,000 \times 10) = 3,475,000$ foot-pounds; and the bending moment at *c* is $(142,500 \times 50) - [(80,000 \times 30) + (60,000 \times 20)] = 3,525,000$ foot-pounds.

From the above it is seen that the greatest bending moment is under the load located at *c*, and its magnitude is 3,525,000 foot-pounds.

From this the flange area required may be calculated by applying formula **14**, as follows:

$$A = \frac{3,525,000}{6 \times 16,000} = 36.71 \text{ square inches.}$$

We will now select a flange section which will have the required net sectional area; referring to the section shown in Fig. 54, the area is found to be 37.88 square inches; this flange will therefore satisfy the requirements.

Having determined the bending moments and the greatest net flange area, begin the diagram shown in Fig. 59 by drawing to any scale the horizontal line *d e* equal in length to the span of the girder; with the same scale locate the

points of application a, b, and c of the concentrated loads. Upwards from the points d, a, b, c, and e, draw indefinite

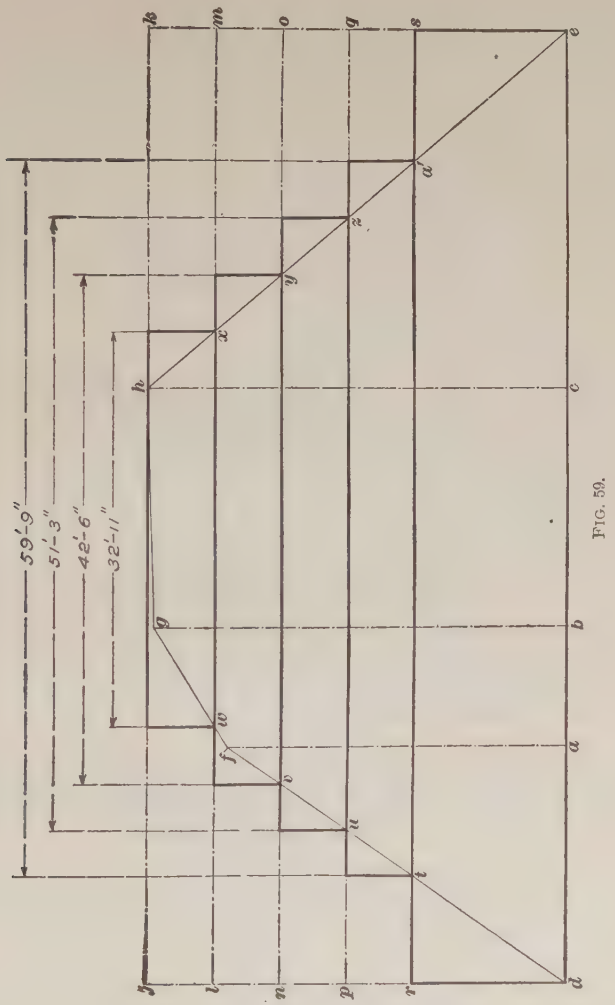


FIG. 59.

perpendicular lines, and on the perpendiculars from a, b, and c, lay off to some convenient scale distances af, bg, and

c h, which represent the respective bending moments at these points.

For example, the bending moment at *a* is 2,850,000 foot-pounds; at *b* it is 3,475,000 foot-pounds; and at *c* it is 3,525,000 foot-pounds; therefore, assuming a scale on which each $\frac{1}{32}$ of an inch represents 50,000 foot-pounds of bending moment, the respective bending moments at the points *a*, *b*, and *c* are represented by lengths *a f* of 57 thirty-seconds, *b g* of $69\frac{1}{2}$ thirty-seconds, and *c h* of $70\frac{1}{2}$ thirty-seconds.

Draw straight lines connecting the points *d*, *f*, *g*, *h*, and *e*.

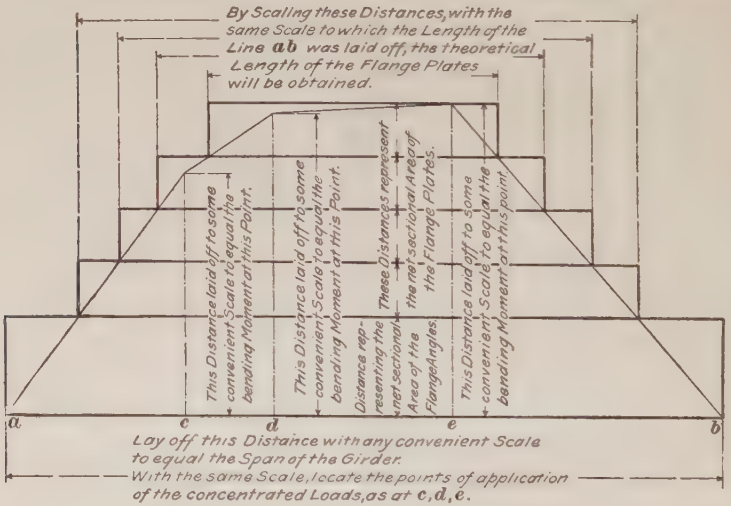

By Scaling these Distances, with the same Scale to which the Length of the Line ab was laid off, the theoretical Length of the Flange Plates will be obtained.
This Distance laid off to some convenient Scale to equal the bending Moment at this Point.
This Distance laid off to some convenient Scale to equal the bending Moment at this Point.
Distance representing the net sectional Area of the Flange Angles.
These Distances represent the net sectional Area of the Flange Plates.
This Distance laid off to some convenient Scale to equal the bending Moment at this point.
a
c
d
e
b
Lay off this Distance with any convenient Scale to equal the Span of the Girder.
With the same Scale, locate the points of application of the concentrated Loads, as at c, d, e.

FIG. 60.

Through the highest point *h*, representing the greatest bending moment, draw the horizontal line *j k*.

The net area of the flange being 37.88 square inches, place the zero mark of any convenient scale on the line *d e*, and slant the scale until the mark which represents 37.88 falls on the line *j k*.

Starting from the zero mark on the scale, lay off a distance which represents he net area of the two flange angles, in this case 13.88 square inches, then divide the remaining distance into equal parts, each of which represents 6 square inches, the net sectional area of the several flange plates.

Through the points just found, draw the horizontal lines *l m*, *n o*, *p q*, and *r s*. Where these horizontal lines intersect the oblique lines at the points *t*, *u*, *v*, *w*, *x*, etc., draw vertical lines until they intersect the next horizontal line above. Then draw in with heavy lines the rectangles representing the flange plates and flange angles.

The theoretical length of the flange plates may now be determined by measuring with the scale to which the span was laid off on the line *d e*.

The length of the top, or first, plate in this case is found to be 32 ft. 11 in.; of the second plate, 42 ft. 6 in.; of the third plate, 51 ft. 3 in.; and of the fourth, or last, plate, 59 ft. 9 in.

In Fig. 60 is shown a diagram which will serve as a general rule for determining the length of the several flange plates of a girder loaded with several concentrated loads.

48. Diagram for a Combination of Concentrated Loads With a Uniformly Distributed Load.—There is still another condition of girder loading which is frequently encountered in practical work, and in which it is necessary to determine the length of the several flange plates by the

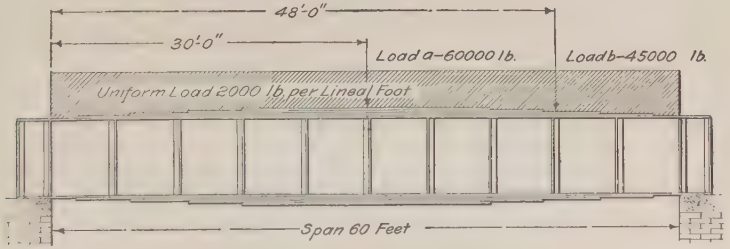


FIG. 61.

graphical method; this condition is produced by a combination of a uniformly distributed load, with several concentrated loads located at different points along the girder.

In order to explain the method for obtaining the length of the flange plates in a girder loaded in this manner, the following problem will be assumed and the diagram will be constructed as was done in previous cases.

Assume the girder to be loaded, as shown in Fig. 61, with a uniformly distributed load, and the two concentrated loads. The flange section shown in Fig. 62 is sufficient to resist the bending moments due to these loads. It is required to determine by the graphical method the theoretical lengths of the several cover-plates making up the flange section.

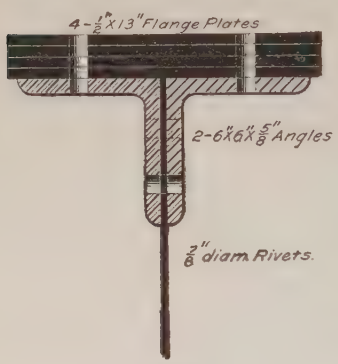


FIG. 62.

Before starting to draw the diagram shown in Fig. 63, it is necessary to make the calculations for the following: The greatest bending moment; the maximum bending moment due to the uniformly distributed load; and the bending moment under each of the concentrated loads, neglecting the uniformly distributed load. These bending moments should be expressed in foot-pounds.

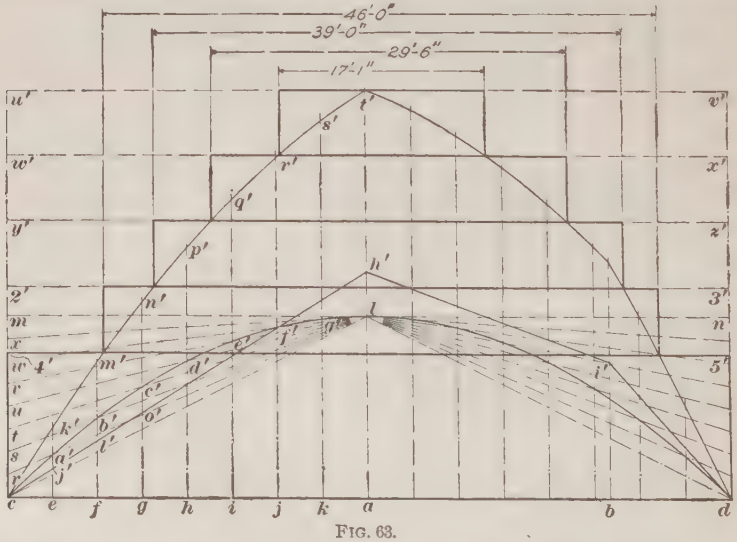


FIG. 63.

The flange area required to successfully resist each of these bending moments should also be calculated.

The calculations in this case have been made in the usual manner, and the results are as follows:

Greatest bending moment..... = 2,070,000 ft.-lb.
Bending moment due to a uniform load = 900,000 ft.-lb.
Bending moment under concentrated load a..... = 1,170,000 ft.-lb.
(Considering the concentrated loads only.)
Bending moment under concentrated load b..... = 792,000 ft.-lb.
(Considering the concentrated loads only.)

Since the depth of the girder is 4 feet, if a unit fiber stress of 15,000 pounds is used; the flange area required to resist the greatest bending moment is

$$A = \frac{2,070,000}{4 \times 15,000} = 34.5 \text{ square inches.}$$

The flange area required to resist the bending moment due to the uniform load is

$$A = \frac{900,000}{4 \times 15,000} = 15 \text{ square inches.}$$

The flange area required to resist the bending moment at the point on the girder where the concentrated load a is situated, considering the concentrated loads only, is

$$A = \frac{1,170,000}{4 \times 15,000} = 19\tfrac{1}{2} \text{ square inches.}$$

The flange area required to resist the bending moment at the point on the girder under the concentrated load b is

$$A = \frac{792,000}{4 \times 15,000} = 13.2 \text{ square inches,}$$

considering as before only the concentrated loads.

As the loads on the girder are not symmetrically placed with regard to the center, it will be necessary to draw the complete diagram. Begin the diagram by drawing to any convenient scale the horizontal line $c\,d$, Fig. 63, equal in length to the span of the girder, and locate the points of application of the concentrated loads at a and b; upwards from the points c, a, b, and d, draw indefinite vertical lines.

Now in accordance with the method explained in Art. **45,** make the construction to determine the curved line representing the bending moment due to the uniform load, as follows:

At the center of the girder draw a vertical line (in this case the center of the girder is found to be at the point where the load *a* is concentrated); divide half the span into any number of equal parts, as at *e*, *f*, *g*, *h*, *i*, etc., and from the points so obtained draw perpendiculars. Lay off on the vertical line passing through the center a distance *a l*, which may represent either the greatest bending moment at this point due to the uniform load, or the flange area required to resist this bending moment, as they are proportional. In this case the net area required in the flange will be used; hence, as the area of the flange required for the uniform load is 15 square inches, if $\frac{1}{16}$ of an inch is assumed to represent 1 square inch of flange area, the distance *a l* will be $\frac{15}{16}$ inch.

Through the point *l*, draw the horizontal line *m n*. Divide the distance *m c* into the number of equal parts that the half of the span was divided into, and from the points *r*, *s*, *t*, *u*, *v*, etc. thus obtained, draw converging lines to the point *l*; where these oblique lines intersect the vertical lines, mark the points *a'*, *b'*, *c'*, *d'*, etc., and through these points draw the curve *c l*. Draw the other half of the curve *l d* in the same manner, thus completing the diagram for the uniform load.

Now draw the diagram for the concentrated loads. On the vertical line *a l*, extended, lay off the distance *a h'*, equal to the net flange area required to support the concentrated load *a*, and on the perpendicular line erected at *b*, lay off the distance *b i'* equal to the flange area required at the point *b* to support the concentrated loads.

It must be remembered that the same scale is to be used as that with which the flange area required for the uniform load diagram was laid off; also that if the vertical distances are laid off to represent the bending moment in the one case, the bending moment should be used in the other, while if the flange area required is used in the one case, it

is self-evident that it should also be used in the other. The student will understand the importance of the above when he proceeds further with the diagram.

Having located the points h' and i', connect with straight lines the points c, h', i', and d, as was done in the previous similar diagrams of concentrated loads. This completes the diagram for the concentrated loads.

The next step in the process is to measure the distance $e\,j'$ with a pair of dividers, and from the point a' on the curve representing the uniform load, lay off on the vertical line the distance $a'\,k'$ equal to $e\,j'$, also lay off from the point b' the distance $b'\,m'$ equal to $f\,l'$, and from the point c' lay off on the vertical line the distance $c'\,n'$ equal to $g\,o'$; continue in this manner through the entire diagram. Having in this manner determined the points k', m', n', p', etc. through the entire diagram, draw in the curve $c\,k'\,m'\,n'$, etc. The point t' is the highest point in the diagram, and its distance from the horizontal line $c\,d$ represents the entire flange area required in the girder to resist the greatest bending due to both the uniform and the concentrated loads.

Through the point t' draw the horizontal line $u'\,v'$, and lay off between the horizontal lines $u'\,v'$ and $c\,d$ the several distances representing the net area of the flange plates and flange angles. Through the points of these divisions, draw the horizontal lines $w'\,x'$, $y'\,z'$, $2'$–$3'$, and $4'$–$5'$. Where these horizontal lines intersect the curved line representing the net area required for the combined uniform and concentrated loads, draw short vertical lines to the next horizontal line above; draw with heavy lines the rectangles representing the flange plates and flange angles; scale the length of the flange plates with the scale to which the span $c\,d$ was laid off, and the theoretical length of the flange plates will be found.

In the girder under consideration, the theoretical length of the top, or first, plate is found to be 17 ft. 1 in.; the length of the second plate, 29 ft. 6 in.; the length of the third plate, 39 ft. 0 in.; and the length of the last, or fourth, plate is 46 ft. 0 in.

RIVET SPACING.

49. Rivets in the End Angles or Stiffeners Over the Abutment.—First, the allowable safe load upon the rivet should be determined. Whether the double shear of the rivet, or the bearing value of the plate around the rivet hole, is greatest, should be found as previously explained. Having obtained the safe allowable load for each rivet, a sufficient number should be placed in the end angles or stiffeners to take care of the entire shear at that point, which on a simple girder is equal to the end reaction at the abutment. For example, let the end reaction of a girder be 100,000 pounds; $\frac{7}{8}$-inch rivets are used and the thickness of the web is $\frac{3}{8}$ inch. With an allowable tensile stress of 12,000 pounds per square inch, the web-bearing value of a $\frac{3}{8}$-inch plate is 6,825 pounds, which is the allowable load on the rivet. The number of rivets required in the two pairs of end angles is, therefore, $100{,}000 \div 6{,}825 = 14.6$, say 16, or 8 rivets in each pair.

50. Rivets in the Stiffeners Between the Abutments.—If possible the rivets in the intermediate stiffeners are usually spaced the same as in the end stiffeners. It is hardly possible to make any calculation of practical value in regard to the number and spacing of these rivets, and in fact no calculation is required; a practical rule is that the pitch of these rivets should never exceed 6 inches, nor should it exceed 16 times the thickness of the leg of the angle.

51. Rivets Connecting Flange Angles With the Web.—It will readily be seen that when a plate girder is loaded, the tendency of the flanges and angles is to slide horizontally past the web; this tendency to slide induces a horizontal flange stress. The rivets connecting the angles to the web resist this tendency, and there must be a sufficient number of rivets to do it safely.

The stress which at any point is transmitted horizontally from the web to the flange is equal to the increment of the flange stress at that point. This increment is found by dividing the maximum shear at any point by the depth of the girder.

For example, assume a girder of 40 feet span, as shown in Fig. 64, with a depth of 4 feet, and a uniformly distributed load of 200,000 pounds. The shearing stress in the girder at the left reaction, or point a, is equal to R_1, in this case 100,000 pounds. At b, 4 feet from R_1, the vertical shear in the girder is $100,000 - (5,000 \times 4) = 80,000$ pounds; at c, 8 feet from R_1, the vertical shear is $100,000 - (5,000 \times 8) = 60,000$ pounds; at d it is $100,000 - (5,000 \times 12) = 40,000$ pounds; at e the shear is 20,000 pounds; and at f the shear is zero.

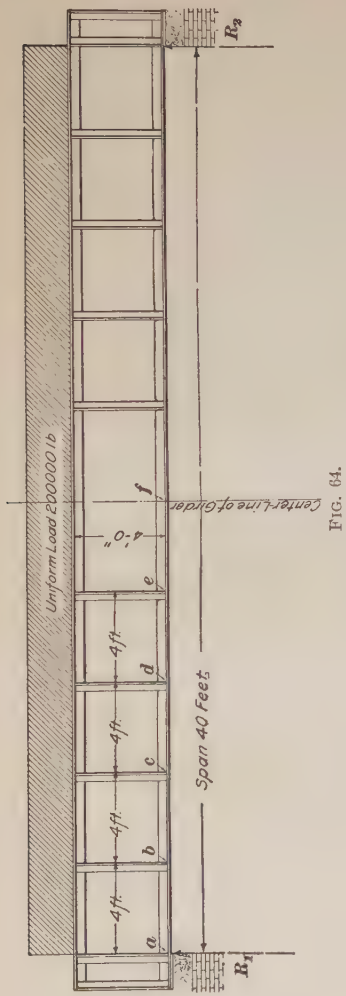


FIG. 64.

From the above results, the rate of increase, per inch of length, of the horizontal stress in the flange at the several points a, b, c, d, e, and f may be obtained by dividing the shear at those points by the depth of the girder in inches. Thus, at the end a of the girder the increase in the horizontal flange stress is $\frac{100,000}{4 \times 12} = 2,083$ pounds per inch of run; at b, $\frac{80,000}{4 \times 12} = 1,667$ pounds per inch of run; at c, $\frac{60,000}{4 \times 12} = 1,250$ pounds per inch of run; at d, $\frac{40,000}{4 \times 12} = 833$ pounds per inch of run; and at e, $\frac{20,000}{4 \times 12} = 417$ pounds per inch of run.

If $\frac{7}{8}$-inch rivets are used, the allowable safe load for each rivet, based on a fiber stress of 12,000 pounds per square inch, with web bearing in a $\frac{3}{8}$-inch plate, is 6,825 pounds. Then at the end, where the increase in stress is 2,083 pounds per inch of run, the rivets must not be spaced farther apart than $6{,}825 \div 2{,}083 = 3.27$ inches, from center to center. At *b*, the maximum allowable pitch of the rivets is $6{,}825 \div 1{,}667 = 4.09$ inches; at *c*, the pitch may be $6{,}825 \div 1{,}250 = 5.46$, or about $5\frac{1}{2}$, inches; and at *d*, $6{,}825 \div 833 = 8.19$ inches. Since, for practical reasons, the rivets in the vertical leg of the flange are spaced the same in both the upper and lower chords, and, since the greatest allowable pitch of rivets in a compression member is 6 inches, it is evident that it is needless to carry the calculation further.

In accordance with the above calculation, the pitch of the rivets between *a* and *b* should be $3\frac{1}{4}$ inches; between *b* and *c* the pitch should be about 4 inches; while between *c* and *d* the theoretical pitch is $5\frac{1}{2}$ inches; since the theoretical pitch at *d* is more than 6 inches, which, for practical reasons, is not allowable, all the rivets between *d* and the center of the girder should be spaced 6 inches from center to center.

52. Effect of Vertical Stress.—Sometimes the vertical as well as the horizontal stress in the flange is taken into account in spacing the rivets, in which case, the resultant of the two stresses is the stress that must be provided for. The vertical stress is due directly to the load resting upon the flange of the girder, which, through the rivets, is transmitted to the web-plate.

In the plate girder shown in Fig. 64, the increase in the horizontal flange stress at the end is, as previously calculated, 2,083 pounds per inch of run; the load upon the girder being uniformly distributed, the vertical stress on the flange, per lineal inch, is equal to the entire load on the girder divided by the span of the girder in inches; it is, therefore, $200{,}000 \div (40 \times 12) = 416$ pounds per inch of run.

The total stress to be resisted by the rivets is, therefore,

equal to the resultant of 2,083 pounds—due to the increase in the horizontal stress on the flange—and the vertical stress of 416 pounds; this resultant is $\sqrt{2{,}083^2 + 416^2} = 2{,}124$ pounds per inch of run. The pitch of the rivets at the end of the girder would then be $6{,}825 \div 2{,}124 = 3.21$, approximately $3\frac{1}{4}$, inches.

At *b*, 4 feet from the end of the girder, the horizontal increment of stress on the flange, as previously calculated, is 1,667 pounds per inch of run, while the vertical stress remains the same; the combined action of these two forces produces a resultant stress upon the rivets of $\sqrt{1{,}667^2 + 416^2} = 1{,}717$ pounds per inch of run, and this divided by the value of one rivet gives $6{,}825 \div 1{,}717 = 3.97$, nearly 4, inches. Similar calculations may be made for each panel point to the center of the girder, or until the pitch exceeds the allowable limit of 6 inches.

The above results show that the values of the pitch in which the vertical stress due to the load is taken into account, are nearly the same as those first obtained; the effect of the vertical stress has, therefore, little influence on the pitch of the rivets, and it is hardly necessary to go into such refinement in the design of an ordinary plate girder.

53. Rivets Spaced According to the Stress Produced by the Bending Moment.—The rivets which connect the flange angles with the web-plate may also be spaced according to the stresses produced on the flanges by the bending moment.

The horizontal stress on the flanges diminishes either way from the point of greatest bending moment towards the end reactions, where it becomes zero, and for any point this stress may be calculated by the application of the principle of moments.

If the bending moment is obtained at any panel point and is divided by the depth of the girder, the stress on the flange at that point will be obtained; and, if this stress is divided by the allowable load upon one rivet, the number of

rivets required between that point and the end reaction will be obtained.

For example, in the girder used in the previous illustration (Fig. 64), the span being 40 feet and the load 200,000 pounds, the bending moment at the center is equal to $\frac{WL}{8} = \frac{200{,}000 \times 40}{8} = 1{,}000{,}000$ foot-pounds; then the depth of the girder being 4 feet, the flange stress at this point is $1{,}000{,}000 \div 4 = 250{,}000$ pounds. The allowable load upon each rivet being 6,825 pounds, the number of rivets between the center and the end reaction is $250{,}000 \div 6{,}825 = 37$ rivets, approximately.

Now, although the number of rivets between the end reaction and the center of the girder has been obtained, the pitch of these rivets is still unknown. The rate at which the horizontal stress in the flange increases varies, being greatest at the ends and least under the position of maximum bending moment, it follows that the rivets should be spaced nearer together at the ends, with an increase in the spacing towards the point of greatest bending moment.

In practical work the rivet spacing is seldom varied in any one panel; if, however, the flange stress is obtained at each of the stiffeners *b*, *c*, *d*, *e*, and *f*, Fig. 64, the number of rivets required between each of these points and the end reaction may be obtained; by finding the difference between these numbers for any two consecutive stiffeners, the number of rivets required in the panel between those stiffeners is arrived at. For example, the stresses on the flange at each of the stiffeners of the girder shown in Fig. 64 are as follows:

	Bending Moment. Foot-Pounds.		Depth of Girder. Feet.		Flange Stress. Pounds.
At *b*,	360,000	÷	4	=	90,000
At *c*,	640,000	÷	4	=	160,000
At *d*,	840,000	÷	4	=	210,000
At *e*,	960,000	÷	4	=	240,000
At *f*,	1,000,000	÷	4	=	250,000

The approximate number of rivets between each stiffener and the reaction R_1 is as follows:

Between b and R_1, $90{,}000 \div 6{,}825 = 13$ rivets.
Between c and R_1, $160{,}000 \div 6{,}825 = 23$ rivets.
Between d and R_1, $210{,}000 \div 6{,}825 = 31$ rivets.
Between e and R_1, $240{,}000 \div 6{,}825 = 35$ rivets.
Between f and R_1, $250{,}000 \div 6{,}825 = 36$ rivets.

Then the number of rivets that are required is:

Between b and a, $13 - 0 = 13$ rivets.
Between c and b, $23 - 13 = 10$ rivets.
Between d and c, $31 - 23 = 8$ rivets.
Between e and d, $35 - 31 = 4$ rivets.
Between f and e, $36 - 35 = 1$ rivet.

Consequently, the pitch between the stiffeners will be as follows:

Between b and a, $48 \div 13 = 3.69$ inches.
Between c and b, $48 \div 10 = 4.80$ inches.
Between d and c, $48 \div 7 = 6.85$ inches.

It is needless to continue the calculation further, because between d and c the theoretical pitch exceeds 6 inches, the limit allowable for a compression member.

By the first method the pitch at each stiffener or panel point is determined, while by the second the average pitch between two consecutive panel points is obtained; in order to compare the two, we will reduce the results obtained by the first to the basis of the second. In the first method the pitch at the several stiffeners or panel points was found to be:

At $a = 3.27$ inches.
At $b = 4.09$ inches.
At $c = 5.46$ inches.
At $d = 8.19$ inches.

From these the average pitch between the several points would be:

Between a and b, $(3.27 + 4.09) \div 2 = 3.68$ inches.
Between b and c, $(4.09 + 5.46) \div 2 = 4.78$ inches.
Between c and d, $(5.46 + 8.19) \div 2 = 6.83$ inches.

These, on comparison, are found to be almost identical with the values 3.69, 4.80, and 6.85 inches obtained by the second method.

54. Rivets Spaced According to the Direct Vertical Shear.—This is a method much used in practical work, and will be found to give safe results, corresponding favorably with those obtained by the previous methods. The method is based on the assumption that at any point the horizontal shear between the flange angles and the web-plate is equal to the vertical shear on the girder; for example, the vertical shear at the end stiffener or point *a*, Fig. 64, is 100,000 pounds; then, according to this method, the shearing stress between the flange angles and the web-plate is 100,000 pounds, distributed over the space between the panel points *a* and *b*, and sufficient rivets should be placed between these points to safely sustain this shear.

The allowable web-bearing load on a $\frac{7}{8}$-inch rivet in a $\frac{3}{8}$-inch plate being 6,825 pounds, the number of rivets required between *a* and *b* is $100{,}000 \div 6{,}825 = 15$, approximately; the vertical shear at *b* is 80,000 pounds, and $80{,}000 \div 6{,}825 = 12$, approximately, the number of rivets to be used between *b* and *c*; the shear at *c* is 60,000 pounds, and $60{,}000 \div 6{,}825 = 9$, approximately, the number of rivets to be used between *c* and *d*; similarly, the number of rivets required between *d* and *e* is found to be 6. According to these results, the pitch of the rivets between *a* and *b* should be $48 \div 15 = 3.2$ inches; between *b* and *c*, $48 \div 12 = 4$ inches; between *c* and *d*, 5.33 inches; and from there on, 6 inches.

Rivet spacing in plate girders is governed so largely by practical considerations, that this method is to be recommended on account of its convenience. It gives safe results which agree closely with those obtained by the more cumbersome methods.

55. Rivet Spacing in the Flange Plates.—In spacing the rivets which bind the several flange plates together, a sufficient number of rivets, spaced from $2\frac{3}{4}$ to 3 inches on

centers, should be used at the ends of each plate to transmit the allowable stress in it to the members below. For the remainder of the plate, the rivets should have the greatest allowable pitch for a compression member; that is, 16 times the thickness of the thinnest outside plate, providing such a distance does not exceed 6 inches. To illustrate:

An intermediate flange plate in a certain girder is $\frac{3}{8}$ in. $\times$ 12 in., the sectional area thus being $4\frac{1}{2}$ square inches. From this area is to be deducted the section cut out by two $\frac{7}{8}$-inch rivet holes, $(1\times\frac{3}{8})\times 2 = \frac{3}{4}$ square inch; then the net area of the cover-plate is $4\frac{1}{2} - \frac{3}{4} = 3\frac{3}{4}$ square inches. Assuming that a safe fiber stress of 15,000 pounds was used in calculating the strength of the girder, the safe strength of the cover-plate is $3\frac{3}{4}\times 15{,}000 = 56{,}250$ pounds. Now the safe load on a $\frac{7}{8}$-inch rivet depends, in this position, on the ordinary bearing value of a $\frac{3}{8}$-inch plate, which, calculated on the basis of a fiber stress of 12,000 pounds, is 5,119 pounds. Hence, the number of rivets required in the end of this cover-plate is $56{,}250 \div 5{,}119 = 10.9$, say 12; this gives 6 on each side of the web, and they should be spaced about 3 inches from center to center. The remaining rivets in this plate may have the greatest allowable pitch until the next cover-plate is reached.

PRACTICAL PROBLEMS.

56. In order to illustrate the application of the rules and formulas given above, the following practical problem will be assumed and worked out:

The floor of a building used for light manufacturing purposes is to be supported by three plate girders, as shown at *a*, *a*, *a*, Fig. 65. The floor is composed of 1-inch yellow-pine flooring laid upon $3'' \times 12''$ hemlock joists, spaced 16-inch centers; these joists are to carry a plastered ceiling on the under side. The live load upon the floor will be 80 pounds per square foot. The girder itself is to extend below the surface of the ceiling and is to be painted, a detail of the construction being shown in Fig. 66.

The total load upon each square foot of floor surface is as follows:

Live load, per square foot of floor surface..	80 pounds.
Lath and plaster, per square foot of floor surface	8 pounds.
1-inch yellow-pine flooring, per square foot of floor surface.....	4 pounds.
Hemlock joist flooring, per square foot of floor surface.....	6 pounds.
Girder (assumed), per square foot of floor surface.....	8 pounds.
Total.....	106 pounds.

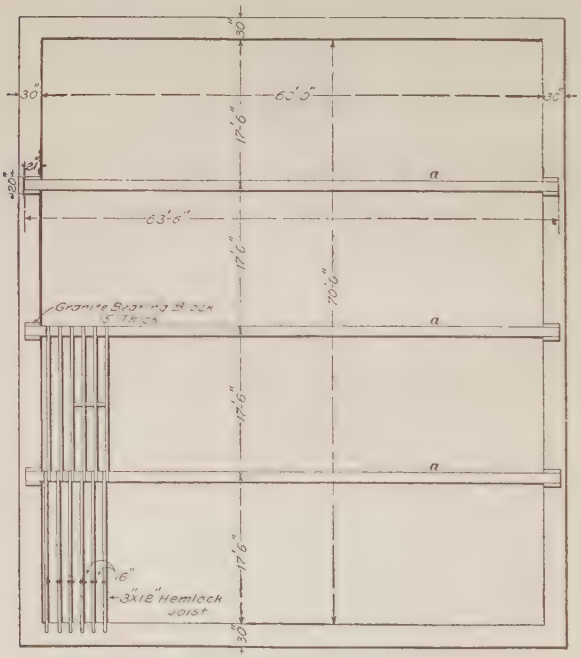


FIG. 65.

The floor area to be supported by one girder is $60 \times 17.5 = 1{,}050$ square feet; and the total uniformly distributed load upon the girder is $1{,}050 \times 106 = 111{,}300$ pounds.

The greatest bending moment on the girder is

$$M = \frac{WL}{8} = \frac{111{,}300 \times 60}{8} = 834{,}750 \text{ foot-pounds.}$$

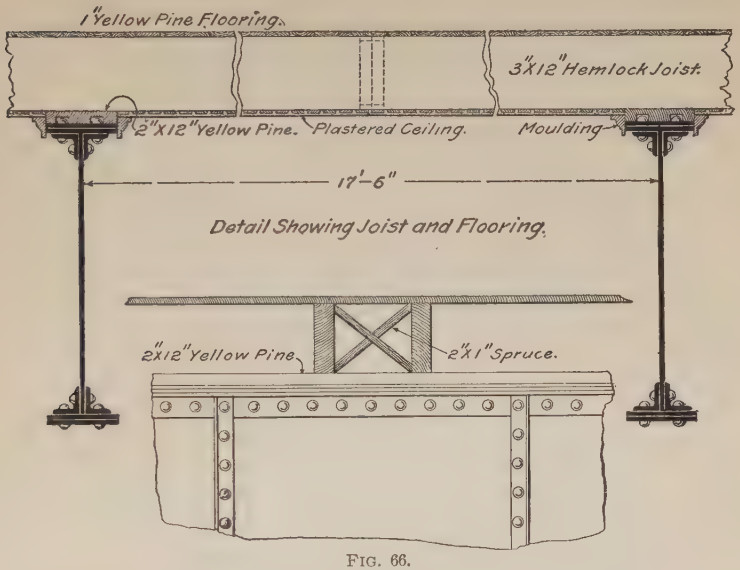


FIG. 66.

The depth of the girder is 4 feet, and the allowable unit fiber stress to be used is 15,000 pounds; therefore, the required flange area may be determined by formula **14,** Art. **43.** Substituting the proper values in the formula, we have

$$A = \frac{834{,}750}{4 \times 15{,}000} = 13.91 \text{ square inches.}$$

Assume a flange composed of two $5'' \times 5'' \times \frac{7}{16}''$ angles and two $\frac{3}{8}'' \times 12''$ flange plates; a sketch of the section with the location of the rivets is shown in Fig. 67. The entire area of the flange is

$2\text{–}\frac{3}{8}'' \times 12''$ plates	= 9	square inches.
$2\text{–}5'' \times 5'' \times \frac{7}{16}''$ angles	= 8.3 6	square inches.
Total, . . .	1 7.3 6	square inches.

The sectional areas cut out for rivet holes are:

$4\text{–}\frac{7}{8}$-inch holes through $\frac{3}{8}$-inch plate	= 1.3 1 2	square inches.
$4\text{–}\frac{7}{8}$-inch holes through $\frac{7}{16}$-inch angles	= 1.5 3 1	square inches.
Total, ...	2.8 4 3	square inches.

The net area of the flange is, therefore, $17.36 - 2.84 = 14.52$ square inches, which, since the required area is 13.91 square inches, is ample, and this section will be adopted.

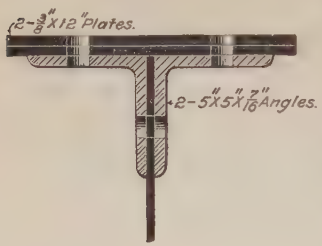


FIG. 67.

We will now determine the thickness of the web-plate. The reaction at either end is equal to one-half of the load, or 55,650 pounds. Assuming that there are eleven $\frac{7}{8}$-inch holes cut in line through the web-plate, the net depth of the plate will be $48 - 11 \times .875 = 38.375$ inches. Using an allowable unit shearing stress of 11,000 pounds, the theoretical thickness of the web-plate, from formula **12,** Art. **38,** is

$$T = \frac{55,650}{38.375 \times 11,000} = .131 \text{ inch.}$$

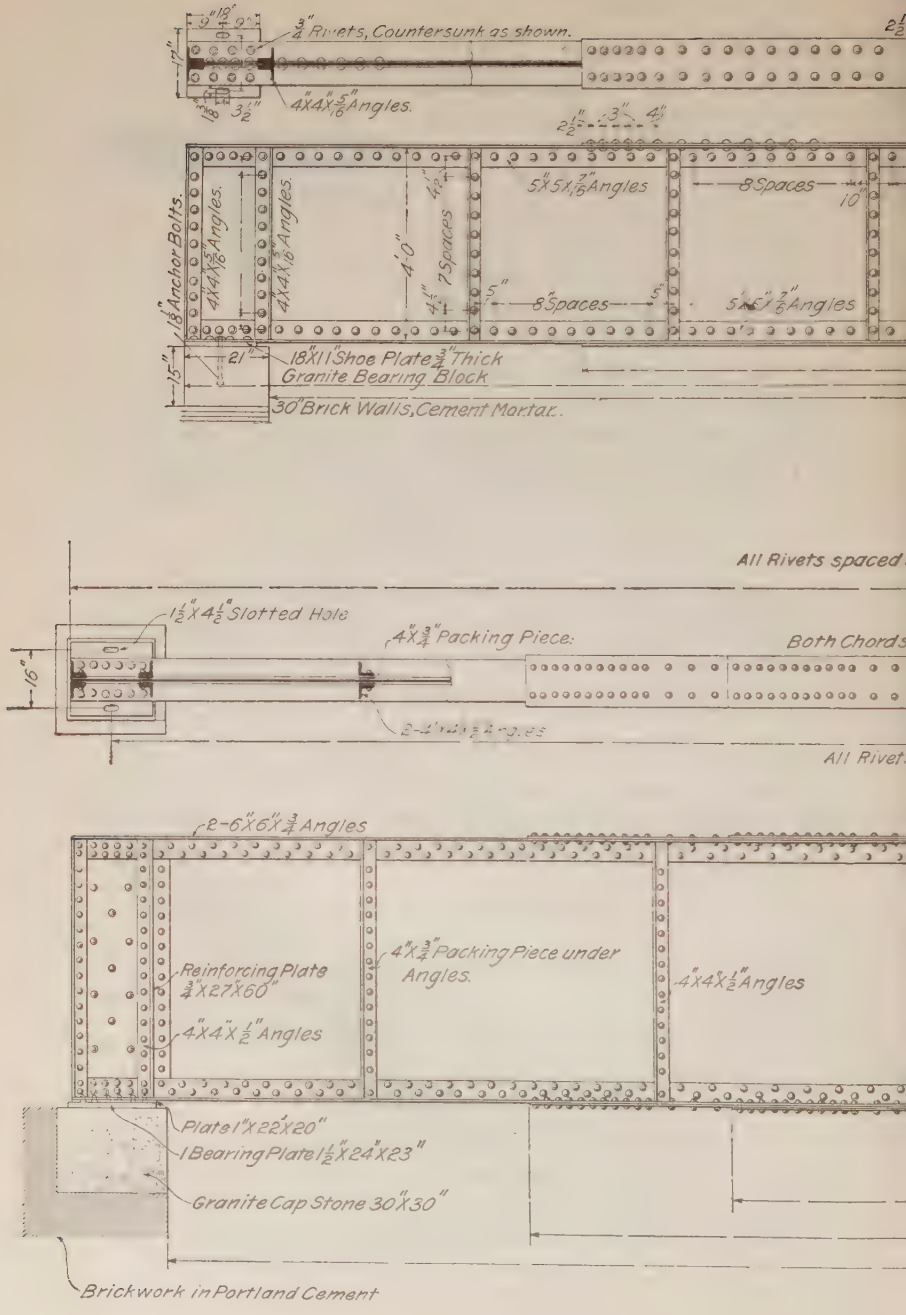
It is not, however, practicable to use this thickness of metal for a web-plate, since it would not provide sufficient bearing value for the rivets. As it is never good practice to use a web-plate less than $\frac{5}{16}$ inch in thickness, this size will be adopted.

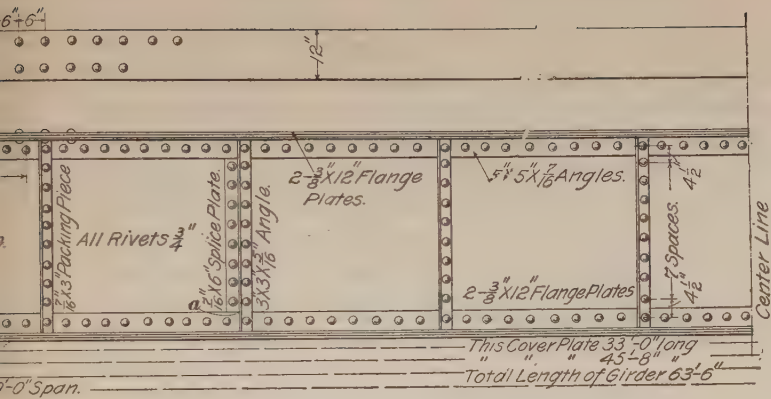
The lengths of the flange plates are now required; they may be determined either by the graphical method or by formula **15,** Art. **44;** using the latter method, the theoretical length of the outside plate is found to be

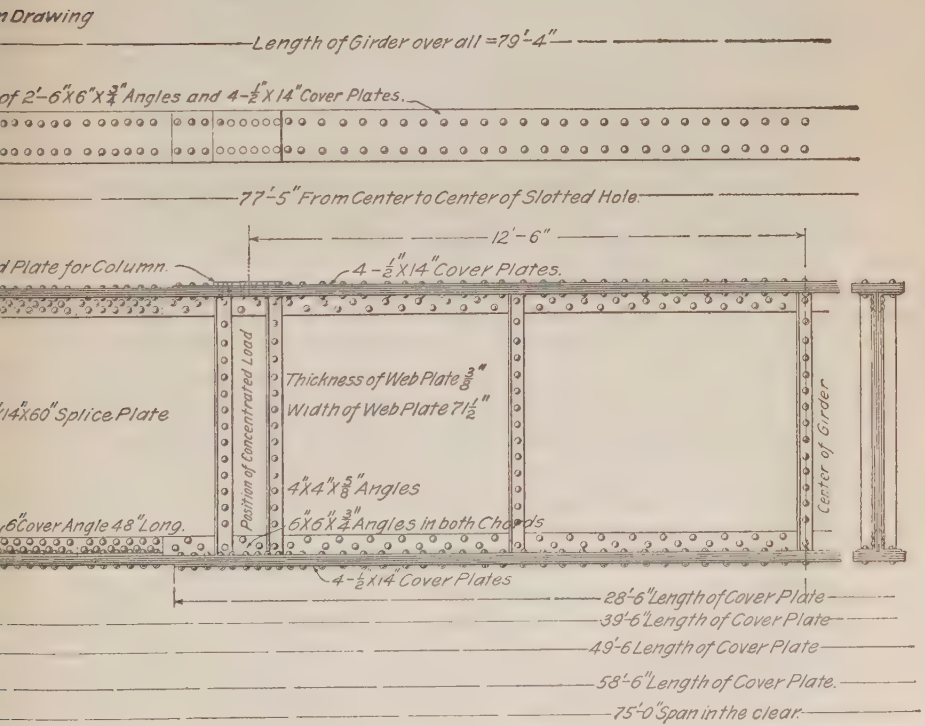
$l = 60\sqrt{\frac{3.844}{14.52}} = 30.87$ feet, or about 30 ft. 10 in., to which is to be added 1 foot at each end to allow for riveting. The total length of the plate is, therefore, 32 ft. 10 in., say 33 feet.

Applying the formula again, the length of the second flange plate is $l = 60\sqrt{\frac{7.688}{14.52}} = 43.66$ feet, or about 43 ft. 8 in.; adding a foot at each end gives us 45 ft. 8 in.

Consider now the size of the four stiffeners at the end of the girder. The reaction at the end of the girder is 55,650 pounds, and the allowable compressive strength of


3/4" Rivets, Countersunk as shown.
4"X4"X5/16" Angles.
5"X5"X7/16" Angles
8 Spaces
8 Spaces
7 Spaces
4'-0"
18" Anchor Bolts.
18"X11" Shoe Plate 3/4" Thick
Granite Bearing Block
30" Brick Walls, Cement Mortar.
All Rivets spaced
1 1/2"X4 1/2" Slotted Hole
4"X3/4" Packing Piece.
Both Chords
16"
2-6"X6"X3/4" Angles
Reinforcing Plate 3/4"X27"X60"
4"X4"X1/2" Angles
4"X3/4" Packing Piece under Angles.
Plate 1"X22"X20"
1 Bearing Plate 1 1/2"X24"X23"
Granite Cap Stone 30"X30"
Brickwork in Portland Cement


6"+6"
12"
All Rivets ¾"
2-⅜"X12" Flange Plates.
5"x5"x7/16 Angles.
7 Spaces. 4½"
Center Line
2-⅜"X12" Flange Plates
This Cover Plate 33'-0" long
" " " 45'-8" "
Total Length of Girder 63'-6"
-0" Span.


n Drawing
Length of Girder over all = 79'-4"
of 2'-6"X6"X¾" Angles and 4-½"X14" Cover Plates.
77'-5" From Center to Center of Slotted Hole
12'-6"
d Plate for Column.
4-½"X14" Cover Plates.
14"X60" Splice Plate
Thickness of Web Plate ⅜"
Width of Web Plate 71½"
4"X4"X⅝" Angles
6"X6"X¾" Angles in both Chords
6" Cover Angle 48" Long.
Position of Concentrated Load
Center of Girder
4-½"X14" Cover Plates
28'-6" Length of Cover Plate
39'-6" Length of Cover Plate
49'-6 Length of Cover Plate
58'-6" Length of Cover Plate.
75'-0" Span in the clear.

the material in the girder will be taken at 13,000 pounds. Then the sectional area required in the four angles composing the stiffeners on the plate girder over the abutments is $55,650 \div 13,000 = 4.28$ square inches. Since it would not be advisable to use smaller than a $4'' \times 4'' \times \frac{5}{16}''$ angle in this position, the sectional area of which is 2.4 square inches (see table "Properties of Angles"), it is evident that there will be ample strength in the four stiffeners. The other stiffeners may be made of $3'' \times 3'' \times \frac{5}{16}''$ angles, which is the smallest size that should be used for any girder requiring intermediate stiffeners.

The rivet spacing, etc. needs no explanation; it would be well, however, for the student to calculate the number of rivets for the several parts and compare the results with the number actually used as shown by the detail drawing, Fig. 68. He will undoubtedly find that more rivets are used than are actually required, but he must bear in mind that there are always practical considerations which influence more or less the design of structural work.

In Fig. 68 it will be noticed that the web-plate is spliced at the point a. The shear at this point is equal to 55,650 pounds, the reaction at R_1 minus the load on the girder between R_1 and the point a under consideration, that is, to $55,650 - 37,150 = 18,500$ pounds. A sufficient number of rivets must be placed on the two sides of the joint to take care of this shear safely.

Fig. 69 shows the design of a heavily loaded girder with long span; this girder was designed to carry a uniformly distributed load of 2,400 pounds per lineal foot and two concentrated loads of 60,000 pounds, placed one on each side of the center of the girder and 12 feet 6 inches therefrom. The unit fiber stress allowable in calculating the flange section was taken at 15,000 pounds.

The student should note particularly the splices on this girder. In this case it was necessary to splice the flange angles; when this is done, care must be taken to weaken the sectional area of the angle as little as possible by the punching of the rivet holes, and a sufficient number of rivets must

also be placed each side of the joint, so that the resistance of the rivet section may equal that of the net section of the flange angles. A careful study should be made of the various other details on this drawing, which represents excellent modern practice.

DEFLECTION OF BEAMS.

57. **Elasticity** is that property which a body possesses of returning to its original form after being strained or distorted by the application of a stress. This property is possessed by all bodies in a greater or less degree. If, after being distorted, a body does not perfectly resume its original form, it is said to have a **permanent set.**

58. **Elasticity of Building Materials.**—It is believed that the elasticity of all solids is more or less imperfect, and that the slightest strain produces a corresponding permanent set. It is customary, however, to consider the elasticity of all building material as practically perfect within certain limits. Under this assumption, stresses, up to a certain limit, may be applied and removed, and the resulting strain or alteration of form will be only temporary, with no appreciable permanent set. Stresses above this limit would, however, cause permanent sets.

59. The **elastic limit** of any material is the maximum unit stress that may be applied to it without causing any apparent permanent set.

To illustrate: Consider a piece of steel wire supported at one end and loaded by a weight suspended from the other. The wire is found to stretch under the action of the load, and by varying the weight or stress, the strain in each case is found to vary in the same proportion, so long as the weight is not greater than one-fourth of the breaking strength of the wire; within this limit it is found upon removing the weight that the wire resumes its original length.

If the load is made considerably greater than one-fourth of

the breaking strength of the wire, it is found that when the load is removed the wire has taken a permanent set; in other words, it will not return to its original length. If the wire remains permanently longer than it was before the load was applied, it has been strained beyond the *limit of elasticity*.

Suppose a weight of 2,000 pounds is hung from the end of a wrought-iron rod having a sectional area of 1 square inch, and the rod stretches about $\frac{1}{13000}$ part of its original length. When the weight is removed, the bar resumes its original length, as far as can be measured by ordinary instruments. Now instead of 2,000 pounds, attach a weight of 24,000 pounds to the rod, and it stretches about $\frac{1}{1000}$ part of its length; when this weight is removed, we find that the bar does not return to its original length, and that it is slightly longer than it was before; that is, the bar has a permanent set.

The unit stress, where the weight upon the rod is just sufficient to produce the least permanent set, is called the **elastic limit.**

60. The **modulus of elasticity** is the ratio of the unit stress to the unit strain for loads within the elastic limit.

For example, if the weight of 2,000 pounds upon the iron bar whose section is 1 square inch produces an elongation of $\frac{1}{13000}$ of the original length of the bar, the unit stress is 2,000 pounds per square inch; the unit strain is $\frac{1}{13000}$; and the modulus of elasticity is $2{,}000 \div \frac{1}{13000} = 26{,}000{,}000$ pounds per square inch.

In most building materials the modulus of elasticity for tension and the modulus for compressive stresses may be considered as practically equal. The modulus of elasticity of the principal building materials used in the construction of beams are given in the table "Modulus of Elasticity of Metals."

61. **Deflection** is the name applied to the distortion or bending produced in a beam when subjected to transverse stresses. If, upon the removal of the transverse stresses or

loads upon the beam, it returns to the straight or original form, the material in the beam has not been strained beyond the elastic limit. On the other hand, if the internal stresses exceed the elastic limit of the material, a permanent set will be given the beam.

62. Stiffness is a measure of the ability of a body to resist bending; this property is very different from the strength of the material or its power to resist rupture.

The stiffness of a structure does not depend so much upon the elasticity of the material of which it is composed as upon its arrangement and form; for example, a floor may be built of shallow and wide joists which will be sufficiently strong to carry a given load, but it will not be nearly so stiff as a floor of equal strength built of narrow and deep ones. This property of stiffness is as important in building construction as mere strength, and the two should be considered together; thus, the floor joists of a building may be strong enough to resist breaking, but so shallow as to lack stiffness, in which case the floor will be springy and vibrate under the footsteps of people walking upon it. If there is a plastered ceiling on the under side of the joists of such a floor, the deflection of the joists may cause the plaster to crack and fall into the room below. Where stiffness is lacking in the rafters of a roof, they will be liable to sag, thereby causing unsightly hollows in the surface of the roof, in which moisture and snow may lodge, which would be very detrimental to the roof covering.

63. Deflection of Beams.—From the foregoing it is evident that not only must the strength of the beams composing a structure be calculated to withstand rupture, but the beams must be stiff or rigid enough to resist bending. It is, therefore, important to be able to calculate the deflection of any beam under its load, and if found excessive, the size of the beam may be increased and the deflection reduced to working limits.

The amount of deflection that exists in beams loaded and

supported in different ways may be calculated by the formulas given in the table "Deflection of Beams." In using these formulas, all the loads should be expressed in pounds and the length in inches. The modulus of elasticity is denoted by E, and the moment of inertia of the section by I.

EXAMPLE 1.—A 10-inch steel I beam, supported at the ends, must support a uniformly distributed load of 10,000 pounds. The span of the beam is 20 feet, and its moment of inertia is 178; there is to be a plastered ceiling on its under side, the allowable deflection of which is $\frac{1}{360}$ inch for each foot of span. Will the deflection of the beam be excessive?

SOLUTION.—The formula for the deflection of a beam of this character from the table "Deflection of Beams" is $D = \frac{5\,W\,L^3}{384\,E\,I}$. From the table "Modulus of Elasticity of Metals," the modulus of elasticity of structural steel is found to be 29,000,000, and this, with the values given in the example, substituted in the formula, gives us the deflection

$$D = \frac{5 \times 10{,}000 \times 240^3}{384 \times 29{,}000{,}000 \times 178} = .34,$$

about $\frac{1}{3}$ of an inch.

Since the allowable deflection for each foot of span is $\frac{1}{360}$ of an inch, the total allowable deflection is $\frac{1}{360}$ of $20 = \frac{2}{3}$ of an inch. This is twice as much as the calculated deflection, and the beam therefore satisfies the required conditions. Ans.

EXAMPLE 2.—A 12″ × 16″ Northern short-leaf, yellow-pine girder must support a symmetrically placed triangular piece of brickwork, which weighs about 12,000 pounds. What will be the deflection of the timber if the span is 20 feet?

SOLUTION.—The formula for the deflection in this case from the table "Deflection of Beams," is $D = \frac{W\,L^3}{60\,E\,I}$. From the table "Modulus of Elasticity of Timber," the value of the modulus of elasticity is found to be 1,200,000. The moment of inertia of the section, from the formula $I = \frac{b\,d^3}{12}$, is

$$I = \frac{12 \times 16^3}{12} = 4{,}096.$$

Then, by substituting the given values, the deflection is

$$D = \frac{12{,}000 \times 240^3}{60 \times 1{,}200{,}000 \times 4{,}096} = .56,$$

about $\frac{9}{16}$ of an inch. Ans.

EXAMPLES FOR PRACTICE.

1. The moment of inertia of a 12-inch steel I beam is 220, and its span is 25 feet. If the ends of the beam are simply supported, what will be its deflection under a concentrated load of 10,000 pounds suspended from its center? Ans. .88 in.

2. A cantilever beam of $12'' \times 16''$ Georgia yellow pine extends from a building wall 10 feet, and is loaded on the end with a concentrated load of 18,000 pounds. What will be the greatest deflection of the beam? Ans. 1.49 in.

3. The span of a 15-inch steel I beam is 30 feet, and the moment of inertia of its section is 429.6. The load upon the beam is uniformly distributed and amounts to 3,000 pounds per lineal foot; if the ends of the beam are firmly fixed, what will be its deflection? Ans. .88 in.

FLITCH-PLATE GIRDERS.

64. The name **flitch-plate girder** is applied to a beam composed of two timbers bolted together with a flat plate of iron or steel sandwiched between them; hence it is often called a **sandwiched girder.**

This girder is used where it is desired to impose a greater weight than the wooden beams will support, and is considered somewhat cheaper than a steel I beam; owing, however, to the present exceedingly low price of structural steel, it is doubtful if there is now much difference in cost in favor of the flitch-plate girder.

The great advantage a beam of this character possesses over a steel rolled section probably lies in the fact that it is partially fireproof; the wood may be charred considerably without losing all its strength, and it will protect the iron or steel plate from the intensity of the heat, thus acting as a non-conductor and fireproofing material for the metal.

It is difficult to so proportion a girder of this character, as to be economical in the use of both the wood and the metal. Theoretically, the iron or steel plate should be so proportioned that, when carrying its share of the load, it deflects equally with the wooden beams, otherwise there is a tendency for the bolts either to shear off or to crush and tear the wood.

The two timbers should be considered as a single beam, and the calculation made for the safe load they will sustain. The plate will then be proportioned to support the remainder of the load coming upon the girder. The timber beams and the plate should be so proportioned that they will both deflect equally under their respective loads. It is, however, almost impossible to realize such a condition as this, as may be seen by referring to the following illustrative example:

65. The span of a flitch-plate girder is 20 feet, and it is composed of two yellow-pine timbers, each 6 in. $\times$ 16 in., with a plate of steel 16 inches wide bolted between them. The girder carries a load of 20,000 pounds concentrated at the center, and is to be so proportioned that its deflection under this load will not be likely to cause plaster cracks. What should be the thickness of the plate?

It will first be necessary to calculate the strength of the two yellow-pine timbers; in order to do this, the section modulus of the combined cross-section of the two timbers will be obtained by the formula $K = \frac{b\,d^2}{6}$, which, upon substituting the values, gives us $K = \frac{12 \times 16^2}{6} = 512$.

Now the safe uniformly distributed load W that a rectangular beam will support, may be obtained by the formula

$$W = \frac{2\,K\,S}{3\,L}. \qquad \textbf{(17.)}$$

This formula is readily derived by combining formula **4** with formula **13,** *Architectural Engineering*, § 5. Note, however, that in formula **4,** the bending moment is in *inch-pounds*, while in formula **13,** *Architectural Engineering*, § 5, it is expressed in *foot-pounds*.

It is known that a beam will support only one-half of this load, if concentrated at the center; therefore, the formula for obtaining the safe concentrated load W at the center of a beam is

$$W = \frac{2KS}{3L} \times \tfrac{1}{2} = \frac{KS}{3L}, \qquad \textbf{(18.)}$$

where K = section modulus of the cross-section;
S = safe transverse strength of the material;
L = span of the beam in feet.

For the beam under consideration, K was found to be 512. S may be obtained by dividing 7,300, the modulus of rupture for yellow pine, as given in Table 6, Art. **61,** *Architectural Engineering*, § 5, by the factor of safety, which will be taken as 5; then, $S = 7{,}300 \div 5 = 1{,}460$ pounds per square inch; L is 20 feet.

With the above values substituted in the formula, the safe load that may be carried by the two pine beams is found to be $W = \frac{512 \times 1{,}460}{3 \times 20} = 12{,}458$ pounds. Since the entire load upon the girder is 20,000 pounds, the load which the steel plate must support is $20{,}000 - 12{,}458 = 7{,}542$ pounds.

The next step is to calculate the deflection of the yellow-pine timbers under their load.

The deflection of a beam supported at both ends and loaded with a concentrated load applied at the center, may be calculated by the formula $D = \frac{WL^3}{48EI}$; see table "Deflection of Beams." The value of E for yellow pine will be taken at 1,200,000; the moment of inertia of the section is $I = \frac{12 \times 16^3}{12} = 4{,}096$; in this case the length of the span is to be in inches, therefore, $L = 20 \times 12 = 240$ inches; and W, according to the problem, is 12,458 pounds. The substitution of these values in the formula gives us $D = \frac{12{,}458 \times 240^3}{48 \times 1{,}200{,}000 \times 4{,}096} = .73$ inch.

The steel plate should now be proportioned so that it will deflect, under its load of 7,542 pounds, a distance equal to .73 inch, the deflection of the wooden girders.

The formula $D = \frac{WL^3}{48EI}$ may be transposed and written

$I = \frac{WL^3}{48\,E\,D}$. By substituting the known values in this formula, we have $I = \frac{7{,}542 \times 240^3}{48 \times 29{,}000{,}000 \times .73} = 102.6$.

Now the depth of the plate is 16 inches, and I for the required section is 102.6. The formula $I = \frac{b\,d^3}{12}$ may be transposed to $b = \frac{I \times 12}{d^3}$, and by substituting the known values in the latter, it becomes $b = \frac{102.6 \times 12}{16^3} = .3$ inch, nearly, the required thickness of the plate.

Before finally adopting this thickness for the plate, however, it should be examined to determine whether the deflection of .73 inch causes too great a fiber stress on the steel. In order to determine the fiber stress upon the plate, the bending moment must be calculated. According to Table 10, Art. **97,** *Architectural Engineering*, § 5, the formula for a beam with a concentrated load at the center is $M = \frac{WL}{4}$; upon substituting the values, we have $M = \frac{7{,}542 \times 20}{4}$ = 37,710 foot-pounds, which, multiplied by 12, gives 452,520 inch-pounds. The section modulus of the section is $K = \frac{.3 \times 16^2}{6} = 12.8$.

Having now the bending moment M and the section modulus K, the unit of fiber stress S may be determined by the formula $S = \frac{M}{K}$; upon substituting the values, we have $S = \frac{452{,}520}{12.8} = 35{,}353$ pounds per square inch. This is entirely too great and should be reduced to about one-third, or say 12,000 pounds per square inch. In order to obtain this lower unit fiber stress, the plate must be about three times as thick, or nearly .9 inch; a $\frac{7}{8}$-inch plate will be strong enough. Changing the thickness of the plate in this manner reduces the deflection of both the timbers and the steel plate below the .73 inch previously determined, and the full

strength of the timbers will not be realized. The reduced deflection will be an advantage in this case, because .73 inch is excessive. The deflection should not be more than $\frac{1}{360}$ of an inch for each foot of span, or $\frac{2}{3}$ of an inch in all. Hence, upon increasing the thickness of the plate, and thus decreasing the deflection, the deflection of the girder will probably be brought within the desired limits.

The bolts holding the timbers and the steel plate together are best located along the neutral axis; for at this point there would practically be no stress upon them. But to space them sufficiently near together along this line would tend to weaken the timber too much, and would be likely to cause the destruction of the beam from longitudinal shearing along this line. Therefore, it is best to alternate them above and below the line of the neutral axis, thus forming

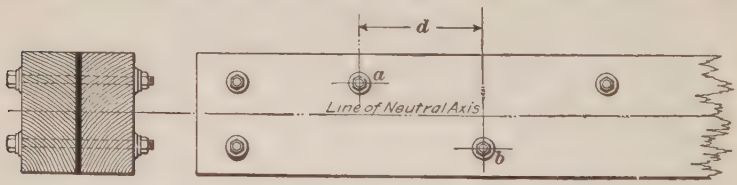


FIG. 70.

two rows of bolts as shown in Fig. 70. The end bolts are doubled, and the horizontal distance d between the bolts a and b should be about equal to the depth of the girder. The bolts in a girder of this character should be about 1 inch in diameter.

ROOF TRUSSES.

DETERMINATION OF STRESSES IN THE FINK TRUSS.

66. The **Polonceau** or **Fink Truss,** shown in Fig. 71 at (*a*), is a favorite form of truss. It is often built of rolled-steel sections; it is also built with wooden rafter members, structural-steel struts, and wrought-iron tension members. Frequently the lower chord of the truss is **cambered** (raised

at the center), as shown at (*b*), Fig. 71. Cambering the lower chord in this manner gives greater headroom under the

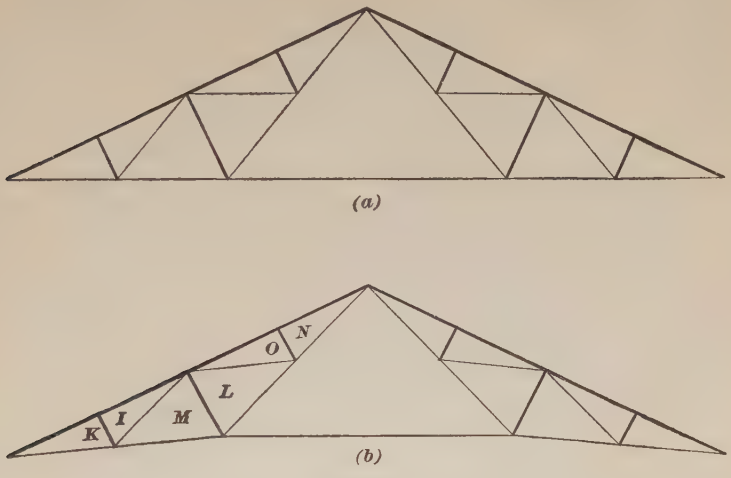


FIG. 71.

truss at the center, and somewhat improves its appearance; but it increases the stresses on all the members except *K I*, *M L*, and *O N*.

67. Diagram for Vertical Loads.—Fig. 72 is a frame diagram showing the vertical loads upon a Fink truss, the span of which is 80 feet, the lower chord, or tie, of the truss being cambered from the horizontal 28 inches. The stress diagram for the vertical loads may be drawn, as shown in Fig. 73, by first drawing the vertical load line *a f*, and laying off to some convenient scale the loads *a b*, *b c*, *c d*, *d e*, and *e f*, designated in the frame diagram, Fig. 72, by *A B*, *B C*, *C D*, *D E*, and *E F*. Since the truss is symmetrically loaded, only one-half of the stress diagram need be drawn, and consequently the loads only as far as *e f* need be laid off on the vertical load line. The reactions R_1 and R_2 are each equal to one-half of the load, or 18,000 pounds; hence, the point *z* is located midway between *e* and *f*, and *z a* represents the reaction R_1, 18,000 pounds.

The stresses around the joint $A\,B\,K\,Z$ may be drawn in the stress diagram by commencing at b and drawing $b\,k$ parallel with $B\,K$, and from z, a line parallel with $K\,Z$, intersecting the first line at k. The polygon of forces around this joint will then be: from a to b, from b to k, from k to z, and from z back again to a, the starting point. From the direction of these forces, the correct direction of the arrowheads in the frame diagram may be marked, and from their direction the kind of stress upon the member is observed.

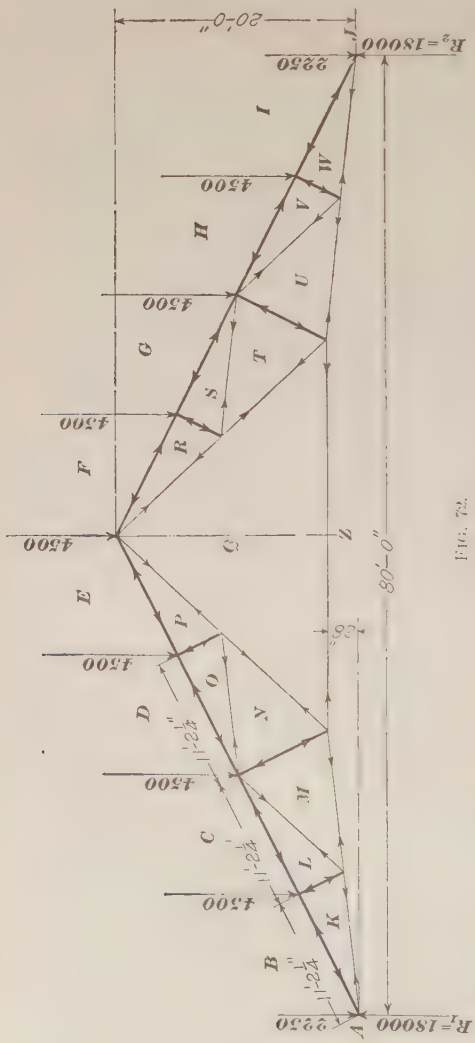


Fig. 72.

The next joint in the truss to analyze is $B\,C\,L\,K$. In the stress diagram begin at c and draw $c\,l$ parallel with $C\,L$ in the frame diagram, then from k, draw a line parallel with $L\,K$ in the frame diagram, until it intersects the line drawn from c at the point l. The polygon of forces around the

joint is from b to c, from c to l, from l to k, and from k back to b, the starting point.

Around the joint $K\,L\,M\,Z$, the stresses are obtained by drawing from l a line parallel with $L\,M$ in the frame

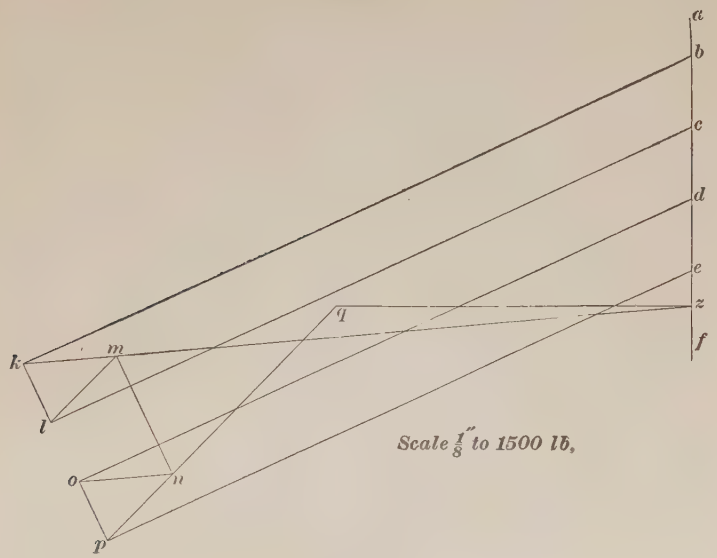


FIG. 73.

diagram, until it intersects the line $z\,k$ at the point m. The polygon of forces around this joint is: from k to l, from l to m, from m to z, and from z back again to k, the starting point.

Difficulty will be encountered upon attempting to analyze either the joint $C\,D\,O\,N\,M\,L$ or $M\,N\,Q\,Z$, for at each of these joints there are three unknown forces or stresses. It is impossible, by the graphical method, to solve the stresses around a joint in a structure where there are more than two unknown forces, consequently some other method of solution must be found.

Upon inspecting the frame diagram, Fig. 72, it will be seen that the joint $D\,E$ is similar to the joint $B\,C$;

the panel load, 4,500 pounds in each case, is supported by two forces, BK and KL, DO and OP, respectively. Since the directions of the forces are respectively parallel, and the panel loads are equal, the stresses in KL and OP are equal, these stresses being due only to a component of their respective panel loads. In a similar manner, it can be shown that the stresses in KL and OP are each held in equilibrium by the pairs of forces, KZ and LM, ON and PQ, and that the stresses in LM and NO are produced by equal components of the equal stresses in KL and OP; therefore, the stresses in LM and NO are equal. This gives us the magnitude of the stress ON, its equal LM having already been determined, and the diagram can be completed as follows: Draw mn parallel to MN of the frame diagram, then draw do parallel with DO, and of such a length that when on is drawn, from its extremity o, the point n will meet the line mn midway between the lines do and ep. This construction makes the length on equal to lm, which is in accordance with the condition that the stresses in LM and ON are equal. Then the polygon of forces around this joint will be: from c to d, from d to o, from o to n, from n to m, from m to l, and from l to c, the starting point.

The remaining joints, when taken in their usual order, offer no difficulty, and the other half of the diagram need not be drawn unless it is desired to check the half of the diagram just completed.

The polygon of forces that has just been traced around the joint $CDONML$ affords a good illustration of the rule, that the forces that meet at a joint must make a closed polygon in the stress diagram.

One of the peculiarities of the stress diagram, Fig. 73, and one that is worthy of note, as it will materially assist in drawing the diagram, is that the triangles lkm and pon are equal and similar to the larger one whose base is mn.

68. The **wind-stress diagram** may now be drawn. First the frame diagram is redrawn, and upon it, as shown in

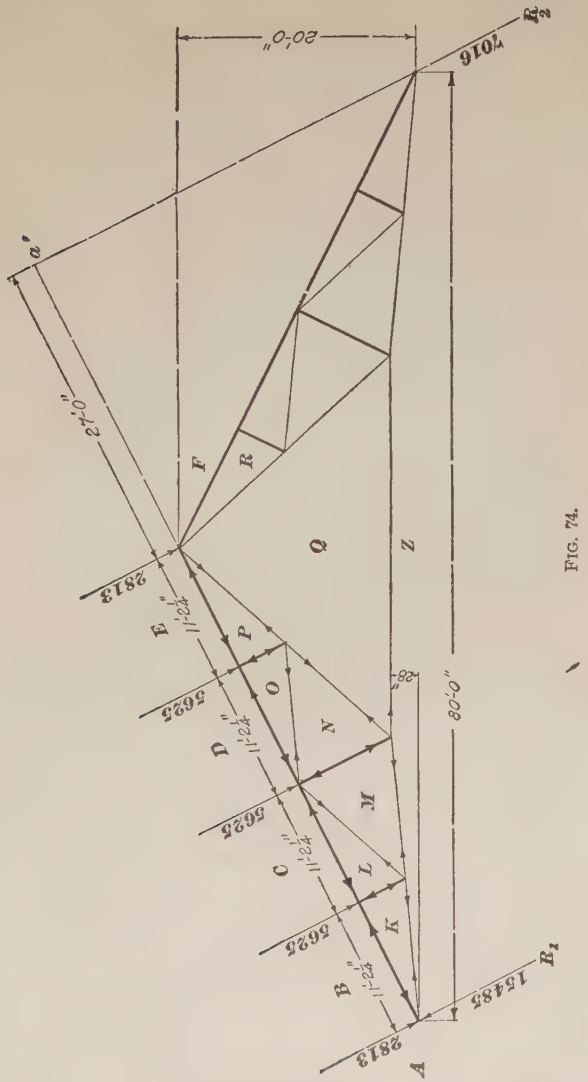


FIG. 74.

Fig. 74, are designated the wind loads acting at the several

joints of the truss, in a direction normal to the slope of the roof.

The reactions R_1 and R_2 may be calculated by the principle of moments. Since the truss is securely fastened at both ends, neither end being free to move in a lateral direction, these reactions will be parallel with the action of the wind on the roof, that is, normal to the slope. The reaction R_2 may first be obtained by extending its line of direction until it intersects the extension of the left-hand rafter member at the point a'. Then by taking the center of moments at the left-hand reaction R_1, the magnitude of the reaction R_2 may be computed.

For convenience, reduce to feet and decimals the distance from each panel point to the point of rotation R_1; the moments about this point will then be as follows:

$5{,}625 \times 11.188 =$ 62,932.50 foot-pounds.
$5{,}625 \times 22.375 =$ 125,859.38 foot-pounds.
$5{,}625 \times 33.563 =$ 188,791.88 foot-pounds.
$2{,}813 \times 44.750 =$ 125,881.75 foot-pounds.
Total, . . 503,465.51 foot-pounds.

The distance of the center of moments of the line of action of the reaction R_2 is $44.75 + 27 = 71.75$ feet, and $503{,}465.51 \div 71.75 = 7{,}016$, the magnitude of the reaction R_2, in pounds.

Since the sum of the wind loads is 22,501 pounds, which is equal to the sum of the reactions, the reaction due to the wind at R_1 is $22{,}501 - 7{,}016 = 15{,}485$ pounds.

The wind-stress diagram, Fig. 75, may now be drawn. First draw the load line a to f parallel to the reactions and direction of the wind loads at the several joints. Then lay off on the load line the loads ab, bc, cd, de, and ef, which are respectively equal to the corresponding loads AB, BC, CD, DE, and EF in the frame diagram. Having located the point f, the magnitude of the reaction R_2, represented by fz, may be laid off upwards (the direction in which

it acts), and the point z is thus located; then upon scaling $z\,a$, it should be found equal to 15,485 pounds, the reaction R_1.

The polygon of external forces will then be: from a to b,

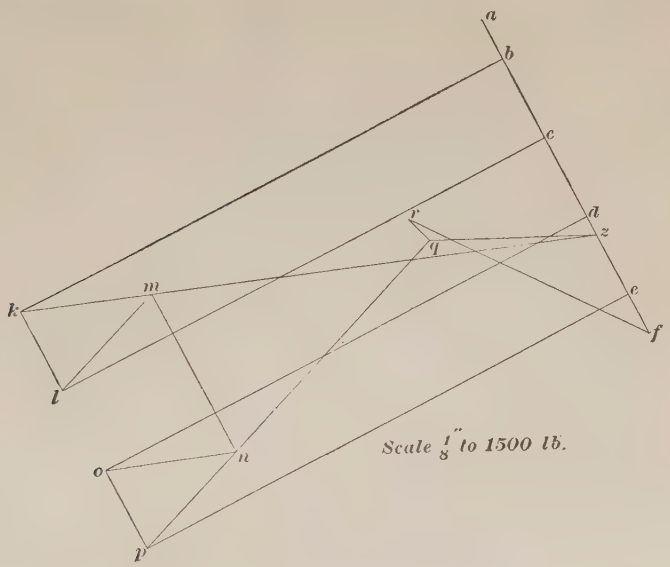


FIG. 75.

from b to c, from c to d, from d to e, from e to f, from f to z, and from z to a, the starting point.

The joint $C\,D\,O\,N\,M\,L$ may be solved in this stress diagram in the same manner as it was solved in the vertical load stress diagram.

The analysis of the stresses around the last joint $E\,F\,R\,Q\,P$ in the frame diagram, is interesting, from the fact that the stress $q\,r$ closes the diagram, and, if the diagram is correctly drawn, it must be parallel to the member $Q\,R$.

Having drawn both the wind and the vertical load stress diagrams, the stresses in the several members in the truss may be obtained by scaling, and their magnitudes may be tabulated as follows:

Member.	Stress Due to Vertical or Dead Load.	Stress Due to Wind Load.	Total Stress.
BK	+46,000	+34,500	+80,500
CL	+44,000	+34,500	+78,500
DO	+42,000	+34,500	+76,500
EP	+40,000	+34,500	+74,500
KL	+ 4,000	+ 5,500	+ 9,500
MN	+ 8,000	+11,250	+19,250
OP	+ 4,000	+ 5,500	+ 9,500
ZK	−41,500	−36,750	−78,250
ZM	−35,500	−28,500	−64,000
ZQ	−21,000	−11,000	−32,000
QN	−15,500	−18,500	−34,000
QP	−21,500	−26,000	−47,500
FR		+17,000	
RQ		− 2,000	
NO	− 5,750	− 8,000	−13,750
LM	− 5,750	− 8,000	−13,750

In the above table, compression is indicated by the plus sign and tension by the minus sign.

THE DESIGN OF A COMPOSITE PIN-CONNECTED ROOF TRUSS.

69. One of the most generally used forms for a composite pin-connected truss is the Polonceau or Fink truss, previously described. A truss of this pattern is usually made of wooden rafter members, structural-steel struts, and wrought-iron tension members. Where the wooden rafter member is oiled, or otherwise finished, and where the iron and steel members are tastily painted, these trusses make a good appearance from the interior of the room or building which they span, and are adapted for supporting the

covering or roof for such buildings as mess halls of barracks or asylums, armory drill halls, railroad stations, etc.

In order to illustrate the method of designing the members and joints in a truss of this character, we will now consider the design of the truss, Fig. 72, the stresses in which were determined by the diagrams Figs. 73 and 75, and tabulated in Art. **68.**

70. The **rafter member** has the greatest compressive stress upon it between the points $A\,B$ and $B\,C$, Fig. 72; this stress is represented in the table of stresses by the stress of 80,500 pounds in the member $B\,K$.

In addition to this compressive stress, there is a bending stress in the rafter due to the construction of the roof, which is composed of sheathing laid upon joists spaced about 14 inches center to center along each rafter. The wind and vertical loads are, by this construction, transmitted to the panel points by the transverse strength of the rafter between these points; it will therefore be necessary, before proportioning the rafter member, to calculate the stresses due to the bending moment produced by the dead and wind loads.

The wind load acts normally to the rafter, but the dead or vertical load does not. If great accuracy were required, it might be advisable to resolve the vertical load so as to determine its component normal to the roof, and add this amount to the wind load to obtain the entire load normal to the rafter; such refinement, however, will not be necessary here, and the vertical load and wind load will be added together directly and considered as a uniformly distributed load upon the rafter member between the panel points.

The sum of the wind and dead loads for a panel is 5,625 $+4,500 = 10,125$ pounds; therefore the bending moment, according to the formula $M = \frac{WL}{8}$, is $M = \frac{10,125 \times 11.188}{8}$ $= 14,160$ foot-pounds, or $14,160 \times 12 = 169,920$ inch-pounds.

Assuming that the rafter member is made of yellow pine, it will be seen by referring to Table 6, Art. **61,** *Architectural Engineering*, § 5, that the modulus of rupture is 7,300; hence, if a factor of safety of 5 is adopted, the safe working

transverse stress in the material will be $7,300 \div 5 = 1,460$ pounds. Since the required section modulus K may be obtained by dividing M (the bending moment in inch-pounds) by the safe unit stress S of the material, as expressed by the formula $K = \frac{M}{S}$, we have $K = \frac{169,920}{1,460}$ $= 116$, the section modulus required to resist the transverse stress upon the rafter member.

Assume that a depth of 14 inches is adopted for the rafter member; then by transposing formula **15,** Art. **101,** *Architectural Engineering*, § 5, $K = \frac{b\,d^2}{6}$ to $b = \frac{K \times 6}{d^2}$, the breadth, or width, of the rafter member required to resist the transverse stress, by substituting the values of K and d, is $b = \frac{116 \times 6}{14^2} = 3\frac{1}{2}$ inches. Thus it is seen that a section of $3\frac{1}{2}$ in. $\times 14$ in. yellow pine is sufficient to take care of the transverse stress upon the rafter member. But in addition to this transverse stress there is a direct compressive stress of 80,500 pounds, as shown by the stress diagram, which must be provided for in the same member, by adding material in the direction of its width.

From Table 6, Art. **61,** *Architectural Engineering*, § 5, the ultimate compressive strength of yellow pine, parallel to the grain, is found to be 4,400 pounds per square inch, and as a factor of safety of 5 is used, the safe compressive strength will be $4,400 \div 5 = 880$ pounds per square inch.

Since the distance from joint to joint is not great, being only about 11 feet, the rafter member between the joints need not be considered as a column, but it may be designed to resist direct compression and the full allowable compressive strength of the material parallel to the grain may be used in calculating the cross-section required. Therefore, $80,500 \div 880 = 91$ square inches is the sectional area of the material which must be added to the rafter member to resist the compressive stress. The depth of the rafter being 14 inches, and the sectional area required 91 square inches, the width to add to the rafter will be $91 \div 14 = 6\frac{1}{2}$ inches.

By combining, the total width of the timber required to resist the two stresses is $3\frac{1}{2} + 6\frac{1}{2} = 10$ inches. The rafter member will therefore be made of one piece of $10'' \times 14''$ yellow-pine timber, which will extend from the heel to the apex of the truss.

71. The **tension rods** should be made of wrought iron with eyes formed on the ends and provided with turnbuckles where required; their sectional area should be calculated to safely carry the loads indicated in the table of stresses.

It is well to make all the tension bars in the lower chord in pairs; the stresses on ZK, ZM, and ZQ, Fig. 72, will thus each be taken care of by two bars, one on each side of the rafter member and strut.

Since the stress in the pair of members ZK is 78,250 pounds, each of the two rods composing this part of the truss is proportioned so as to sustain $\frac{1}{2}$ of 78,250, or 39,125 pounds. We will assume that a quality of wrought iron is used whose ultimate tensile strength is 52,000 pounds per square inch, and, owing to the reliability of the material composing them, a factor of safety of 4 is sufficient in the iron tension members; the allowable tensile strength of the wrought iron is, therefore, $52{,}000 \div 4 = 13{,}000$ pounds per square inch, and the required sectional area of one rod is $39{,}125 \div 13{,}000 = 3$ square inches. If square rods are used, as they will be in this case, a $1\frac{3}{4}'' \times 1\frac{3}{4}''$ rod will be found adequate, as it has a sectional area of $1.75 \times 1.75 = 3.06$ square inches.

The size of the other tension rods may be found in a similar manner, but as the stresses upon LM and NO, Fig. 72, are light, these members may be made of a single tension bar; these bars and also QZ must be provided with turnbuckles, which will be required to tighten the whole structure and take care of any slack in the truss due to inaccuracy in construction. In designing the members provided with turnbuckles, care should be taken to see that the rods are upset on the ends, so as to realize a sectional area at the root of the screw thread equal to the required sectional area of the rod.

72. The **steel struts** may be proportioned by the formula for calculating the strength of structural-steel columns with hinged ends, see formula **7**, Art. **13**; all the values in the formula are known except the square of the radius of gyration, which may be obtained by the method explained in Art. **11**. Assume some convenient section which will be thought to have the required resistance. In this case the section shown in Fig. 76 is assumed, it being convenient for making the various connections to the tension rods and wooden rafter member.

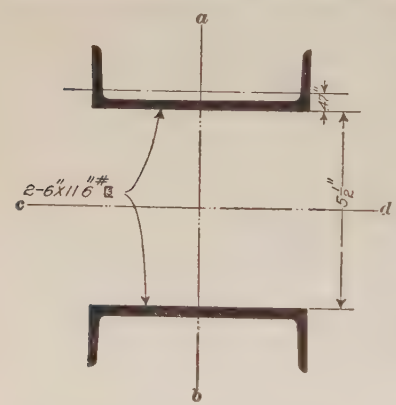


FIG. 76.

In order to calculate the least value of R^2, it will be necessary to calculate the least moment of inertia of the section, which will be done in accordance with the principles given in Art. **7**.

The properties of this section are not given in the tables, but the moment of inertia of one of these channels, with respect to the axis $a\,b$, is 15.47. Since the neutral axis $a\,b$ of the section passes through the center of gravity of the two channels, the moment of inertia of the section, with respect to this axis, is equal to the sum of the moment of inertia of the channels, with respect to the same axis, that is, to $15.47 \times 2 = 30.94$.

The moment of inertia I' of the channel, with respect to an axis through its center of gravity parallel to $c\,d$, is .82, the area of the section of the channel is 3.35 square inches, and the distance of its center of gravity from the back of the web is .47 inch. The distance of the axis through the center of gravity of the channel, from the neutral axis $c\,d$ of the column section, is, therefore, $5\frac{1}{2} \div 2 + .47 = 3.22$ inches. By applying formula **1**, Art. **7**, the moment of inertia of one of the channels, with respect to the axis $c\,d$, is

$I' = .82 + 3.35 \times 3.22^2 = 35.55$. For the entire section, the moment of inertia, with respect to $c\,d$, is equal to the sum of the moments of its parts, that is, to $35.55 \times 2 = 71.1$.

From these calculations, it is evident that the least moment of inertia is on the axis $a\,b$, and is equal to 30.94. Then, by substituting the values of the least moment of inertia and the total area of the section in formula **6,** the square of the least radius of gyration is $R^2 = \frac{30.94}{6.70} = 4.62$.

According to Table 6, Art. **61,** *Architectural Engineering*, § 5, the ultimate compressive strength of structural steel is 52,000 pounds per square inch; the length of the longest column in the truss is 8 feet, or 96 inches. By substituting in formula **7,** Art. **13,** the ultimate compressive resistance of the section is $S = \frac{52{,}000}{1 + \left(\frac{96^2}{18{,}000 \times 4.62}\right)} = 46{,}840$ pounds per square inch.

Since the area of the section is 6.70 square inches, the ultimate resistance of the column will be $46{,}840 \times 6.70 = 313{,}828$ pounds; if a factor of safety of 4 is used in this column, its safe resistance will be $313{,}828 \div 4 = 78{,}457$ pounds.

Since the stress in this long strut or column is only 19,250 pounds, it can readily be seen that it has several times the required strength. It is, however, deemed advisable to use this size of column, as the detailing where it joins the rafter member and also the pin connections at the lower end demand that channels of this size be used.

73. On account of the facilities in making the connections, and because, in a case of this kind, it is generally advisable to adopt the same rolled sections throughout, wherever possible, the **short struts** should be made of the same rolled sections as the long strut; this method saves labor in the shop and facilitates assembling and erection in the field.

74. The **size** of the **pins** is yet to be determined. This was so thoroughly treated in Arts. **29–31** that no further explanation will be required here. It is sufficient to say that

upon thorough examination of the several pinned joints, it was decided that a 3-inch pin would be sufficiently strong to resist any bending, shearing, or bearing stresses that would be applied to them.

The correct design of the castings at the heel and apex of the truss is more a matter of experience and good judgment than of calculation.

75. The student should carefully study the details of the truss, shown in Fig. 77, which have been designed according to the preceding diagrams and calculations. He should observe how all the connections are made and especially the details of the pin connections and the castings at the apex and heel of the truss.

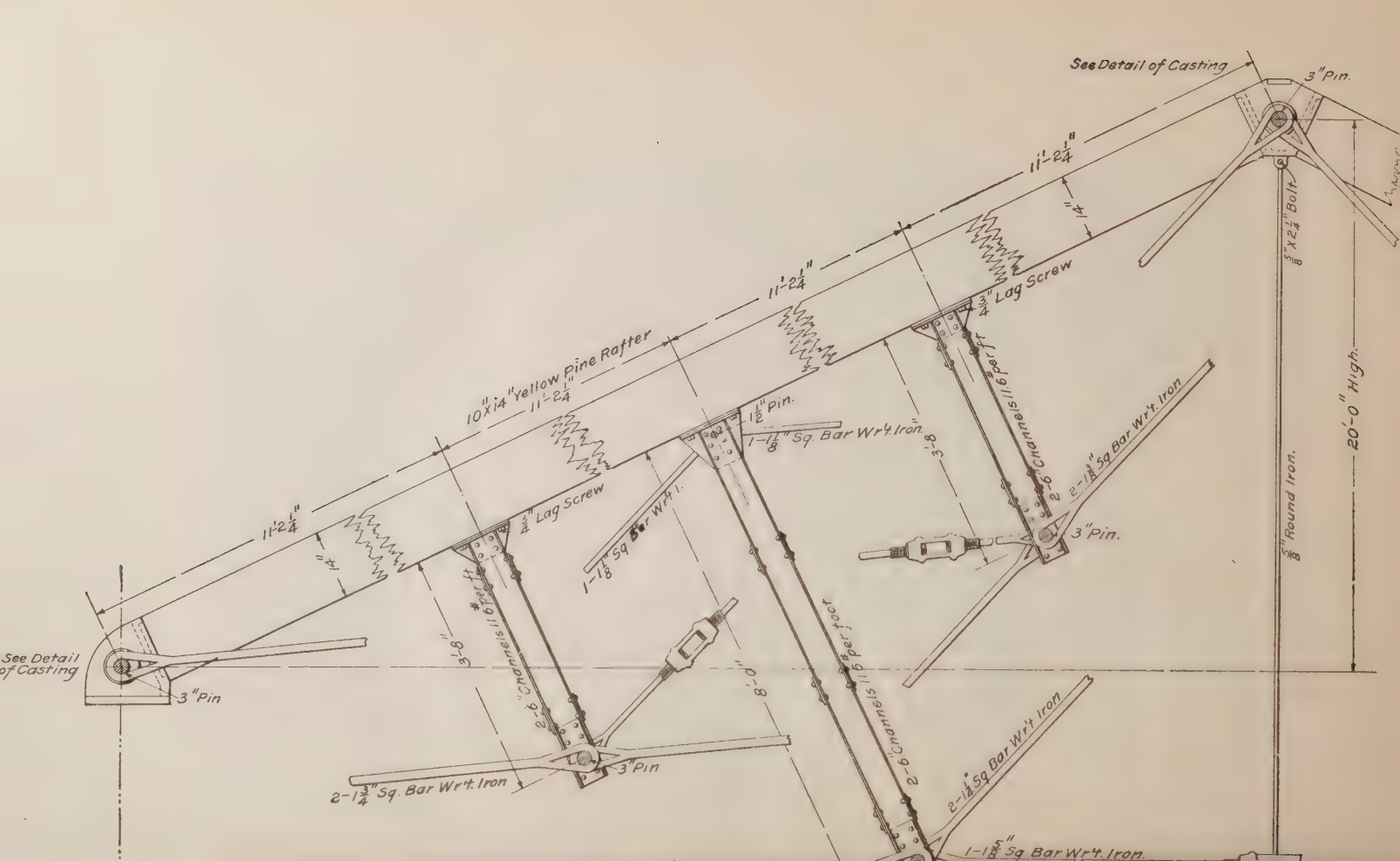
The light rod at the center of the truss has no stress on it, but is used simply to support the lower central tie-member and prevent it from sagging.

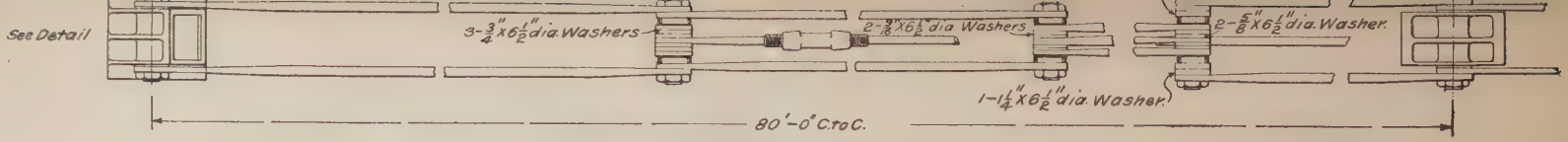
In designing compression members made up of two channels tied together with plates, as shown in Fig. 77, the ties should not be farther apart than 16 times the width of the flange; for example, in this case, the width of the flange of the channels composing the column is about $1\frac{3}{4}$ inches, 16 times this width is 28 inches; hence, the distance between the two pieces or ties should not, in this column, be over 28 inches. An inch one way or the other, however, would make very little difference.

THE DESIGN OF A STRUCTURAL-STEEL ROOF TRUSS.

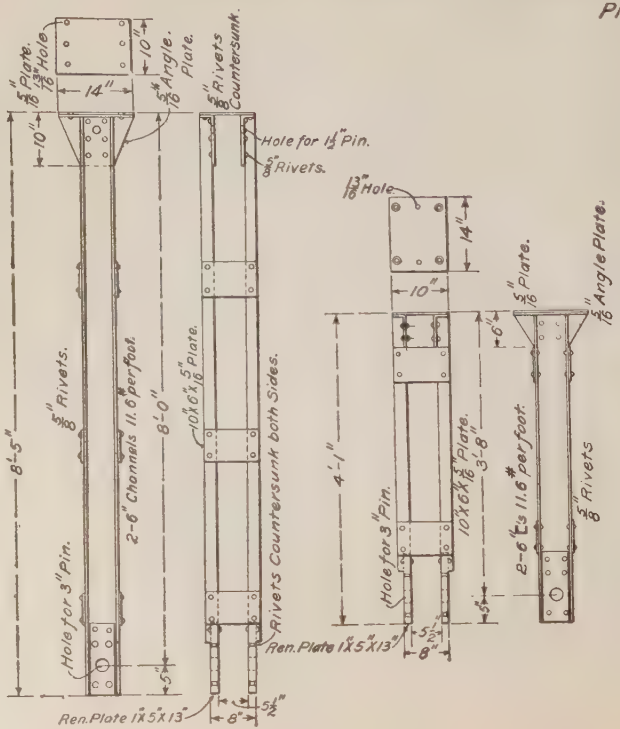
76. The material most generally used in the construction of roof trusses for the support of the roof covering of modern buildings is structural steel. The rolled sections chiefly used in their construction are angles and plates, though any of the other steel sections may be adapted to special cases.

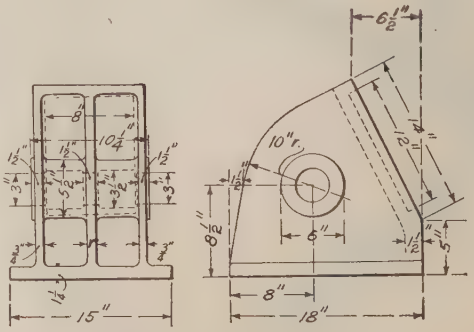
When a structural-steel roof truss is made of angles and plates, the angles are usually connected in pairs with the plate between them, as shown in Fig. 78. When this


See Detail of Casting
3" Pin.
11'-2¼"
14"
11'-2¼"
10"X14" Yellow Pine Rafter
11'-2¼"
¾" Lag Screw
1½" Pin.
1-1⅛" Sq. Bar Wr't Iron.
3'-8"
2-6" Channels 11.6# Per Ft.
2-1⅛" Sq Bar Wr't Iron
3" Pin.
⅝" Round Iron.
20'-0" High.
¾" Lag Screw
1-1⅛" Sq. Bar Wr't I.
3'-8"
2-6" Channels 11.6# Per ft.
See Detail of Casting
3" Pin
8'-0"
2-6" Channels 11.6# per foot
3" Pin
2-1¾" Sq. Bar Wr't Iron
2-1⅛" Sq Bar Wr't Iron
1-1⅛" Sq. Bar Wr't Iron.

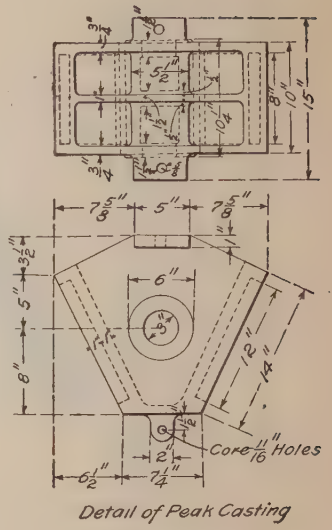


Plan of Tension Members.





Detail of Foot Castings



Detail of Peak Casting

FIG. 77.

construction is adopted, the joints and connections of the several members may be made quite conveniently.

Assume that it is desirable to construct the Fink roof truss,

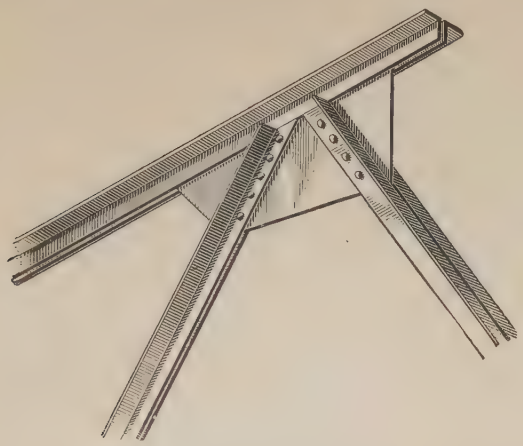


FIG. 78.

previously described and designed as a pin-connected truss, of structural steel. The stresses will be the same as given in the table, Art. **68,** and the general dimensions as given in Fig. 72.

The frame diagram may be redrawn and the stresses marked upon the several members, as shown in Fig. 79.

77. The **rafter member** should be made in one length from the heel to the apex of the truss and proportioned to suit the greatest stress upon it, which in this case is 80,500 pounds. Generally in steel construction, the roof covering is supported on purlins which are placed at the panel points. There is, therefore, no bending stress in the rafter, and it is subjected to compression only.

In this case the rafter is in compression only, and the portion between each panel point will be regarded as a column whose length is equal to the distance from center to center of the joints. Assume the size of a pair of angles, which judgment and experience dictates as being adequate to support the stress upon the member; then by the formula for

structural-steel columns with fixed ends, determine the strength of the assumed section. If its strength, as found by the formula, is equal to, or slightly in excess of, the strength required to resist the stress in the member, the section may be adopted; providing, of course, that suitable connections can be made to it.

In this case it was decided to assume a section made of two $5'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ angles, placed back to back with the long legs vertical, and about $\frac{5}{16}$ inch apart. The length of the column is 11 ft. $2\frac{1}{4}$ in., say 134 inches; and from the table "Radii of Gyration for Two Angles Back to Back," the radius of gyration of a section composed of two $5'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$

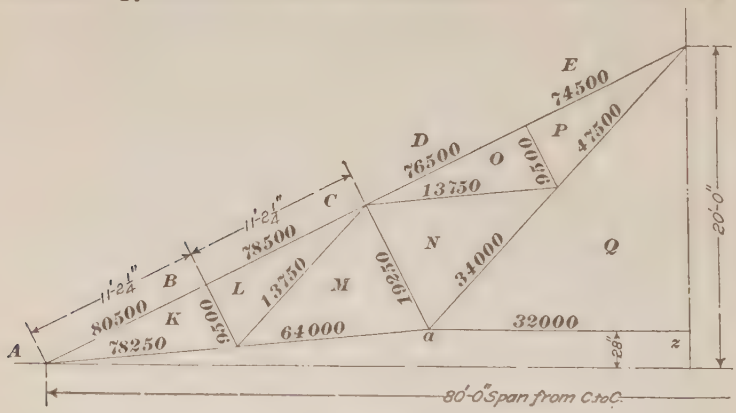


FIG. 79.

angles, with the long legs back to back and $\frac{1}{2}$ inch apart, is found to be 1.51, a value sufficiently exact for our purpose. Substituting these values, in formula **9**, the ultimate compressive strength of the column is $S = \dfrac{52{,}000}{1 + \left(\dfrac{134^2}{36{,}000 \times 1.51^2}\right)}$ = 42,600 pounds per square inch of section; dividing by 4, the factor of safety adopted, the allowable strength of the column per square inch of section becomes 10,650 pounds. The area of a $5'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ angle, according to the table "Areas of Angles," is 4 square inches; the area of the entire section is, therefore, $4 \times 2 = 8$ square inches, and the

allowable strength of the member, $10{,}650 \times 8 = 85{,}200$ pounds. Since the stress on the member is only 80,500 pounds, it is evident that the assumed section will fulfil the requirements, and may be used for the rafter member.

78. The **main strut,** or the member MN, is the next compression member of any considerable size; its length is 8 ft. 7 in., or 103 inches; it is assumed that a section composed of two $3'' \times 2'' \times \frac{1}{4}''$ angles, placed back to back at a distance apart of $\frac{5}{16}$ inch, will suit this position in the truss; the value of R for two $3'' \times 2'' \times \frac{1}{4}''$ angles placed back to back and $\frac{1}{2}$ inch apart, from the table "Radii of Gyration for Two Angles Placed Back to Back," is found to be .93, which is a close enough approximation for our purpose.

Then in this case $S = \dfrac{52{,}000}{1 + \left(\dfrac{103^2}{36{,}000 \times .93^2}\right)} = 39{,}000$ pounds,

the ultimate strength of the section per square inch of area; since the area of the two $3'' \times 2'' \times \frac{1}{4}''$ angles is 2.38 square inches and the required factor of safety is 4, the allowable strength of the section will be $(39{,}000 \div 4) \times 2.38 = 23{,}200$ pounds. Since the stress in the member is only 19,250 pounds, it is evident that this section will be sufficiently strong.

79. The stress on the **short struts** KL and OP is so small that any calculation of their required section would only result in obtaining a section composed of such small shapes that their use would not be practical in a truss of this size and character.

Here it is well to call attention to the practical principle that in selecting the section of a member where the stresses are light, care must be taken to choose such a section as will best fulfil the requirements of the construction, disregarding the fact that it will be stronger than is actually required to sustain the load. For example, it is considered good practice to make the leg of the angle held by a rivet not less in width than three times the diameter of the rivet; accordingly,

a $\frac{3}{4}$-inch rivet should not be used in an angle leg whose width is less than three times $\frac{3}{4}$ inches, or $2\frac{1}{4}$ inches.

In the case of the truss under consideration, it was deemed advisable to use two $2\frac{1}{2}'' \times 2'' \times \frac{1}{4}''$ angles for each of the struts KL and OP.

80. The **tension members** in the truss are to be composed of rolled sections similar to those used in the struts; that is, two angles back to back and far enough apart to allow the $\frac{5}{16}$-inch **gusset plate**, forming the means of connection at the joints, to slip between them.

As far as practicable, tension members, in common with the struts, are made up of one continuous section; thus, irrespective of the fact that the stress in ZK is greater than in ZM, these members should both be made of one continuous pair of angles; by so doing the labor is minimized, thus effecting a saving that would more than offset the cost of the superfluous material in the member ZM, besides producing a much more pleasing appearance.

In the truss under consideration, ZK and ZM will be made of the same pair of angles; QN and QP will also be composed of a single pair of angles; care being taken to proportion the section to withstand the greater stress.

The stress in the member ZK is 78,250 pounds; the allowable tensile strength of structural steel, if the ultimate stress is taken at 60,000 pounds, and a factor of safety of 4 is used, will be $60{,}000 \div 4 = 15{,}000$ pounds per square inch. Then $78{,}250 \div 15{,}000 = 5.21$, the sectional area in square inches that will be required in the pair of angles forming the member ZK after deducting the sectional area cut out by one rivet hole in each angle.

From the table "Areas of Angles," a $3'' \times 3'' \times \frac{9}{16}''$ angle is seen to have a sectional area of 3.06 square inches; two angles will then have a sectional area of twice 3.06, or 6.12 square inches. Since the thickness of these angles is $\frac{9}{16}$ inch and a $\frac{3}{4}$-inch rivet is to be used, the area of the section to be deducted for one rivet hole is $.875 \times .5625 = .49$ square inch, and the net sectional area in the two angles is, therefore,

$6.12 - 2 \times .49 = 5.14$ square inches. This is slightly under the sectional area demanded by the calculation, but, as a low ultimate tensile strength was assumed, it will be safe to use this section for the members ZK and ZM.

The other tension members in the truss may be proportioned in the same manner, care being taken to deduct from the section area of the member the sectional area removed for rivet holes; also, that the rolled sections adopted will satisfy the practical demands of the construction.

81. The number of rivets required at the several joints in the truss should be carefully calculated. Since the method of calculation is the same for all the joints, only one will be considered here, though the student should analyze the others for himself.

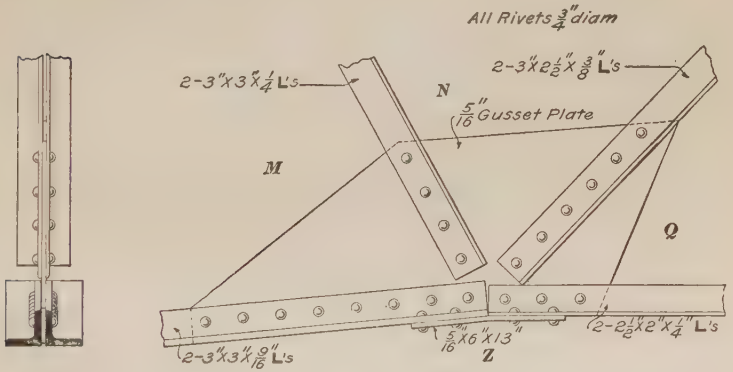


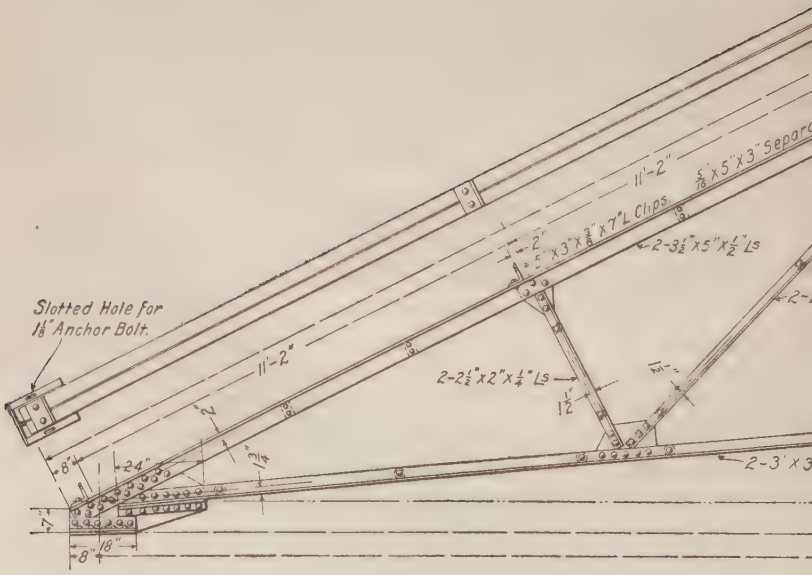
FIG. 80.

Fig. 80 represents a detail of the joint at a, Fig. 79. The stress in the member MN is 19,250 pounds, and a sufficient rivet section must be provided at the end of this member to resist this stress. By referring to the table "Values of Rivets," and using an allowable stress per square inch of 15,000 pounds, the least value of a $\frac{3}{4}$-inch rivet in this connection is found to be the web bearing of a $\frac{5}{16}$-inch plate, which is 6,094 pounds; then $19{,}250 \div 6{,}094 = 3.15$, or say 4, the number of rivets required in this connection.

The member NQ is subjected to a stress of 34,000 pounds, the least value for the rivets is the same as before, and the number required in the end of this member is $34{,}000 \div 6{,}094 = 5.56$, say 6 rivets.

The stress in the member ZM is 64,000 pounds, which will require a large number of rivets. Using 8 rivets in the vertical legs and gusset plate, as shown in Fig. 80, the value of each is 6,094 pounds and the value of eight is $6{,}094 \times 8 = 48{,}752$ pounds. This deducted from 64,000 pounds, the entire stress in the member, leaves 15,248 pounds yet to be provided for by the use of a splice plate attached to the horizontal legs of the angles. The value of one rivet in the horizontal legs of the angles forming the member QZ is the ordinary bearing value of a $\frac{3}{4}$-inch rivet in a $\frac{1}{4}$-inch plate, which, according to the table "Values of Rivets," is 3,656 pounds, with an allowable stress of 15,000 pounds. The value of four rivets will then be $3{,}656 \times 4 = 14{,}624$ pounds, a little less than the amount to be taken care of, but the difference is so small that it may be disregarded and for the sake of symmetry the same number of rivets will be used in connecting the splice plate to the member MZ. The number of rivets required to connect the member ZQ to the gusset plate may be readily obtained.

82. Fig. 81 shows the usual shop drawing of the truss just designed; the student should pay particular attention to the details of the connections. **Separators** should be placed in both the tension and compression members; they are placed in the tension members to prevent the angles from striking against each other when the trusses are subjected to vibrations, and also to join the two angles of a member, so that the work will arrive at the point of erection in a convenient form, ready to put together. The separators are placed in the compression members so as to insure against any tendency to bend them apart, and so that they will act in unison. The spacing of these separators is more a matter of judgment on the part of the designer than anything else, though any spacing over 8 times the least



Slotted Hole for
1⅛" Anchor Bolt.
11'-2"
11'-2"
5/16" x 5" x 3" Separa
5" x 3" x 3/8" x 7" L Clips.
2-3½" x 5" x ½" Ls
2-2½" x 2" x ¼" Ls
2-3' x 3
8"
24"
18"
8"
7"

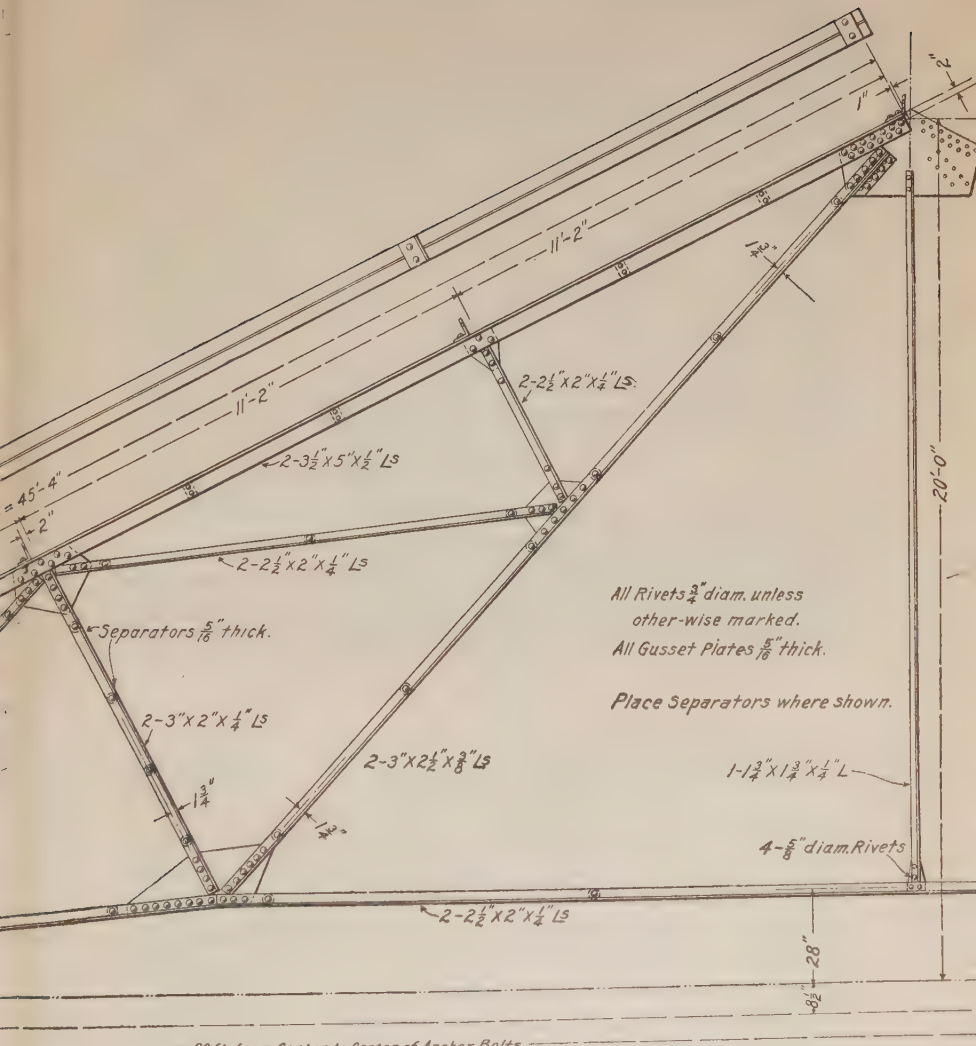

11'-2"
11'-2"
45'-4"
2"
1"
2"
1¼"
2-2½"x2"x¼" Ls
2-3½"x5"x½" Ls
2-2½"x2"x¼" Ls
20'-0"
All Rivets ¾" diam. unless other-wise marked.
All Gusset Plates ⅝" thick.
Place Separators where shown.
Separators 5/16" thick.
2-3"x2"x¼" Ls
2-3"x2½"x⅜" Ls
1¾"
1¼"
1-1¾"x1¾"x¼" L
4-⅝" diam. Rivets
2-2½"x2"x¼" Ls
28"
9½"
80 ft from Center to Center of Anchor Bolts.

FIG. 81.

dimension of the member is not to be recommended. Separators should always be placed near the end of a pair of angles to be connected to a gusset plate, as it will hold them the right distance apart in shipment and facilitate erection in the field. The $1\frac{3}{4}'' \times 1\frac{3}{4}'' \times \frac{1}{4}''$ angle, joining the apex of the truss and its lower chord, is required only to support the chord angles and prevent them from sagging. When there is considerable stress in this member, sagging is unlikely to occur, but it is the usual practice, and a good one, to introduce some support for this long and usually light member. It will be noticed that one size of rivets is used throughout this truss wherever practicable; this is a point in economical construction that should always be observed. The ends of the angles and members, when practicable, should be cut off square, and the gusset plates should be designed so that they may be formed with as few cuts as possible, and unless it is desired to make the truss somewhat fanciful, these cuts should never be other than straight lines.

In designing roof trusses, and, in fact, all structural-steel constructions, care should be taken to see that the several sections into which each may be divided are not so large that they cannot be shipped by railroad or other transportation at hand; this is important and should be carefully considered.

GENERAL NOTES REGARDING THE DESIGN OF A ROOF TRUSS.

83. Lateral Bracing.—Trusses forming the principal support of a roof, if of any considerable size, should be braced together in the planes of the rafters so as to secure them against any tendency of the wind, when blowing in a direction perpendicular to the gable ends, to produce lateral movement. If the roof sheathing is laid close and is well nailed, it will sufficiently stiffen trusses of moderate span. The heels of trusses are sometimes fastened securely to the walls, especially in those buildings where the wind is liable to get under the roof. When so secured there is a tendency

for the wind to reverse the stresses in the members of a roof provided with a light covering, and this reversal should be provided for in the design of the truss.

84. Factors of Safety.—Since due allowance must be made for unforeseen and unknown defects of material and workmanship, and for unknown stresses which are liable to occur, it is necessary to proportion the several parts of a structure so that they will be able to resist, without failure, much larger forces than those obtained from the stress diagram.

In roof trusses, however, the stresses can be calculated with more certainty than in the case of a bridge or a machine, and their application is more steady in its nature, and, therefore, not so severe on the material. For this reason it is permissible to allow unit stresses, in the design of roof trusses, some fifty per cent. in excess of those considered allowable in first-class bridges.

85. Tension Members.—The strength of a tension member is that of the smallest cross-section. It is, therefore, good practice to *upset* or enlarge the ends of long bolts or tension rods with screw ends, so that the cross-section at the root of the thread will be at least equal to the sectional area of the main part of the rod.

The central axis of the cross-section of ties and struts should coincide with the line of action of the thrust or pull, as otherwise the piece will be greatly weakened and dangerous bending moments will be developed in the structure. To calculate the net sectional area required in any tension member, the force or load upon it should be divided by the safe working tensile strength of the material, and allowance must be made in the area of the cross-section for the cutting of bolt and rivet holes.

86. Compression members whose lengths do not exceed six times the least dimension may be proportioned by the method followed for the tie as just explained;

but, when the length of the truss is increased, there is a tendency to yield sidewise when compressed, and the sectional area must be increased or the unit stress diminished. Hence, the column formulas given in Arts. **13–15** must be used. Pieces subjected to alternate compression and tension should have a materially larger section than would be required for either stress alone. Cast iron is seldom used in the best work for anything but short compression pieces, packing blocks, and pedestals.

87. Members in Trusses Subjected to Transverse Stresses.—In determining the resisting moment of a member subjected to transverse stresses, or in calculating the section required at the point of maximum bending moment, due allowance must be made for portions cut away on the tension side in attaching fastenings, or in making connections; similar allowance must also be made on the compression side, unless the holes are completely filled by the rivets, in which case no deduction need be made from the sectional area of the member.

88. Members in Trusses Subjected to Both Transverse and Direct Stresses.—The rafter members in a roof truss, likewise members in other structures, are often called upon to resist both a bending and a direct stress. Such pieces must first be designed to safely resist the bending moment, and then their transverse dimensions must be increased so that the added material will have sectional area sufficient to resist the direct pull or thrust. Should the direct force be a compressive stress, it will be well to test the size of the piece by the proper column formula.

89. Pins and Eyes.—In proportioning the pins and eyes of tension bars, the diameter of the pin should be from three-fourths to four-fifths of the width of the bar in flats, and one and one-fourth times the diameter of the bar in rounds. The sectional area of the metal around the eye should be fifty per cent. in excess of that of the rod or bar. When

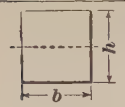
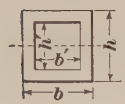
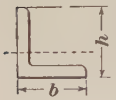
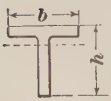
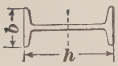
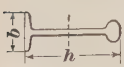
flat bars are used, their thickness should be not less than one-fourth of their width; this will secure a good bearing surface on the pin. The size of a pin is usually decided by the bending moment on it, consequently the assembled pieces on a pin should be packed close together, and opposing members should be brought as nearly in line with one another as possible.

90. Details.—The designer should carefully examine each joint and connection in the structure, and consider such practical points as means of shipment, erection in the field, etc. Care should be taken in designing all connections, to so place the rivets and bolts as to realize their full strength, and at the same time not cut away too much of the material of the members connected. All members and joints should be examined for tension, compression, shearing, and bending, and proportioned accordingly. The strength of the joints and connections between the members of a structure are of as much importance as the strength of the members themselves; the strength of any structure depends upon the strength of its weakest point, and its failure at a joint or connection is as fatal as the failure of any of its members.

The student will find the handbooks issued by the various steel mills of great value to him in the prosecution of his work. They contain many useful tables giving the properties of rolled sections, with information as to their use and application.

ELEMENTS OF USUAL SECTIONS.

NOTE.—Moments refer to horizontal axis through center of gravity. This table is intended for convenient application where extreme accuracy is not important. The values for the last seven sections and those marked * are approximate. A = area of section; in case of hollow section, a = area of interior space.

Section.	Moment of Inertia. I	Section Modulus. K	Distance of Base from Center of Gravity.	Square of Least Radius of Gyration. R^2	Least Radius of Gyration. R
	$\frac{b\,h^3}{12}$	$\frac{b\,h^2}{6}$	$\frac{h}{2}$	$\frac{(Least\ side)^2}{12}$	$\frac{Least\ side}{3.46}$
	$\frac{b\,h^3 - b'\,h'^3}{12}$	$\frac{b\,h^3 - b'\,h'^3}{6\,h}$	$\frac{h}{2}$	$\frac{h^2 + h'^2}{12}$ *	$\frac{h + h'}{4.89}$ *
	$\frac{A\,D^2}{16}$	$\frac{A\,D}{8}$	$\frac{D}{2}$	$\frac{D^2}{16}$	$\frac{D}{4}$
	$\frac{AD^2 - ad^2}{16}$	$\frac{AD^2 - ad^2}{8\,D}$	$\frac{D}{2}$	$\frac{D^2 + d^2}{16}$	$\frac{D + d}{5.64}$ *
	$\frac{b\,h^3}{36}$	$\frac{b\,h^2}{24}$	$\frac{h}{3}$	The smaller: $\frac{h^2}{18}$ or $\frac{b^2}{24}$	The smaller: $\frac{h}{4.24}$ or $\frac{b}{4.9}$
	$\frac{A\,h^2}{10.2}$	$\frac{A\,h}{7.2}$	$\frac{h}{3.3}$	$\frac{b^2}{25}$	$\frac{b}{5}$
	$\frac{A\,h^2}{9.5}$	$\frac{A\,h}{6.5}$	$\frac{h}{3.5}$	$\frac{(h\,b)^2}{13\,(h^2 + b^2)}$	$\frac{h\,b}{2.6\,(h + b)}$
	$\frac{A\,h^2}{19}$	$\frac{A\,h}{9.5}$	$\frac{h}{2}$	$\frac{h^2}{22.5}$	$\frac{h}{4.74}$
	$\frac{A\,h^2}{11.1}$	$\frac{A\,h}{8}$	$\frac{h}{3.3}$	$\frac{b^2}{22.5}$	$\frac{b}{4.74}$
	$\frac{A\,h^2}{6.66}$	$\frac{A\,h}{3.33}$	$\frac{h}{2}$	$\frac{b^2}{21}$	$\frac{b}{4.58}$
	$\frac{A\,h^2}{7.34}$	$\frac{A\,h}{3.67}$	$\frac{h}{2}$	$\frac{b^2}{12.5}$	$\frac{b}{3.54}$
	$\frac{A\,h^2}{6.9}$	$\frac{A\,h}{4}$	$\frac{h}{2.3}$	$\frac{b^2}{36.5}$	$\frac{b}{6}$

VALUES OF RIVETS.

Rivet Section.		Minimum Distance from Edge of Plate.		Ordinary Bearing.					Web Bearing.				
Diam.	Area.	End.	Side.	Plate Thickness.	Allowable Stress per Square Inch on High-Test Iron.				Plate Thickness.	Allowable Stress per Square Inch on High-Test Iron.			
					1,000	10,000	12,000	15,000		1,000	10,000	12,000	15,000
½″	.196	⅞ 15/16 1.0 1 1/16	1 1/16 1 1/8 1 3/16	¼″ Plate 5/16″ “	81.25 121.28	813 1,219	975 1,463	1,219 1,828	⅛″ Plate 3/16″ “ ¼″ “ 5/16″ “ ⅜″ “ 7/16″ “ ½″ “	108.34 162.50 216.67 270.84 325.00 379.17 433.34	1,083 1,625 2,167 2,708 3,250 3,792 4,335	1,300 1,950 2,600 3,250 3,900 4,550 5,200	1,625 2,438 3,250 4,063 4,875 5,687 6,500
				Single Shear.	141.56	1,416	1,699	2,123	Double Shear.	283.12	2,831	3,397	4,247
⅝″	.307	15/16 1.0 1 1/16 1 1/8 1 3/16	1 3/16 1 1/4 1 5/16	¼″ Plate 5/16″ “ ⅜″ “ 7/16″ “	101.56 152.34 203.13	1,016 1,523 2,031	1,219 1,828 2,437	1,523 2,285 3,046	⅛″ Plate 3/16″ “ ¼″ “ 5/16″ “ ⅜″ “ 7/16″ “ ½″ “	135.42 203.13 270.84 338.55 406.26 473.97 541.68	1,354 2,031 2,708 3,386 4,063 4,740 5,417	1,625 2,438 3,250 4,063 4,875 5,688 6,500	2,031 3,047 4,063 5,078 6,094 7,109 8,125
				Single Shear.	221.73	2,217	2,661	3,326	Double Shear.	443.46	4,435	5,322	6,652
¾″	.442	1.0 1 1/16 1 1/8 1 3/16 1 1/4 1 5/16	1 5/16 1 3/8 1 7/16	¼″ Plate 5/16″ “ ⅜″ “ 7/16″ “	121.88 182.81 243.75 304.69	1,219 1,828 2,438 3,047	1,463 2,193 2,925 3,656	1,828 2,742 3,656 4,570	⅛″ Plate 3/16″ “ ¼″ “ 5/16″ “ ⅜″ “ 7/16″ “ ½″ “	162.50 243.75 325.00 406.25 487.50 568.75 650.00	1,625 2,438 3,250 4,063 4,875 5,688 6,500	1,950 2,925 3,900 4,875 5,850 6,825 7,800	2,438 3,656 4,875 6,094 7,313 8,531 9,750
				Single Shear.	319.24	3,192	3,830	4,788	Double Shear.	638.47	6,385	7,662	9,577
⅞″	.601	1 1/16 1 1/8 1 3/16 1 1/4 1 5/16 1 3/8 1 7/16 1 1/2	1 7/16 1 1/2 1 9/16	¼″ Plate 5/16″ “ ⅜″ “ 7/16″ “ ½″ “	142.19 213.28 284.37 355.47 426.57	1,422 2,133 2,844 3,555 4,266	1,706 2,559 3,413 4,266 5,119	2,133 3,199 4,266 5,332 6,398	⅛″ Plate 3/16″ “ ¼″ “ 5/16″ “ ⅜″ “ 7/16″ “ ½″ “	189.58 284.37 379.17 473.96 568.75 663.54 758.33 853.12	1,896 2,844 3,792 4,740 5,688 6,635 7,583 8,531	2,275 3,413 4,550 5,687 6,825 7,962 9,100 10,237	2,844 4,266 5,687 7,109 8,531 9,953 11,375 12,796
				Single Shear.	434.07	4,341	5,209	6,511	Double Shear.	868.15	8,682	10,418	13,022

Ordinary bearing = 1½ × unit compressive stress × bearing area. Web bearing = 2 × unit compressive stress × bearing area.
Shear = ⅞ × unit compressive stress × section. Compression = 12/13 × allowable tensile stress.

AREAS OF ANGLES.

With Equal Legs.

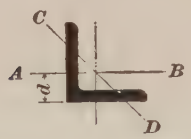
Size in Inches.	Thickness in Inches.											
	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	1
6×6			4.36	5.06	5.75	6.43	7.11	7.78	8.44	9.06	9.74	11.0
5×5			3.61	4.18	4.75	5.31	5.86	6.42	6.94	7.47	7.99	9.0
4×4			2.86	3.31	3.75	4.18	4.61	5.03	5.44	5.84		
3½×3½			2.48	2.87	3.25	3.62	3.99	4.34	4.69	5.03		
3×3	1.44	1.78	2.11	2.43	2.75	3.06	3.36	3.65				
2¾×2¾	1.31	1.62	1.92	2.21	2.50							
2½×2½	1.19	1.47	1.73	2.00	2.25							
2¼×2¼	1.06	1.31	1.55	1.78	2.00							
2×2	0.94	1.15	1.36	1.56								
1¾×1¾	0.81	1.00	1.17	1.30								
1½×1½	0.69	0.84	0.99									

With Unequal Legs.

7×3½				4.40	5.00	5.59	6.17	6.75	7.31	7.87	8.42	9.50
6×4			3.61	4.18	4.75	5.30	5.86	6.41	6.94	7.47	7.99	9.00
6×3½			3.42	3.96	4.50	5.03	5.55	6.06	6.56	7.06	7.55	8.50
5×4			3.23	3.74	4.25	4.74	5.23	5.72	6.18	6.65	7.11	8.00
5×3½			3.05	3.52	4.00	4.46	4.92	5.37	5.81	6.25	6.67	
5×3			2.86	3.30	3.75	4.17	4.60	5.03	5.44	5.84		
4½×3			2.67	3.09	3.50	3.90	4.30	4.68	5.06	5.43		
4×3½			2.67	3.09	3.50	3.90	4.30	4.68	5.06	5.43		
4×3		2.09	2.48	2.87	3.25	3.62	3.98	4.34	4.69	5.03		
3½×3		1.93	2.30	2.65	3.00	3.34	3.67	4.00	4.31	4.62		
3½×2½	1.44	1.78	2.11	2.43	2.75	3.06	3.36	3.65				
3×2½	1.31	1.62	1.92	2.21	2.50	2.78						
3×2	1.19	1.46	1.73	1.99	2.25							
2½×2	1.06	1.31	1.55	1.78	2.00							
2¼×1½	0.88	1.07	1.27	1.45	1.63							
2×1⅜	0.78											

PROPERTIES OF ANGLES.

Equal Legs.



Dimensions.	Thickness.	Weight per Foot.	Area of Section.	Distance d of Center of Gravity from Back of Flange.	Moment of Inertia, Axis AB.	Radius of Gyration, Axis AB.	Radius of Gyration, Axis CD.
In.	In.	Lb.	Sq. In.	In.	I	R	R'
6×6	$\frac{7}{8}$	33.1	9.74	1.82	31.92	1.81	1.17
6×6	$\frac{7}{16}$	17.2	5.06	1.66	17.68	1.87	1.19
5×5	$\frac{7}{8}$	27.2	7.99	1.57	17.75	1.49	0.98
5×5	$\frac{3}{8}$	12.3	3.61	1.39	8.74	1.56	0.99
4×4	$\frac{13}{16}$	19.9	5.84	1.29	8.14	1.18	0.80
4×4	$\frac{5}{16}$	8.2	2.40	1.12	3.71	1.24	0.82
$3\frac{1}{2}\times3\frac{1}{2}$	$\frac{13}{16}$	17.1	5.03	1.17	5.25	1.02	0.69
$3\frac{1}{2}\times3\frac{1}{2}$	$\frac{3}{8}$	8.5	2.48	1.01	2.87	1.07	0.70
3×3	$\frac{5}{8}$	11.4	3.36	0.98	2.62	0.88	0.59
3×3	$\frac{1}{4}$	4.9	1.44	0.84	1.24	0.93	0.60
$2\frac{3}{4}\times2\frac{3}{4}$	$\frac{1}{2}$	8.5	2.50	0.87	1.67	0.82	0.54
$2\frac{3}{4}\times2\frac{3}{4}$	$\frac{1}{4}$	4.5	1.31	0.78	0.93	0.85	0.55
$2\frac{1}{2}\times2\frac{1}{2}$	$\frac{1}{2}$	7.7	2.25	0.81	1.23	0.74	0.49
$2\frac{1}{2}\times2\frac{1}{2}$	$\frac{1}{4}$	4.1	1.19	0.72	0.70	0.77	0.50
$2\frac{1}{4}\times2\frac{1}{4}$	$\frac{1}{2}$	6.8	2.00	0.74	0.87	0.66	0.48
$2\frac{1}{4}\times2\frac{1}{4}$	$\frac{1}{4}$	3.7	1.06	0.66	0.51	0.69	0.46
2×2	$\frac{7}{16}$	5.3	1.56	0.66	0.54	0.59	0.39
2×2	$\frac{3}{16}$	2.5	0.72	0.57	0.28	0.62	0.40

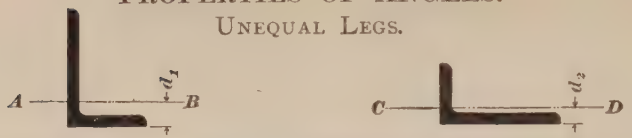
PROPERTIES OF ANGLES—*Continued.*

EQUAL LEGS.

Dimensions.	Thickness.	Weight per Foot.	Area of Section.	Distance *d* of Center of Gravity from Back of Flange.	Moment of Inertia, Axis *A B*.	Radius of Gyration, Axis *A B*.	Radius of Gyration, Axis *C D*.
In.	In.	Lb.	Sq. In.	In.	*I*	*R*	*R'*
$1\frac{3}{4}\times1\frac{3}{4}$	$\frac{7}{16}$	4.6	1.30	0.59	0.350	0.51	0.35
$1\frac{3}{4}\times1\frac{3}{4}$	$\frac{3}{16}$	2.1	0.62	0.51	0.180	0.54	0.36
$1\frac{1}{2}\times1\frac{1}{2}$	$\frac{3}{8}$	3.4	0.99	0.51	0.190	0.44	0.31
$1\frac{1}{2}\times1\frac{1}{2}$	$\frac{3}{16}$	1.8	0.53	0.44	0.110	0.46	0.32
$1\frac{1}{4}\times1\frac{1}{4}$	$\frac{5}{16}$	2.4	0.69	0.42	0.090	0.36	0.25
$1\frac{1}{4}\times1\frac{1}{4}$	$\frac{1}{8}$	1.0	0.30	0.35	0.044	0.38	0.26
1×1	$\frac{1}{4}$	1.5	0.44	0.34	0.037	0.29	0.20
1×1	$\frac{1}{8}$	0.8	0.24	0.30	0.022	0.31	0.21
$\frac{7}{8}\times\frac{7}{8}$	$\frac{3}{16}$	1.0	0.29	0.29	0.019	0.26	0.18
$\frac{7}{8}\times\frac{7}{8}$	$\frac{1}{8}$	0.7	0.21	0.26	0.014	0.26	0.19
$\frac{3}{4}\times\frac{3}{4}$	$\frac{3}{16}$	0.8	0.25	0.26	0.012	0.22	0.16
$\frac{3}{4}\times\frac{3}{4}$	$\frac{1}{8}$	0.6	0.17	0.23	0.009	0.23	0.17

PROPERTIES OF ANGLES.

Unequal Legs.



Dimensions.	Thickness.	Weight per Foot.	Area of Section.	Distances of Center of Gravity from Back of Flange.		Moments of Inertia. I.		Radii of Gyration. R.		
						Axis AB.	Axis CD.	Axis AB.	Axis CD.	Least Radius; Axis Diagonal.
In.	In.	Lb.	Sq.In.	d_1	d_2					
6×4	$\frac{7}{8}$	27.2	7.99	2.12	1.12	27.73	9.75	1.86	1.11	.88
6×4	$\frac{3}{8}$	12.3	3.61	1.94	0.94	13.47	4.90	1.93	1.17	.88
6×3½	$\frac{7}{8}$	25.7	7.55	2.22	0.97	26.38	6.55	1.87	0.93	.78
6×3½	$\frac{3}{8}$	11.7	3.42	2.04	0.79	12.86	3.34	1.94	0.99	.77
5×4	$\frac{7}{8}$	24.2	7.11	1.71	1.21	16.42	9.23	1.52	1.14	.88
5×4	$\frac{3}{8}$	11.0	3.23	1.53	1.03	8.14	4.67	1.59	1.20	.86
5×3½	$\frac{7}{8}$	22.7	6.67	1.79	1.04	15.67	6.21	1.53	0.96	.77
5×3½	$\frac{3}{8}$	10.4	3.05	1.61	0.86	7.78	3.18	1.60	1.02	.76
5×3	$\frac{13}{16}$	19.9	5.84	1.86	0.86	13.98	3.71	1.55	0.80	.66
5×3	$\frac{5}{16}$	8.2	2.40	1.68	0.68	6.26	1.75	1.61	0.85	.66
4½×3	$\frac{13}{16}$	18.5	5.43	1.65	0.90	10.33	3.60	1.38	0.81	.67
4½×3	$\frac{3}{8}$	9.1	2.67	1.49	0.74	5.50	1.98	1.44	0.86	.66
4×3½	$\frac{13}{16}$	18.5	5.43	1.36	1.11	7.77	5.49	1.19	1.01	.74
4×3½	$\frac{3}{8}$	9.1	2.67	1.21	0.96	4.18	2.99	1.25	1.06	.73
4×3	$\frac{13}{16}$	17.1	5.03	1.44	0.94	7.34	3.47	1.21	0.83	.66
4×3	$\frac{5}{16}$	7.1	2.09	1.26	0.76	3.38	1.65	1.27	0.89	.65
3½×3	$\frac{13}{16}$	15.7	4.62	1.23	0.98	4.98	3.33	1.04	0.85	.65
3½×3	$\frac{5}{16}$	6.6	1.93	1.06	0.81	2.33	1.58	1.10	0.90	.63
3½×2½	$\frac{11}{16}$	12.4	3.65	1.27	0.77	4.13	1.72	1.06	0.67	.58
3½×2½	$\frac{1}{4}$	4.9	1.44	1.11	0.61	1.80	0.78	1.12	0.74	.55
3¼×2	$\frac{9}{16}$	9.0	2.64	1.21	0.59	2.64	0.75	1.00	0.53	.45
3¼×2	$\frac{1}{4}$	4.3	1.25	1.09	0.48	1.36	0.40	1.04	0.57	.44
3×2½	$\frac{9}{16}$	9.5	2.78	1.02	0.77	2.28	1.42	0.91	0.72	.54
3×2½	$\frac{1}{4}$	4.5	1.31	0.91	0.66	1.17	0.74	0.95	0.75	.53

PROPERTIES OF ANGLES—*Continued.*

UNEQUAL LEGS.

Dimensions.	Thickness.	Weight per Foot.	Area of Section.	Distances of Center of Gravity from Back of Flange.		Moments of Inertia. *I.*		Radii of Gyration. *R.*		
						Axis *A B.*	Axis *C D.*	Axis *A B.*	Axis *C D.*	Least Radius; Axis Diagonal.
In.	In.	Lb.	Sq.In.	d_1	d_2					
3 × 2	$\frac{1}{2}$	7.7	2.25	1.08	0.58	1.92	0.67	0.92	0.55	.47
3 × 2	$\frac{1}{4}$	4.0	1.19	0.99	0.49	1.09	0.39	0.95	0.56	.46
$2\frac{1}{2}$ × 2	$\frac{1}{2}$	6.8	2.00	0.88	0.63	1.14	0.64	0.75	0.56	.44
$2\frac{1}{2}$ × 2	$\frac{3}{16}$	2.8	0.81	0.76	0.51	0.51	0.29	0.79	0.60	.43
$2\frac{1}{4}$ × $1\frac{1}{2}$	$\frac{1}{2}$	5.5	1.63	0.86	0.48	0.82	0.26	0.71	0.40	.39
$2\frac{1}{4}$ × $1\frac{1}{2}$	$\frac{3}{16}$	2.3	0.67	0.75	0.37	0.34	0.12	0.72	0.43	.40
2 × $1\frac{3}{8}$	$\frac{1}{4}$	2.7	0.78	0.69	0.37	0.37	0.12	0.63	0.39	.30
2 × $1\frac{3}{8}$	$\frac{3}{16}$	2.1	0.60	0.66	0.35	0.24	0.09	0.63	0.40	.29

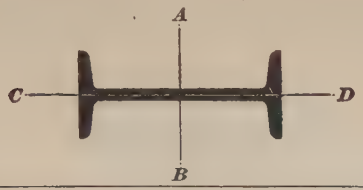
PROPERTIES OF Z BARS.

Depth of Web.	Width of Flange.	Thickness of Metal.	Weight per Foot.	Area of Section.	Moment of Inertia, Axis *A B.*	Radius of Gyration, Inches, Axis *A B.*	Moment of Inertia, Axis *C D.*	Radius of Gyration, Inches, Axis *C D.*	Least Radius of Gyration, Inches, Axis Diagonal.
In.	In.	In.	Lb.	Sq.In.	*I*	*R*	*I*	*R*	*R'*
6	$3\frac{1}{2}$	$\frac{3}{8}$	15.6	4.59	25.32	2.35	9.11	1.41	0.83
$6\frac{1}{16}$	$3\frac{9}{16}$	$\frac{7}{16}$	18.3	5.39	29.80	2.35	10.95	1.43	0.84
$6\frac{1}{8}$	$3\frac{5}{8}$	$\frac{1}{2}$	21.0	6.19	34.36	2.36	12.87	1.44	0.84
6	$3\frac{1}{2}$	$\frac{9}{16}$	22.7	6.68	34.64	2.28	12.59	1.37	0.81
$6\frac{1}{16}$	$3\frac{9}{16}$	$\frac{5}{8}$	25.4	7.46	38.86	2.28	14.42	1.39	0.82
$6\frac{1}{8}$	$3\frac{5}{8}$	$\frac{11}{16}$	28.0	8.25	43.18	2.29	16.34	1.41	0.84

PROPERTIES OF Z BARS—*Continued.*

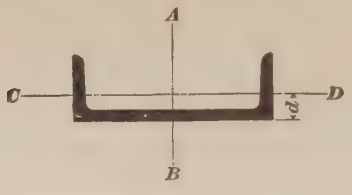
Depth of Web.	Width of Flange.	Thickness of Metal.	Weight per Foot.	Area of Section.	Moment of Inertia, Axis *A B*.	Radius of Gyration, Inches, Axis *A B*.	Moment of Inertia, Axis *C D*.	Radius of Gyration, Inches, Axis *C D*.	Least Radius of Gyration, Inches, Axis Diagonal.
In.	In.	In.	Lb.	Sq. In.	*I*	*R*	*I*	*R*	*R'*
6	3 1/2	3/4	29.3	8.63	42.12	2.21	15.44	1.34	0.81
6 1/16	3 9/16	13/16	32.0	9.40	46.13	2.22	17.27	1.36	0.82
6 1/8	3 5/8	7/8	34.6	10.17	50.22	2.22	19.18	1.37	0.83
5	3 1/4	5/16	11.6	3.40	13.36	1.98	6.18	1.35	0.75
5 1/16	3 5/16	3/8	13.9	4.10	16.18	1.99	7.65	1.37	0.76
5 1/8	3 3/8	7/16	16.4	4.81	19.07	1.99	9.20	1.38	0.77
5	3 1/4	1/2	17.8	5.25	19.19	1.91	9.05	1.31	0.74
5 1/16	3 5/16	9/16	20.2	5.94	21.83	1.91	10.51	1.33	0.75
5 1/8	3 3/8	5/8	22.6	6.64	24.53	1.92	12.06	1.35	0.76
5	3 1/4	11/16	23.7	6.96	23.68	1.84	11.37	1.28	0.73
5 1/16	3 5/16	3/4	26.0	7.64	26.16	1.85	12.83	1.30	0.75
5 1/8	3 3/8	13/16	28.3	8.33	29.31	1.88	14.36	1.31	0.76
4	3 1/16	1/4	8.2	2.41	6.28	1.62	4.23	1.33	0.67
4 1/16	3 1/8	5/16	10.3	3.03	7.94	1.62	5.46	1.34	0.68
4 1/8	3 3/16	3/8	12.4	3.66	9.63	1.62	6.77	1.36	0.69
4	3 1/16	7/16	13.8	4.05	9.66	1.55	6.73	1.29	0.66
4 1/16	3 1/8	1/2	15.8	4.66	11.18	1.55	7.96	1.31	0.67
4 1/8	3 3/16	9/16	17.9	5.27	12.74	1.55	9.26	1.33	0.69
4	3 1/16	5/8	18.9	5.55	12.11	1.48	8.73	1.25	0.66
4 1/16	3 1/8	11/16	20.9	6.14	13.52	1.48	9.95	1.27	0.67
4 1/8	3 3/16	3/4	22.9	6.75	14.97	1.49	11.24	1.29	0.69
3	2 11/16	1/4	6.7	1.97	2.87	1.21	2.81	1.19	0.55
3 1/16	2 3/4	5/16	8.4	2.48	3.64	1.21	3.64	1.21	0.56
3	2 11/16	3/8	9.7	2.86	3.85	1.16	3.92	1.17	0.55
3 1/16	2 3/4	7/16	11.4	3.36	4.57	1.17	4.75	1.19	0.56
3	2 11/16	1/2	12.5	3.69	4.59	1.12	4.85	1.15	0.55
3 1/16	2 3/4	9/16	14.2	4.19	5.26	1.12	5.70	1.17	0.56

PROPERTIES OF I BEAMS.



Depth of Beam.	Weight per Foot.	Area.	Thickness of Web.	Width of Flange.	Moment of Inertia, Axis *A B*.	Radius of Gyration, Axis *A B*.	Moment of Inertia, Axis *C D*.	Radius of Gyration, Axis *C D*.
In.	Lb.	Sq. In.	In.	In.	*I*	*R*	*I'*	*R'*
20	90	26.4	.78	6.75	1,506.10	7.55	42.30	1.27
20	80	23.5	.69	6.38	1,345.10	7.55	33.20	1.19
20	75	22.1	.66	6.16	1,246.90	7.53	28.20	1.13
20	65	19.1	.50	6.00	1,148.60	7.76	25.50	1.16
15	75	22.1	.81	6.29	720.40	5.72	34.60	1.25
15	$66\frac{2}{3}$	19.7	.65	6.13	676.30	5.87	31.70	1.27
15	60	17.6	.52	6.00	637.70	6.02	29.20	1.29
15	50	14.7	.45	5.75	529.70	6.00	21.00	1.20
15	42	12.4	.40	5.50	429.60	5.90	14.00	1.08
12	55	16.1	.63	6.00	358.10	4.72	25.20	1.25
12	40	11.8	.39	5.50	281.30	4.90	16.80	1.20
12	$31\frac{1}{2}$	9.3	.35	5.13	220.50	4.88	10.30	1.04
10	40	11.8	.58	5.21	178.50	3.89	13.50	1.07
10	33	9.7	.37	5.00	161.30	4.08	11.80	1.10
10	30	8.8	.45	4.89	134.50	3.90	8.10	0.96
10	25	7.3	.31	4.75	122.50	4.06	7.30	0.99
9	27	7.9	.31	4.75	110.60	3.72	9.10	1.07
9	$23\frac{1}{3}$	6.9	.35	4.58	89.00	3.60	5.90	0.93
9	21	6.2	.27	4.50	84.30	3.70	5.56	0.95
8	27	7.9	.48	4.56	77.60	3.14	6.91	0.93
8	22	6.4	.29	4.38	69.70	3.30	6.02	0.97
8	18	5.2	.25	4.13	56.80	3.30	3.95	0.87
7	20	5.7	.28	4.09	47.60	2.89	4.86	0.92
7	15	4.4	.23	3.88	37.10	2.89	3.12	0.84
6	15	4.3	.25	3.52	26.40	2.47	2.74	0.79
6	12	3.6	.22	3.38	21.70	2.47	1.91	0.73
5	13	3.8	.26	3.13	15.70	2.06	1.98	0.72
5	$9\frac{3}{4}$	2.9	.21	3.00	12.10	2.06	1.29	0.67
4	10	2.9	.39	2.69	6.84	1.53	0.89	0.55
4	$7\frac{1}{2}$	2.2	.20	2.50	5.86	1.63	0.70	0.56
4	6	1.8	.18	2.19	4.59	1.61	0.38	0.47

PROPERTIES OF CHANNELS.



Depth of Channel.	Weight per Foot.	Area.	Thickness of Web.	Width of Flange.	Moment of Inertia, Axis $A\,B$.	Radius of Gyration, Inches, Axis $A\,B$.	Moment of Inertia, Axis $C\,D$.	Radius of Gyration, Inches, Axis $C\,D$.	Distance d.
In.	Lb.	Sq. In.	In.	In.	I	R	I'	R'	In.
15	40	11.80	.47	3.63	371.60	5.62	11.500	.99	.89
15	33	9.70	.40	3.38	304.20	5.64	7.900	.92	.79
12	27	7.90	.38	3.13	161.00	4.54	5.730	.86	.78
12	20	5.90	.28	2.88	124.70	4.59	3.690	.79	.69
10	20	5.90	.31	2.88	85.50	3.81	3.750	.80	.74
10	15	4.40	.25	2.60	66.82	3.89	2.490	.74	.66
9	16	4.70	.28	2.56	57.09	3.48	2.590	.74	.66
9	13	3.80	.23	2.36	45.48	3.46	1.640	.64	.57
8	13	3.80	.25	2.22	35.56	3.07	1.470	.62	.58
8	10	3.00	.20	2.08	28.20	3.08	1.000	.58	.52
7	13	3.80	.28	2.22	27.35	2.69	1.890	.71	.62
7	9	2.61	.20	2.00	19.05	2.70	.814	.56	.51
6	17	4.85	.38	2.41	25.43	2.28	2.390	.70	.78
6	12	3.48	.28	2.19	18.70	2.32	1.380	.63	.65
6	8	2.35	.20	1.94	12.75	2.33	.710	.55	.52
5	9	2.59	.25	1.91	9.67	1.93	.810	.55	.57
5	6	1.76	.18	1.66	6.53	1.93	.390	.47	.45
4	8	2.31	.27	1.86	5.47	1.54	.685	.55	.59
4	5	1.46	.17	1.59	3.59	1.57	.293	.45	.46

RADII OF GYRATION FOR TWO ANGLES.

PLACED BACK TO BACK, SHORT LEG VERTICAL.

UNEQUAL LEGS.

The different radii of gyration are indicated in the figures by arrowheads.

Size. Inches.	Thickness. Inches.	Radii of Gyration.			
		R_0	R_1	R_2	R_3
6×4	$\frac{7}{8}$	1.19	2.94	3.13	3.23
6×4	$\frac{3}{8}$	1.17	2.74	2.92	3.02
$5\times3\frac{1}{2}$	$\frac{3}{4}$	1.01	2.39	2.58	2.68
$5\times3\frac{1}{2}$	$\frac{3}{8}$	1.02	2.27	2.45	2.55
5×3	$\frac{3}{4}$	.86	2.50	2.69	2.79
5×3	$\frac{5}{16}$	.85	2.33	2.51	2.61
$4\frac{1}{2}\times3$	$\frac{3}{4}$	.86	2.18	2.38	2.46
$4\frac{1}{2}\times3$	$\frac{5}{16}$	.87	2.06	2.25	2.33
$4\times3\frac{1}{2}$	$\frac{3}{4}$	1.05	1.85	2.04	2.14
$4\times3\frac{1}{2}$	$\frac{5}{16}$	1.07	1.73	1.91	2.00
4×3	$\frac{5}{8}$	.83	1.84	2.03	2.13
4×3	$\frac{5}{16}$	.89	1.79	1.97	2.07
$3\frac{1}{2}\times3$	$\frac{5}{8}$	.87	1.57	1.76	1.87
$3\frac{1}{2}\times3$	$\frac{5}{16}$	.90	1.53	1.71	1.81
$3\frac{1}{2}\times2\frac{1}{2}$	$\frac{9}{16}$	.72	1.66	1.85	1.95
$3\frac{1}{2}\times2\frac{1}{2}$	$\frac{1}{4}$	.74	1.58	1.76	1.86
$3\times2\frac{1}{2}$	$\frac{9}{16}$	.73	1.40	1.59	1.69
$3\times2\frac{1}{2}$	$\frac{1}{4}$	.75	1.32	1.49	1.60
3×2	$\frac{1}{2}$	.55	1.42	1.62	1.72
3×2	$\frac{1}{4}$	.57	1.39	1.57	1.68

RADII OF GYRATION FOR TWO ANGLES.

PLACED BACK TO BACK, LONG LEG VERTICAL.

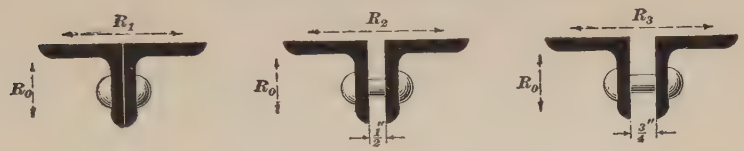
UNEQUAL LEGS.

The different radii of gyration are indicated in the figures by arrowheads.

Size. Inches.	Thickness. Inches.	Radii of Gyration.			
		R_0	R_1	R_2	R_3
6×4	$\frac{7}{8}$	1.95	1.68	1.87	1.97
6×4	$\frac{3}{8}$	1.93	1.50	1.67	1.76
$5\times3\frac{1}{2}$	$\frac{3}{4}$	1.59	1.44	1.63	1.73
$5\times3\frac{1}{2}$	$\frac{3}{8}$	1.60	1.34	1.51	1.61
5×3	$\frac{3}{4}$	1.62	1.23	1.42	1.52
5×3	$\frac{5}{16}$	1.61	1.09	1.26	1.36
$4\frac{1}{2}\times3$	$\frac{3}{4}$	1.43	1.25	1.44	1.55
$4\frac{1}{2}\times3$	$\frac{5}{16}$	1.45	1.13	1.31	1.40
$4\times3\frac{1}{2}$	$\frac{3}{4}$	1.24	1.53	1.72	1.83
$4\times3\frac{1}{2}$	$\frac{5}{16}$	1.26	1.41	1.58	1.69
4×3	$\frac{5}{8}$	1.23	1.20	1.39	1.50
4×3	$\frac{5}{16}$	1.27	1.17	1.35	1.45
$3\frac{1}{2}\times3$	$\frac{5}{8}$	1.06	1.27	1.46	1.56
$3\frac{1}{2}\times3$	$\frac{5}{16}$	1.10	1.21	1.39	1.49
$3\frac{1}{2}\times2\frac{1}{2}$	$\frac{9}{16}$	1.10	1.04	1.23	1.34
$3\frac{1}{2}\times2\frac{1}{2}$	$\frac{1}{4}$	1.12	.96	1.17	1.24
$3\times2\frac{1}{2}$	$\frac{9}{16}$	.93	1.07	1.27	1.37
$3\times2\frac{1}{2}$	$\frac{1}{4}$	.95	1.00	1.18	1.28
3×2	$\frac{1}{2}$	.92	.80	1.00	1.10
3×2	$\frac{1}{4}$	.96	.75	.93	1.04
$2\frac{1}{4}\times1\frac{1}{2}$	$\frac{5}{16}$	.70	.60	.79	.91
$2\frac{1}{4}\times1\frac{1}{2}$	$\frac{3}{16}$	.72	.57	.75	.86

RADII OF GYRATION FOR TWO ANGLES.

PLACED BACK TO BACK.



EQUAL LEGS.

The different radii of gyration are indicated in the figures by arrowheads.

Size. Inches.	Thickness. Inches.	Radii of Gyration.			
		R_0	R_1	R_2	R_3
6×6	$\frac{7}{8}$	1.87	2.64	2.83	2.92
6×6	$\frac{3}{8}$	1.88	2.49	2.66	2.75
5×5	$\frac{3}{4}$	1.55	2.20	2.38	2.48
5×5	$\frac{3}{8}$	1.56	2.09	2.27	2.36
4×4	$\frac{13}{16}$	1.24	1.83	2.03	2.12
4×4	$\frac{5}{16}$	1.24	1.67	1.85	1.94
$3\frac{1}{2}\times3\frac{1}{2}$	$\frac{5}{8}$	1.04	1.51	1.70	1.81
$3\frac{1}{2}\times3\frac{1}{2}$	$\frac{5}{16}$	1.08	1.46	1.65	1.74
3×3	$\frac{5}{8}$	.94	1.40	1.59	1.69
3×3	$\frac{1}{4}$	.93	1.25	1.43	1.53
$2\frac{1}{2}\times2\frac{1}{2}$	$\frac{1}{2}$	.76	1.12	1.31	1.42
$2\frac{1}{2}\times2\frac{1}{2}$	$\frac{1}{4}$	.77	1.05	1.25	1.34
$2\frac{1}{4}\times2\frac{1}{4}$	$\frac{1}{2}$	.70	1.05	1.25	1.35
$2\frac{1}{4}\times2\frac{1}{4}$	$\frac{3}{16}$	.69	.94	1.12	1.22
2×2	$\frac{1}{2}$	.62	.95	1.15	1.26
2×2	$\frac{3}{16}$	.62	.84	1.03	1.13

MODULI OF ELASTICITY.

METALS.

Iron (cast),	12,000,000
Iron (wrought shapes),	27,000,000
Iron (rerolled bars),	26,000,000
Steel (casting),	30,000,000
Steel (structural),	29,000,000

TIMBER.

Chestnut,	1,000,000
Cypress,	900,000
Cedar,	700,000
Hemlock,	900,000
Oak (White),	1,100,000
Pine (White),	1,000,000
Pine (Southern, Long-leaf, or Georgia Yellow Pine),	1,700,000
Pine (Douglass, Oregon and Washington Fir, or Yellow Pine),	1,400,000
Pine (Northern Short-leaf Yellow Pine),	1,200,000
Pine (Red),	1,200,000
Pine (Norway),	1,200,000
Pine (Red, Ontario, Canadian),	1,400,000
Redwood (California),	700,000
Spruce and Eastern Fir,	1,200,000
Spruce (California),	1,200,000

RESISTING MOMENTS OF PINS.

With Extreme Fiber Stresses
Varying from 15,000 to 25,000 Pounds per Square Inch.

Diameter of Pin in Inches.	Area of Pin in Square Inches.	Moments in Inch-Pounds for Fiber Strains of				Diameter of Pin in Inches.
		15,000 lb. per Square Inch.	20,000 lb. per Square Inch.	22,000 lb. per Square Inch.	25,000 lb. per Square Inch.	
1	0.785	1,470	1,960	2,160	2,450	1
1 1/8	0.994	2,100	2,800	3,080	3,500	1 1/8
1 1/4	1.227	2,880	3,830	4,220	4,790	1 1/4
1 3/8	1.485	3,830	5,100	5,620	6,380	1 3/8
1 1/2	1.767	4,970	6,630	7,290	8,280	1 1/2
1 5/8	2.074	6,320	8,430	9,270	10,500	1 5/8
1 3/4	2.405	7,890	10,500	11,570	13,200	1 3/4
1 7/8	2.761	9,710	12,900	14,240	16,200	1 7/8
2	3.142	11,800	15,700	17,280	19,600	2
2 1/8	3.547	14,100	18,800	20,730	23,600	2 1/8
2 1/4	3.976	16,800	22,400	24,600	28,000	2 1/4
2 3/8	4.430	19,700	26,300	28,900	32,900	2 3/8
2 1/2	4.909	23,000	30,700	33,700	38,400	2 1/2
2 5/8	5.412	26,600	35,500	39,000	44,400	2 5/8
2 3/4	5.940	30,600	40,800	44,900	51,000	2 3/4
2 7/8	6.492	35,000	46,700	51,300	58,300	2 7/8
3	7.069	39,800	53,000	58,300	66,300	3
3 1/8	7.670	44,900	59,900	65,900	74,900	3 1/8
3 1/4	8.296	50,600	67,400	74,100	84,300	3 1/4
3 3/8	8.946	56,600	75,500	83,000	94,400	3 3/8
3 1/2	9.621	63,100	84,200	92,600	105,200	3 1/2
3 5/8	10.321	70,100	93,500	102,900	116,900	3 5/8
3 3/4	11.045	77,700	103,500	113,900	129,400	3 3/4
3 7/8	11.793	85,700	114,200	125,600	142,800	3 7/8
4	12.566	94,200	125,700	138,200	157,100	4
4 1/8	13.364	103,400	137,800	151,600	172,300	4 1/8
4 1/4	14.186	113,000	150,700	165,800	188,400	4 1/4
4 3/8	15.033	123,300	164,400	180,800	205,500	4 3/8
4 1/2	15.904	134,200	178,900	196,800	223,700	4 1/2
4 5/8	16.800	145,700	194,300	213,700	242,800	4 5/8
4 3/4	17.721	157,800	210,400	231,500	263,000	4 3/4
4 7/8	18.665	170,600	227,500	250,200	284,400	4 7/8

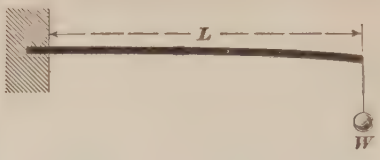
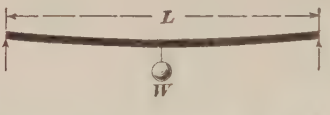
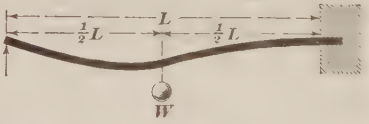
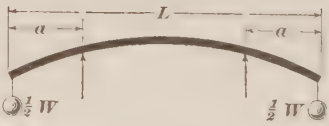
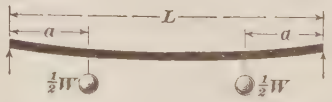
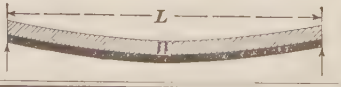
RESISTING MOMENTS OF PINS—*Continued.*

Diameter of Pin in Inches.	Area of Pin in Square Inches.	Moments in Inch-Pounds for Fiber Strains of				Diameter of Pin in Inches.
		15,000 lb. per Square Inch.	20,000 lb. per Square Inch.	22,000 lb. per Square Inch.	25,000 lb. per Square Inch.	
5	19.635	184,100	245,400	270,000	306,800	5
5 1/8	20.629	198,200	264,300	290,700	330,400	5 1/8
5 1/4	21.648	213,100	284,100	312,500	355,200	5 1/4
5 3/8	22.691	228,700	304,900	335,400	381,100	5 3/8
5 1/2	23.758	245,000	326,700	359,300	408,300	5 1/2
5 5/8	24.850	262,100	349,500	384,400	436,800	5 5/8
5 3/4	25.967	280,000	373,300	410,600	466,600	5 3/4
5 7/8	27.109	298,600	398,200	438,000	497,700	5 7/8
6	28.274	318,100	424,100	466,500	530,200	6
6 1/8	29.465	338,400	451,200	496,300	564,000	6 1/8
6 1/4	30.680	359,500	479,400	527,300	599,200	6 1/4
6 3/8	31.919	381,500	508,700	559,600	635,900	6 3/8
6 1/2	33.183	404,400	539,200	593,100	674,000	6 1/2
6 5/8	34.472	428,200	570,900	628,000	713,700	6 5/8
6 3/4	35.785	452,900	603,900	664,200	754,800	6 3/4
6 7/8	37.122	478,500	638,000	701,800	797,500	6 7/8
7	38.485	505,100	673,500	740,800	841,900	7
7 1/8	39.871	532,700	710,200	781,200	887,800	7 1/8
7 1/4	41.282	561,200	748,200	823,000	935,300	7 1/4
7 3/8	42.718	590,700	787,600	866,300	984,500	7 3/8
7 1/2	44.179	621,300	828,400	911,200	1,035,400	7 1/2
7 5/8	45.664	652,900	870,500	957,500	1,088,100	7 5/8
7 3/4	47.173	685,500	914,000	1,005,300	1,142,500	7 3/4
7 7/8	48.707	719,200	958,900	1,054,800	1,198,700	7 7/8
8	50.265	754,000	1,005,300	1,105,800	1,256,600	8
8 1/8	51.849	789,900	1,053,200	1,158,500	1,316,500	8 1/8
8 1/4	53.456	826,900	1,102,500	1,212,800	1,378,200	8 1/4
8 3/8	55.088	865,100	1,153,400	1,268,800	1,441,800	8 3/8
8 1/2	56.745	904,400	1,205,800	1,326,400	1,507,300	8 1/2
8 5/8	58.426	944,900	1,259,800	1,385,800	1,574,800	8 5/8
8 3/4	60.132	986,500	1,315,400	1,446,900	1,644,200	8 3/4
8 7/8	61.862	1,029,400	1,372,500	1,509,800	1,715,700	8 7/8

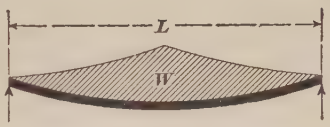
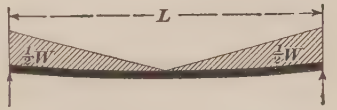
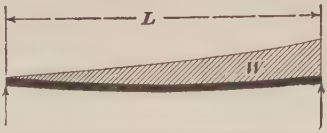
RESISTING MOMENTS OF PINS—*Continued.*

Diameter of Pin in Inches.	Area of Pin in Square Inches.	Moments in Inch-Pounds for Fiber Strains of				Diameter of Pin in Inches.
		15,000 lb. per Square Inch.	20,000 lb. per Square Inch.	22,000 lb. per Square Inch.	25,000 lb. per Square Inch.	
9	63.617	1,073,500	1,431,400	1,574,500	1,789,200	9
9 1/8	65.397	1,118,900	1,491,900	1,641,100	1,864,800	9 1/8
9 1/4	67.201	1,165,500	1,554,000	1,709,400	1,942,500	9 1/4
9 3/8	69.029	1,213,400	1,617,900	1,779,600	2,022,300	9 3/8
9 1/2	70.882	1,262,600	1,683,400	1,851,800	2,104,300	9 1/2
9 5/8	72.760	1,313,100	1,750,800	1,925,900	2,188,500	9 5/8
9 3/4	74.662	1,364,900	1,819,900	2,001,900	2,274,900	9 3/4
9 7/8	76.590	1,418,100	1,890,800	2,079,900	2,363,500	9 7/8
10	78.540	1,472,600	1,963,500	2,159,900	2,454,400	10
10 1/4	82.520	1,585,900	2,114,500	2,325,900	2,643,100	10 1/4
10 1/2	86.590	1,704,700	2,273,000	2,500,200	2,841,200	10 1/2
10 3/4	90.760	1,829,400	2,439,300	2,683,200	3,049,100	10 3/4
11	95.030	1,960,100	2,613,400	2,874,800	3,266,800	11
11 1/4	99.400	2,096,800	2,795,700	3,075,400	3,494,800	11 1/4
11 1/2	103.870	2,239,700	2,986,300	3,284,800	3,732,800	11 1/2
12	113.100	2,544,700	3,392,900	3,732,200	4,241,200	12

DEFLECTION OF BEAMS.

Description.	Mode of Loading. Lengths in Inches. Loads in Pounds.	Greatest Deflection in Inches.
One end firmly fixed, other end loaded.		$\frac{W L^3}{3 E I}$
Supported at both ends, loaded at the center.		$\frac{W L^3}{48 E I}$
Supported at both ends, loaded any place.		$\frac{W a b \sqrt{3 a (2L-a)^3}}{27 L E I}$
One end fixed, other end supported, loaded at center.		$\frac{3 W L^3}{322 E I}$
Both ends fixed, loaded at center.		$\frac{W L^3}{192 E I}$
Loaded at each end, two supports between ends.		For overhang: $\frac{W a}{12 E I}(3 a L - 4 a^2)$ Between supports: $\frac{W a}{16 E I}(L - 2 a)^2$
Both ends supported, two symmetrical loads.		$\frac{W a}{48 E I}(3 L^2 - 4 a^2)$
One end fixed, load uniformly distributed.		$\frac{W L^3}{8 E I}$
Both ends supported, load uniformly distributed.		$\frac{5 W L^3}{384 E I}$

DEFLECTION OF BEAMS—*Continued.*

Description.	Mode of Loading. Lengths in Inches. Loads in Pounds.	Greatest Deflection in Inches.
Both ends fixed, load uniformly distributed.		$\frac{WL^3}{384EI}$
One end fixed, load distributed, increasing uniformly towards the fixed ends.		$\frac{WL^3}{15EI}$
Both ends supported, load distributed, increasing uniformly towards the center.		$\frac{WL^3}{60EI}$
Both ends supported, load distributed, decreasing uniformly towards the center.		$\frac{3WL^3}{320EI}$
Both ends supported, load increasing uniformly towards one end.		$\frac{47WL^3}{3,600EI}$

A SERIES

OF

QUESTIONS AND EXAMPLES

RELATING TO THE SUBJECTS
TREATED OF IN THIS VOLUME.

It will be noticed that the questions and examples contained in the following pages are divided into sections corresponding to the sections of the text of the preceding pages, so that each section has a headline which is the same as the headline of the section to which the questions refer. No attempt should be made to answer any questions or to work any examples until the corresponding part of the text has been carefully studied.

ARITHMETIC.

(SECTION 1.)

EXAMINATION QUESTIONS.

(1) What is arithmetic?

(2) What is a number?

(3) What is the difference between a concrete number and an abstract number?

(4) Define notation and numeration.

(5) Write each of the following numbers in words:
(*a*) 980; (*b*) 605; (*c*) 28,284; (*d*) 9,006,042; (*e*) 850,317,002; (*f*) 700,004.

(6) Represent in figures the following expressions:
(*a*) Seven thousand six hundred. (*b*) Eighty-one thousand four hundred two. (*c*) Five million four thousand seven. (*d*) One hundred eight million ten thousand one. (*e*) Eighteen million six. (*f*) Thirty thousand ten.

(7) What is the sum of $3,290 + 504 + 865,403 + 2,074 + 81 + 7$?
Ans. 871,359.

(8) $709 + 8,304,725 + 391 + 100,302 + 300 + 909 =$ what?
Ans. 8,407,336.

(9) Find the difference between the following:
(*a*) 50,962 and 3,338; (*b*) 10,001 and 15,339.
Ans. { (*a*) 47,624. (*b*) 5,338.

For notice of the copyright, see page immediately following the title page.

(10) (*a*) $70,968 - 32,975 = ?$ (*b*) $100,000 - 98,735 = ?$

Ans. { (*a*) 37,993. (*b*) 1,265.

(11) The greater of two numbers is 1,004 and their difference is 49; what is their sum? Ans. 1,959.

(12) From $5,962 + 8,471 + 9,023$ take $3,874 + 2,039$.

Ans. 17,543.

(13) A man willed $125,000 to his wife and two children; to his son he gave $44,675, to his daughter $26,380, and to his wife the remainder. What was his wife's share?

Ans. $53,945.

(14) Find the products of the following:

(*a*) $526,387 \times 7$; (*b*) $700,298 \times 17$; (*c*) $217 \times 103 \times 67$.

Ans. { (*a*) 3,684,709. (*b*) 11,905,066. (*c*) 1,497,517.

(15) If your watch ticks once every second, how many times will it tick in one week? Ans. 604,800 times.

(16) If a monthly publication contains 24 pages in each issue, how many pages will there be in 8 yearly volumes?

Ans. 2,304.

(17) An engine and boiler in a manufactory are worth $3,246. The building is worth three times as much, plus $1,200, and the tools are worth twice as much as the building, plus $1,875. (*a*) What is the value of the building and tools? (*b*) What is the value of the whole plant?

Ans. { (*a*) $34,689. (*b*) $37,935.

(18) Solve the following by cancelation:

(*a*) $\frac{72 \times 48 \times 28 \times 5}{96 \times 15 \times 7 \times 6} = ?$ (*b*) $\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$

Ans. { (*a*) 8. (*b*) 32.

(19) If a mechanic earns $1,500 a year for his labor, and his expenses are $968 per year, in what time can he save enough to buy 28 acres of land at $133 an acre?

Ans. 7 yr.

(20) A freight train ran 365 miles in one week, and 3 times as far, lacking 246 miles, the next week; how far did it run the second week? Ans. 849 mi.

(21) If the driving wheel of a locomotive is 16 feet in circumference, how many revolutions will it make in going from Philadelphia to Pittsburg, the distance between which is 354 miles, there being 5,280 feet in one mile?

Ans. 116,820 rev.

(22) What is the quotient of:

(*a*) 589,824 ÷ 576? (*b*) 369,730,620 ÷ 43,911? (*c*) 2,527,525 ÷ 505? (*d*) 4,961,794,302 ÷ 1,234?

Ans. (*a*) 1,024.
(*b*) 8,420.
(*c*) 5,005.
(*d*) 4,020,903.

(23) A man paid $444 for a horse, wagon, and harness. If the horse cost $264 and the wagon $153, how much did the harness cost? Ans. $27.

(24) What is the product of:

(*a*) 1,024 × 576? (*b*) 5,005 × 505? (*c*) 43,911 × 8,420?

Ans. (*a*) 589,824.
(*b*) 2,527,525.
(*c*) 369,730,620.

(25) If a man receives 30 cents an hour for his wages, how much will he earn in a year, working 10 hours a day and averaging 25 days per month? Ans. $900.

ARITHMETIC.

(SECTION 2.)

EXAMINATION QUESTIONS.

(26) What is a fraction?

(27) What are the terms of a fraction?

(28) What does the denominator show?

(29) What does the numerator show?

(30) How do you find the value of a fraction?

(31) Is $\frac{13}{8}$ a proper or an improper fraction, and why?

(32) Write three mixed numbers.

(33) Reduce the following fractions to their lowest terms: $\frac{4}{8}$, $\frac{4}{16}$, $\frac{8}{32}$, $\frac{32}{64}$. Ans. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{2}$.

(34) Reduce 6 to an improper fraction whose denominator is 4. Ans. $\frac{24}{4}$.

(35) Reduce $7\frac{7}{8}$, $13\frac{5}{16}$, and $10\frac{3}{4}$ to improper fractions. Ans. $\frac{63}{8}$, $\frac{213}{16}$, $\frac{43}{4}$.

(36) What is the value of each of the following: $\frac{13}{2}$, $\frac{17}{4}$, $\frac{69}{16}$, $\frac{16}{8}$, $\frac{67}{64}$? Ans. $6\frac{1}{2}$, $4\frac{1}{4}$, $4\frac{5}{16}$, 2, $1\frac{3}{64}$.

(37) Solve the following:

(*a*) $35 \div \frac{5}{16}$; (*b*) $\frac{9}{16} \div 3$; (*c*) $\frac{17}{2} \div 9$; (*d*) $\frac{113}{64} \div \frac{7}{16}$; (*e*) $15\frac{3}{4} \div 4\frac{3}{8}$.

Ans. (*a*) 112. (*b*) $\frac{3}{16}$. (*c*) $\frac{17}{18}$. (*d*) $4\frac{1}{28}$. (*e*) $3\frac{3}{5}$.

For notice of copyright, see page immediately following the title page.

(38) $\frac{1}{8}+\frac{2}{8}+\frac{5}{8}=?$ Ans. 1.

(39) $\frac{1}{4}+\frac{3}{8}+\frac{5}{16}=?$ Ans. $\frac{15}{16}$.

(40) $42+31\frac{5}{8}+9\frac{7}{16}=?$ Ans. $83\frac{1}{16}$.

(41) An iron plate is divided into four sections: the first contains $29\frac{3}{4}$ square inches; the second, $50\frac{5}{8}$ square inches; the third, 41 square inches; and the fourth, $69\frac{3}{16}$ square inches. How many square inches are in the plate?

Ans. $190\frac{9}{16}$ sq. in.

(42) Find the value of each of the following:

$$(a)\ \frac{\frac{7}{3}}{16};\quad (b)\ \frac{\frac{15}{32}}{\frac{5}{8}};\quad (c)\ \frac{\frac{4+3}{2+6}}{5}.$$

Ans. $\begin{cases} (a) & 37\frac{1}{3}. \\ (b) & \frac{3}{4}. \\ (c) & \frac{7}{40}. \end{cases}$

(43) The numerator of a fraction is 28, and the value of the fraction $\frac{7}{8}$; what is the denominator? Ans. 32.

(44) What is the difference between (*a*) $\frac{7}{8}$ and $\frac{7}{16}$? (*b*) 13 and $7\frac{7}{16}$? (*c*) $312\frac{9}{16}$ and $229\frac{5}{32}$?

Ans. $\begin{cases} (a) & \frac{7}{16}. \\ (b) & 5\frac{9}{16}. \\ (c) & 83\frac{13}{32}. \end{cases}$

(45) If a man travels $85\frac{5}{12}$ miles in one day, $78\frac{9}{15}$ miles in another day, and $125\frac{17}{35}$ miles in another day, how far did he travel in the three days? Ans. $289\frac{211}{420}$ mi.

(46) From $573\frac{4}{5}$ tons take $216\frac{5}{8}$ tons. Ans. $357\frac{7}{40}$ T.

(47) At $\frac{3}{8}$ of a dollar a yard, what will be the cost of $9\frac{1}{4}$ yards of cloth? Ans. $3\frac{15}{32}$ dollars.

(48) Multiply $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{11}$ of $\frac{19}{20}$ of 11 by $\frac{7}{8}$ of $\frac{5}{6}$ of 45.

Ans. $109\frac{13}{128}$.

(49) How many times is $\frac{2}{3}$ contained in $\frac{3}{4}$ of 16?

Ans. 18 times.

(50) Bought $211\frac{1}{4}$ pounds of old lead for $1\frac{7}{8}$ cents per pound. Sold a part of it for $2\frac{1}{2}$ cents per pound, receiving for it the same amount as I paid for the whole. How many pounds did I have left? Ans. $52\frac{13}{16}$ lb.

ARITHMETIC.

(SECTION 3.)

EXAMINATION QUESTIONS.

(51) Write out in words the following numbers: .08, .131, .0001, .000027, .0108, and 93.0101.

(52) How do you place decimals for addition and subtraction?

(53) Give a rule for multiplication of decimals.

(54) Give a rule for division of decimals.

(55) State the difference between a fraction and a decimal.

(56) State how to reduce a fraction to a decimal.

(57) Reduce the following fractions to equivalent decimals: $\frac{1}{2}$, $\frac{7}{8}$, $\frac{5}{32}$, $\frac{65}{100}$, and $\frac{125}{1000}$.

Ans. { .5. .875. .15625. .65. .125.

(58) Solve the following:

(*a*) $\dfrac{32.5+.29+1.5}{4.7+9}$;

(*b*) $\dfrac{1.283\times\overline{8+5}}{2.63}$;

(*c*) $\dfrac{\overline{589+27}\times 163-8}{25+39}$;

(*d*) $\dfrac{\overline{40.6+7.1}\times(3.029-1.874)}{6.27+8.53-8.01}$.

Ans. { (*a*) 2.5029. (*b*) 6.3418. (*c*) 1,491.875. (*d*) 8.1139.

(59) How many inches in .875 of a foot? Ans. $10\frac{1}{2}$ in.

For notice of copyright, see page immediately following the title page.

(60) What decimal part of a foot is $\frac{3}{16}$ of an inch?

Ans. .015625.

(61) A cubic inch of water weighs .03617 of a pound. What is the weight of a body of water whose volume is 1,500 cubic inches? Ans. 54.255 lb.

(62) If by selling a carload of coal for $82.50, at a profit of $1.65 per ton, I make enough to pay for 72.6 feet of fencing at $.50 a foot, how many tons of coal were in the car? Ans. 22 T.

(63) Divide 17,892 by 231, and carry the result to four decimal places. Ans. 77.4545+.

(64) What is the value of the following expression carried to three decimal places:

$$\frac{74.26\times24\times3.1416\times19\times19\times350}{33,000\times12\times4} = ?$$ Ans. 446.619—.

(65) Express: (*a*) .7928 in 64ths; (*b*) .1416 in 32ds; (*c*) .47915 in 16ths.

Ans. (*a*) $\frac{51}{64}$. (*b*) $\frac{5}{32}$. (*c*) $\frac{8}{16}$.

(66) Work out the following examples:

(*a*) 709.63 − .8514; (*b*) 81.963 − 1.7; (*c*) 18 − .18; (*d*) 1 − .001; (*e*) 872.1 − (.8721 + .008); (*f*) (5.028 + .0073) − (6.704 − 2.38).

Ans. (*a*) 708.7786. (*b*) 80.263. (*c*) 17.82. (*d*) .999. (*e*) 871.2199. (*f*) .7113.

(67) Work out the following:

(*a*) $\frac{7}{8}$ − .807; (*b*) .875 − $\frac{3}{8}$; (*c*) ($\frac{5}{32}$ + .435) − ($\frac{21}{100}$ − .07); (*d*) What is the difference between the sum of 33 millionths and 17 thousandths, and the sum of 53 hundredths and 274 thousandths?

Ans. (*a*) .068. (*b*) .5. (*c*) .45125. (*d*) .786967.

(68) What is the sum of .125, .7, .089, .4005, .9, and .000027? Ans. 2.214527.

(69) $927.416 + 8.274 + 372.6 + 62.07938 = ?$

Ans. 1,370.36938.

(70) Add 17 thousandths, 2 tenths, and 47 millionths.

Ans. .217047.

(71) Find the products of the following expressions:

(*a*) $.013 \times .107$; (*b*) $203 \times 2.03 \times .203$; (*c*) $2.7 \times 31.85 \times (3.16 - .316)$; (*d*) $(107.8 + 6.541 - 31.96) \times 1.742$.

Ans. (*a*) .001391. (*b*) 83.65427. (*c*) 244.56978. (*d*) 143.507702.

(72) Solve the following:

(*a*) $(\frac{7}{16} - .13) \times \overline{.625 + \frac{5}{8}}$; (*b*) $(\frac{19}{32} \times .21) - (.02 \times \frac{3}{16})$; (*c*) $(\frac{13}{4} + .013 - 2.17) \times \overline{13\frac{1}{4} - 7\frac{5}{16}}$.

Ans. (*a*) .384375. (*b*) .1209375. (*c*) 6.4896875.

(73) Solve the following:

(*a*) $.875 \div \frac{1}{2}$; (*b*) $\frac{7}{8} \div .5$; (*c*) $\frac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125}$.

Ans. (*a*) 1.75. (*b*) 1.75. (*c*) .5.

(74) Find the value of the following expression:

$$\frac{1.25 \times 20 \times 3}{\overline{87 + (11 \times 8)}} \cdot \overline{459 + 32}$$

Ans. $210\frac{3}{7}$.

(75) From 1 plus .001 take .01 plus .000001.

Ans. .990999.

ARITHMETIC.

(SECTION 4.)

EXAMINATION QUESTIONS.

(1) What is 25 per cent. of 8,428 lb.? Ans. 2,107 lb.

(2) What is 1 per cent. of $100? Ans. $1.

(3) What is ½ per cent. of $35,000? Ans. $175.

(4) What per cent. of 50 is 2? Ans. 4%.

(5) What per cent. of 10 is 10? Ans. 100%.

(6) Solve the following:

(*a*) Base = $2,522 and percentage = $176.54. What is the rate? (*b*) Percentage = 16.96 and rate = 8 per cent. What is the base? (*c*) Amount = 216.7025 and base = 213.5. What is the rate? (*d*) Difference = 201.825 and base = 207. What is the rate?

Ans. (*a*) 7%. (*b*) 212. (*c*) 1½%. (*d*) 2½%.

(7) A farmer gained 15% on his farm by selling it for $5,500. What did it cost him? Ans. $4,782.61.

(8) A man receives a salary of $950. He pays 24% of it for board, 12½% of it for clothing, and 17% of it for other expenses. How much does he save in a year? Ans. $441.75.

(9) If 37½ per cent. of a number is 961.38, what is the number? Ans. 2,563.68.

For notice of copyright, see page immediately following the title page.

(10) A man owns $\frac{3}{4}$ of a property. 30% of his share is worth $1,125. What is the whole property worth?

Ans. $5,000.

(11) What sum diminished by 35% of itself equals $4,810?

Ans. $7,400.

(12) A merchant's sales amounted to $197.55 on Monday, and this sum was 12½% of his sales for the week. How much were his sales for the week? Ans. $1,580.40.

(13) The distance between two stations on a certain railroad is 16.5 miles, which is 12½% of the entire length of the road. What is the length of the road? Ans. 132 mi.

(14) After paying 60% of my debts I find that I still owe $35. What was my whole indebtedness? Ans. $87.50.

(15) Reduce 28 rd. 4 yd. 2 ft. 10 in. to inches.

Ans. 5,722 in.

(16) Reduce 5,722 in. to higher denominations.

Ans. 28 rd. 4 yd. 2 ft. 10 in.

(17) How many seconds in 5 weeks and 3.5 days?

Ans. 3,326,400 sec.

(18) How many pounds, ounces, pennyweights, and grains are contained in 13,750 gr.?

Ans. 2 lb. 4 oz. 12 pwt. 22 gr.

(19) Reduce 4,763,254 links to miles.

Ans. 595 mi. 32 ch. 54 li.

(20) Reduce 764,325 cu.in. to cu.yd.

Ans. 16 cu.yd. 10 cu.ft. 549 cu.in.

(21) What is the sum of 2 rd. 2 yd. 2 ft. 3 in.; 4 yd. 1 ft. 9 in.; 2 ft. 7 in.? Ans. 3 rd. 2 yd. 2 ft. 1 in.

(22) What is the sum of 3 gal. 3 qt. 1 pt. 3 gi.; 6 gal. 1 pt. 2 gi.; 4 gal. 1 gi.; 8 qt. 5 pt.? Ans. 16 gal. 3 qt. 2 gi.

(23) What is the sum of 240 gr. 125 pwt. 50 oz. and 3 lb.?

Ans. 7 lb. 8 oz. 15 pwt.

(24) What is the sum of 11° 16′ 12″; 13° 19′ 30″; 20° 25″; 26′ 29″; 16° 17′ 11″? Ans. 55° 19′ 47″.

(25) What is the sum of 130 rd. 5 yd. 1 ft. 6 in.; 215 rd. 2 ft. 8 in.; 304 rd. 4 yd. 11 in.? Ans. 2 mi. 10 rd. 5 yd. 7 in.

(26) What is the sum of 21 A. 67 sq.ch. 3 sq.rd. 21 sq.li.; 28 A. 78 sq.ch. 2 sq.rd. 23 sq.li.; 47 A. 6 sq.ch. 2 sq.rd. 18 sq.li.; 56 A. 59 sq.ch. 2 sq.rd. 16 sq.li.; 25 A. 38 sq.ch. 3 sq.rd. 23 sq.li.; 46 A. 75 sq.ch. 2 sq.rd. 21 sq.li.?
Ans. 255 A. 3 sq.ch. 14 sq.rd. 122 sq.li.

(27) From 20 rd. 2 yd. 2 ft. 9 in. take 300 ft.
Ans. 2 rd. 1 yd. 2 ft. 9 in.

(28) From a farm containing 114 A. 80 sq.rd. 25 sq.yd., 75 A. 70 sq.rd. 30 sq.yd. are sold. How much remains?
Ans. 39 A. 9 sq.rd. $25\frac{1}{4}$ sq.yd.

(29) From a hogshead of molasses, 10 gal. 2 qt. 1 pt. are sold at one time, and 26 gal. 3 qt. at another time. How much remains? Ans. 25 gal. 2 qt. 1 pt.

(30) If a person were born June 19, 1850, how old would he be August 3, 1892? Ans. 42 yr. 1 mo. 14 da.

(31) A note was given August 5, 1890, and was paid June 3, 1892. What length of time did it run?
Ans. 1 yr. 9 mo. 28 da.

(32) What length of time elapsed from 16 min. past 10 o'clock A. M., July 4, 1883, to 22 min. before 8 o'clock P. M., Dec. 12, 1888? Ans. 5 yr. 5 mo. 8 da. 9 hr. 22 min.

(33) If 1 iron rail is 17 ft. 3 in. long, how long would 51 rails be, if placed end to end? Ans. 53 rd. $1\frac{1}{2}$ yd. 9 in.

(34) Multiply 3 qt. 1 pt. 3 gi. by 4.7.
Ans. 4 gal. 2 qt. 1.7 gi.

(35) Multiply 3 lb. 10 oz. 13 pwt. 12 gr. by 1.5.
Ans. 5 lb. 10 oz. 6 gr.

(36) How many bushels of apples are contained in 9 bbl., if each barrel contains 2 bu. 3 pk. 6 qt.?

Ans. 26 bu. 1 pk. 6 qt.

(37) Multiply 7 T. 15 cwt. 10.5 lb. by 1.7.

Ans. 13 T. 3 cwt. 67.85 lb.

(38) Divide 358 A. 57 sq.rd. 6 sq.yd. 2 sq.ft. by 7.

Ans. 51 A. 31 sq.rd. 8 sq.ft.

(39) Divide 282 bu. 3 pk. 1 qt. 1 pt. by 12.

Ans. 23 bu. 2 pk. 2 qt. $\frac{1}{4}$ pt.

(40) How many iron rails, each 30 ft. long, are required to lay a railroad track 23 mi. long? Ans. 8,096 rails.

(41) How many boxes, each holding 1 bu. 1 pk. 7 qt., can be filled from 356 bu. 3 pk. 5 qt. of cranberries?

Ans. 243 boxes.

(42) If 16 square miles are equally divided into 62 farms, how much land will each contain?

Ans. 165 A. 25 sq.rd. 24 sq.yd. 3 sq.ft. 80+ sq.in.

ARITHMETIC.

(SECTION 5.)

EXAMINATION QUESTIONS.

(43) What is the square of 108? Ans. 11,664.

(44) Find the fifth power of 9. Ans. 59,049.

(45) What is the value of $.0133^3$? Ans. .000002352637.

(46) In what respect does evolution differ from involution?

(47) Extract the square root of 90. Ans. 9.4868+.

(48) Find the value of $(3\frac{3}{4})^3$. Ans. $52\frac{47}{64}$, or 52.734375.

(49) What is the cube root of 92,416? Ans. 45.212—.

(50) Find the value of $\sqrt{502,681}$. Ans. 709.

(51) What is the value of $\sqrt[3]{\frac{27}{64}}$? Ans. $\frac{3}{4}$.

(52) From the cube of 4 take the cube root of 8. Ans. 62.

(53) What is the value of $\sqrt[3]{\frac{3}{8}}$? Ans. .72112+.

(54) Extract the square root of .3364. Ans. .58.

(55) Find the square root of 3.1416. Ans. 1.7725—.

(56) What number multiplied by itself equals 114.9184? Ans. 10.72.

(57) Extract the square root of 3,486,784. Ans. 1,867.3—.

(58) Find the square root of .00041209. Ans. .0203.

For notice of copyright, see page immediately following the title page.

ARITHMETIC.

(SECTION 6.)

EXAMINATION QUESTIONS.

Find the value of x in the following:

(59) $11.7 : 13 :: 20 : x$. Ans. 22.22+.

(60) (*a*) $20+7 : 10+8 :: 3 : x$; (*b*) $12^2 : 100^2 :: 4 : x$.

Ans. { (*a*) 2. (*b*) 277.7+.

(61) (*a*) $\frac{4}{x} = \frac{7}{21}$; (*b*) $\frac{x}{24} = \frac{8}{16}$; (*c*) $\frac{2}{10} = \frac{x}{100}$; (*d*) $\frac{15}{45} = \frac{60}{x}$; (*e*) $\frac{10}{150} = \frac{x}{600}$.

Ans. { (*a*) $x = 12$. (*b*) $x = 12$. (*c*) $x = 20$. (*d*) $x = 180$. (*e*) $x = 40$.

(62) $x : 5 :: 27 : 12.5$. Ans. $10\frac{4}{5}$.

(63) $45 : 60 :: x : 24$. Ans. 18.

(64) $x : 35 :: 4 : 7$. Ans. 20.

(65) $9 : x :: 6 : 24$. Ans. 36.

(66) $\sqrt[3]{1,000} : \sqrt[3]{1,331} = 27 : x$. Ans. 29.7.

(67) $64 : 81 = 21^2 : x^2$. Ans. 23.625.

(68) $7+8 : 7 = 30 : x$. Ans. 14.

For notice of copyright, see page immediately following the title page.

(69) A man whose steps measure 2 ft. 5 in. takes 2,480 steps in walking a certain distance. How many steps of 2 ft. 7 in. will be required for the same distance?

Ans. 2,320 steps.

(70) If a horse travels 12 mi. in 1 hr. 36 min., how far will he travel at the same rate in 15 hr.? Ans. 112.5 mi.

(71) If a column of mercury 27.63 in. high weighs .76 of a pound, what will be the weight of a column of mercury having the same diameter, 29.4 in. high? Ans. .808+ lb.

(72) If 2 gal. 3 qt. 1 pt. of water will last a man 5 da., how long will 5 gal. 3 qt. last him, if he drinks at the same rate? Ans. 10 da.

(73) Heat from a burning body varies inversely as the square of the distance from it. If a thermometer held 6 ft. from a stove shows a rise in temperature of 24°, how many degrees rise in temperature would it indicate if held 12 ft. from the stove? Ans. 6°.

(74) If a pile of wood 12 ft. long, 4 ft. wide, and 3 ft. high is worth $12, what is the value of a pile of wood 15 ft. long, 5 ft. wide, and 6 ft. high? Ans. $37.50.

(75) If 100 gal. of water run over a dam in 2 hr., how many gallons will run over the dam in 14 hr. 28 min.?

Ans. $723\frac{1}{3}$ gal.

(76) If a cistern 28 ft. long, 12 ft. wide, 10 ft. deep holds 798 bbl. of water, how many barrels of water will a cistern hold that is 20 ft. long, 17 ft. wide, and 6 ft. deep?

Ans. $484\frac{1}{2}$ bbl.

(77) If a railway train runs 444 mi. in 8 hr. 40 min., in what time can it run 1,060 mi. at the same rate of speed?

Ans. 20 hr. 41.44 min.

(78) If sound travels at the rate of 6,160 ft. in $5\frac{1}{2}$ sec., how far does it travel in 1 min.? Ans. 67,200 ft.

(79) If 5 men by working 8 hours a day can do a certain amount of work, how many men by working 10 hours a day can do the same work? Ans. 4 men.

(80) If a man travels 540 miles in 20 days of 10 hours each, how many hours a day must he travel to cover 630 miles in 25 days? Ans. $9\frac{1}{3}$ hr.

(81) Referring to example 4, Art. **168,** *Arithmetic*, § 2, what is the horsepower of an engine whose cylinder is 30 inches in diameter, piston speed, 660 feet per minute, and mean effective pressure, 42 pounds per square inch? Ans. 594 horsepower.

(82) The weight of a cubic inch of cast iron is .261 pound. Referring to Art. **164,** *Arithmetic*, § 2, what is the weight of a solid cast-iron cylinder whose diameter is 12 inches and length is 60 inches? Ans. 1,771.11 lb.

(83) Referring to Art. **167,** *Arithmetic*, § 2, what is the centrifugal force of a 40-pound body revolving in a circle having a radius of 10 inches, at a speed of 18 feet per second? Ans. 484.7 lb.

FORMULAS.

(ARTS. 1–21. SEC. 3.)

EXAMINATION QUESTIONS.

$A = 5$ $\quad h = 200$
$B = 10$ $\quad x = 12$
$i = 3.5$ $\quad D = 120$

Work out the solutions to the following formulas, using the above values for the letters:

(1) $C = \dfrac{D-x}{B+i}$. Ans. $C = 8$.

(2) $Q = \dfrac{A\,h+D}{2\,x+6}+D$. Ans. $Q = 157\frac{1}{3}$.

(3) $r = \dfrac{3.246\,B\,h}{\dfrac{A\,x+h}{A\,i-B}}$. Ans. $r = 187.269+$.

(4) $v = \sqrt{\dfrac{A\,D}{i\,B+1.5}}$. Ans. $v = 4.05+$.

(5) $u = \sqrt[3]{\dfrac{B\,x}{.00018\,h(A^2-x)}}$. Ans. $u = 6.35+$.

(6) $f = \dfrac{10(h-D)^2}{\sqrt[3]{D}+A}$. Ans. $f = 12{,}800$.

For notice of the copyright, see page immediately following the title page.

(7) $g = \dfrac{(B-A)^2 - \sqrt[3]{D+A}}{A^3-(1+D)}$. Ans. $g = 5$.

(8) $k = \sqrt{\dfrac{A\,B^2}{\sqrt[3]{A\,h}}}$. Ans. $k = 7.071+$.

(9) $T = \sqrt{\dfrac{A^2\left[490+\dfrac{(h\,x)^2}{D^2}\right]}{h+\dfrac{x}{D}(A^2-B)^3}}$. Ans. $T = 10$

GEOMETRY AND MENSURATION.

(ARTS. 1–158. SEC. 4.)

EXAMINATION QUESTIONS.

(See note at bottom of page 12, Instruction Paper.)

(1) Fig. 1 represents a center—half a regular hexagon—for a 5-foot semicircular arch. (*a*) What is the angle of bevel (*A B O*) to which the ends of the planks must be cut? (*b*) What length of plank, as *F D*, is needed for each piece? (*c*) What is the length of the inner side, as *A B*, of each plank, after cutting, if the width is 8 inches?

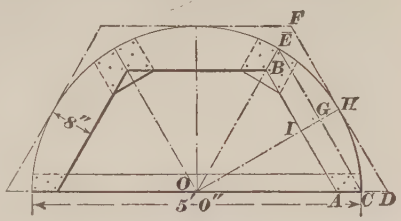


FIG. 1.

Ans. { (*a*) 60°. (*b*) 2.89 ft. = 2 ft. 10.7 in. (*c*) 2.12 ft. = 2 ft. 1½ in.

(2) How many feet of molding are needed to go around an 8-sided room, in which the distance between the parallel sides is 12 feet, and between the opposite angles, 13 feet?

Ans. 40 ft.

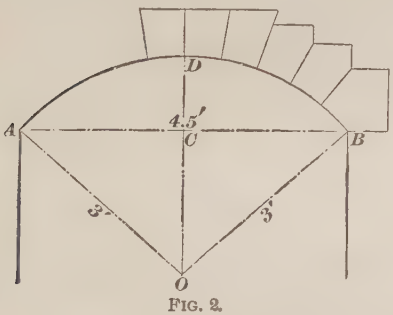


FIG. 2.

(3) *A D B*, Fig. 2, represents an arch of 3 feet radius. (*a*) The span *A B* being 4½ feet, what is the

For notice of copyright, see page immediately following the title page.

rise CD? (b) If the radius OA, and rise CD were given, show how to find AB. Ans. (a) 1.02 ft. = 1 ft. $\frac{1}{4}$ in.

(4) In staking out a building, the points A, B, C, D, E, and F are located, the distances being as shown in Fig. 3. To check these measurements, it is desired to measure the diagonals FB, FD, EA, and EC. What are their lengths? Ans. $FB = 30$ ft.; $FD = 33.11$ ft.; $EA = 38.42$ ft.; $EC = 18.44$ ft.

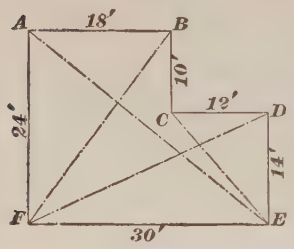


FIG. 3.

(5) Explain why the sides of a regular hexagon are equal to the radius of the circumscribing circle.

(6) In a half-pitch (or 45°) roof of 24-foot span, show that the length along the roof from side wall to ridge = $\sqrt{288}$ feet.

(7) Show by a sketch how to divide a line 6 inches long into 7 equal parts.

(8) In Fig. 4, what is the length of the ridge plate AB, which is 10 feet above CD, the latter being 36 feet long, and the angles DCA and CDB being 45°? Ans. 16 ft.

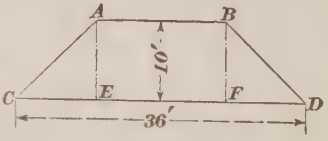


FIG. 4.

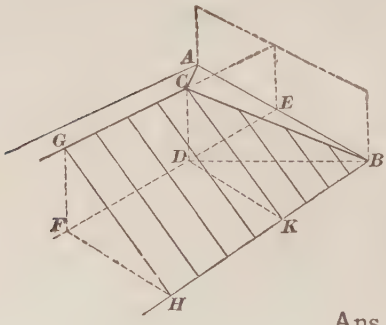


FIG. 5.

(9) Fig. 5 represents a roof whose span AB is 22 feet. The slopes are half-pitch, and the distance from E to D (under C) is 11 feet. (a) Find the length of a "common rafter," as GH. (b) Find the length of a "hip rafter," as BC.

Ans. (a) 15.56 ft. = 15 ft. $6\frac{3}{4}$ in.
(b) 19.05 ft. = 19 ft. $\frac{5}{8}$ in.

(10) In Fig. 6, the distances from A to C, and from B to D, across a stream, are required. The line $A\,B$ is 210 feet long, and E is the middle point. $E\,F$, parallel to $B\,D$, is 86 feet long; and $E\,G$, parallel to $A\,C$, is 90 feet long. What are the lengths of $A\,C$ and $B\,D$?

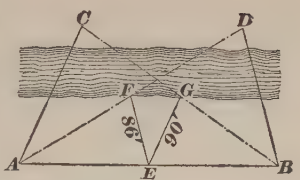


Fig. 6.

Ans. { $A\,C$, 180 ft. / $B\,D$, 172 ft.

(11) In a right triangle, one of the acute angles is 37°. (*a*) What is the other acute angle? (*b*) What is the angle formed by producing one side of the given angle?

Ans. { (*a*) 53°. / (*b*) 143°.

(12) What is the angle included between two adjacent sides of a regular nonagon (nine-sided polygon)?

Ans. 140°.

(13) In Fig. 7, the span $A\,C$ of a semicircular arch is 16 feet, and the distance $A\,E$ to the top of the masonry is 11 feet. How thick is the stonework at $D\,F$, 4 feet from the vertical line $A\,E$? Ans. 4.07 ft.

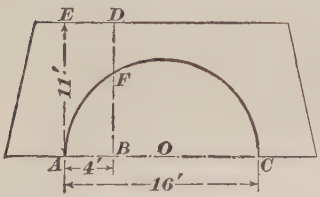


Fig. 7.

(14) Show how to lay out, with a tape, a line 12 feet long, perpendicular to another at its middle point, the length of the latter being 32 feet.

(15) The corners of a cast-iron plate, 15 inches square and $1\frac{1}{4}$ inches thick, are rounded off by quarter circles of 2-inch radius. Estimating cast iron at .26 pound per cubic inch, how much less does this plate weigh than if the corners were square? Ans. 1.11 lb.

(16) A brick pier is 30 inches square at the base, 18 inches square at the top, and 6 feet high. Figuring $22\frac{1}{2}$ brick per cubic foot, how many brick are required for 18 piers? Ans. 9,922.5.

(17) Fig. 8 represents a cross-section of a "Phœnix" column, made of wrought iron, of which a strip 1 foot long and 1 inch square in section weighs 3.33 pounds. Disregarding curved corners and edges, but adding 2 per cent. to the weight of the iron for rivet heads, what is the weight of the column per foot of length?

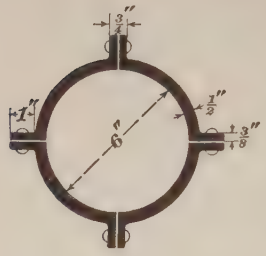


FIG. 8.

Ans. 44.87 lb.

(18) The freight rate from a sandstone quarry to a town is 21 cents per hundred pounds. The following pieces of dimension stone were shipped. Allowing 140 pounds to the cubic foot, what were the charges?

1 piece 4′ 6″ × 3′ 9″ × 15″. 2 pieces 5′ 6″ × 4′ 0″ × 15″.
1 piece 4′ 3″ × 3′ 6″ × 15″. 1 piece 5′ 0″ × 3′ 9″ × 15″.
1 piece 5′ 3″ × 4′ 0″ × 15″. 1 piece 4′ 9″ × 4′ 6″ × 15″.
3 pieces 3′ 3″ × 3′ 6″ × 15″. 2 pieces 4′ 6″ × 4′ 3″ × 15″.

Ans. $76.90.

(19) Fig. 9 represents the plan and end views of a roof.

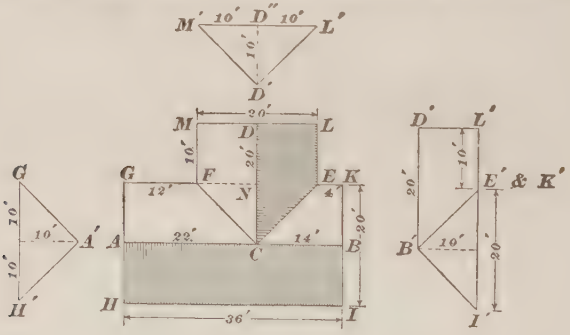


FIG. 9.

Knowing that the ridges $A\,B$ and $C\,D$ are 10 feet above the eaves, find (a) the total area of the roof, and (b) the length of the "valleys" $F\,C$ and $E\,C$.

Ans. { (a) 1,300.88 sq. ft.
{ (b) 17.32 ft. = 17 ft. $3\frac{7}{8}$ in.

(20) Figuring $7\frac{1}{2}$ gallons per cubic foot, how many gallons will be discharged per minute through a 4-inch pipe, if the water flows 5 feet per second? Take the area of pipe as .09 square foot. Ans. 202.5 gal.

(21) What length of lead pipe, weighing .41 pound per cubic inch, $1\frac{3}{4}$ inches outside diameter and $\frac{1}{8}$ inch thick, will be needed to make a 3-pound weight? **Ans. 11.6 in**

(22) The span $A\,B$ of the arch shown in Fig. 10 is 6 feet 8 inches, and the rise $C\,D$ is 8 inches. (*a*) Find the radius. (*b*) If there are 13 ring stones, what is the bottom width of each?

Ans. { (*a*) 8 ft. 8 in. (*b*) 6.32 in.

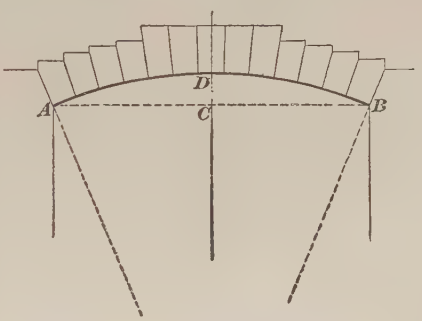


Fig. 10.

(23) In Fig. 10, find the radius by the principle that a perpendicular from any point on a circumference to a diameter is a mean proportional between the two parts of the diameter.

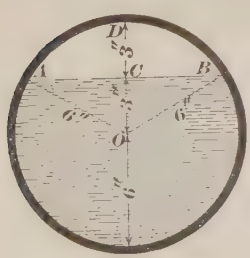


Fig. 11.

(24) It is required, in a question of water supply, to determine the area of the water section in a 12-inch pipe flowing 9 inches deep; that is, what is the area below the surface $A\,B$, Fig. 11? (See Art. **106.**) Ans. 91.11 sq. in.

(25) Find the total feet B. M. in the following bill of material:

3 pieces 3″ × 10″; 16′ 0″ long.
8 pieces 3″ × 10″; 12′ 6″ long.
6 pieces 3″ × 8″; 14′ 6″ long.
10 pieces 3″ × 8″; 9′ 0″ long.
4 pieces 3″ × 6″; 10′ 6″ long.
20 pieces 3″ × 6″; 7′ 0″ long.
8 pieces 4″ × 6″; 18′ 0″ long.
4 pieces 4″ × 6″; 6′ 6″ long.
52 pieces 2″ × 4″; 18′ 0″ long.
15 pieces 2″ × 4″; 9′ 6″ long.
600 ft. B. M. 1″ × 6″ flooring.
200 ft. B. M. 2″ plank.

Ans. 2,856 ft. B. M.

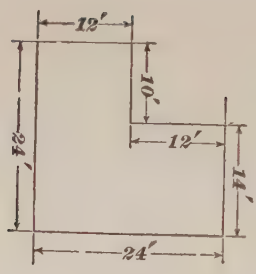


FIG. 12.

(26) Fig. 12 represents the plan of a house which is to be faced with pressed brick. The walls are 22 ft. high, and there are two gables, one 14 ft. × 7 ft. high, and the other 12 ft. × 7 ft. high. Deducting 10 windows, 3 ft. × 7 ft.; 10 windows, 3 ft. × 6 ft.; and 4 doors, 3 ft. 6 in. × 8 ft.; and allowing 7 brick to each square foot of surface, how many brick will be required? Ans. 11,907.

(27) The major and minor axes of an elliptic window frame are 8 feet and 5 feet, respectively. The sash is 4 inches wide all around. What is the area of the glass?

Ans. 24.93 sq. ft.

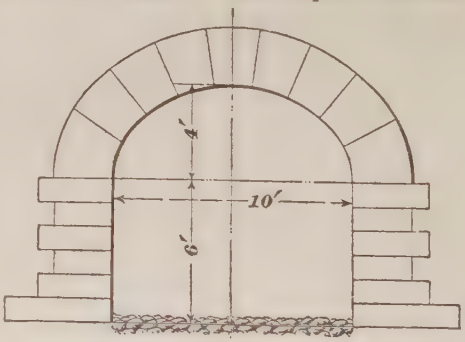


FIG. 13.

(28) A culvert is required to have an area of 88 square feet. How much more than this area is the cross-section shown in Fig. 13, the arch being semi-elliptical? Ans. 3.42 sq. ft.

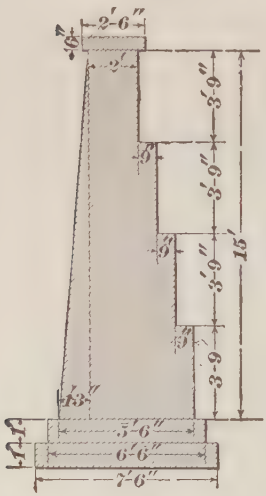


FIG. 14.

(29) How many cubic yards are there in a retaining wall 16 feet long, having the cross-section shown in Fig. 14? Ans. 42.36 cu. yd.

(30) The top of the frame of a window $3\frac{1}{2}$ feet in width is to be bent to a circular arc, the rise of which is 3 inches. What length of piece will be required?

Ans. 42.57 in. = 3 ft. $6\frac{9}{16}$ in.

(31) Compute the cubic yards of excavation for the cellar, 9 feet deep, of the building shown in Fig. 3, increasing all the dimensions, *except* BC and CD, by 2 feet, to give room to lay the masonry. (The reason for not increasing BC and CD will be seen by drawing lines around the plan 1 foot outside the wall lines.) Ans. 237.33 cu. yd.

(32) The length of a building with a plain roof is 32 feet, and the width is 24 feet. The height of the gables is $\frac{5}{8}$ the width. The roof projects over the walls 15 inches (measured on the roof) at ends and eaves. (*a*) How many squares (100 sq. ft.) of slating in the roof area? (*b*) The slates being 12 inches wide and exposed $8\frac{1}{2}$ inches to the weather, how many will be required per square?

Ans. { (*a*) 14.12 squares. (*b*) 141+ slates.

(33) The dome of a cupola is hemispherical, and its diameter is 3 feet. Deducting 5 square feet for windows, etc., what is the area of the remaining surface?

Ans. 9.14 sq. ft.

(34) Estimating cast iron at 450 pounds per cubic foot, what is the weight per 12-foot length of pipe, 10 inches inside diameter, the thickness being $\frac{1}{2}$ inch? Ans. 616.5 lb.

(35) The outside of a circular tower 28 feet high and 8 feet in diameter is to be shingled. (*a*) How many squares (100 sq. ft.) of shingling are required? (*b*) If the shingles average 6 inches wide and are laid 5 inches to the weather, how many will be required to the square?

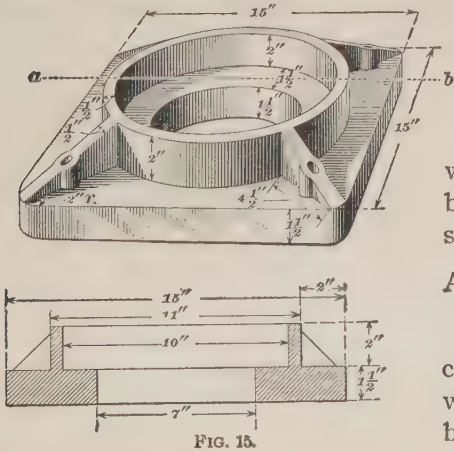


FIG. 15.

Ans. { (*a*) 7.04 squares. (*b*) 480.

(36) At .26 pound per cubic inch, what is the weight of the cast-iron base shown in Fig. 15?

Figure the 4 ribs as plain, with no allowance for swelled parts nor deduction for holes. The corners of the base are rounded to a 2-inch radius. Ans. 82.32 lb.

(37) A bridge pier 28 feet high is 12 ft. × 30 ft. at the base, and 7 ft. × 22 ft. at the top. What will be the cost of the masonry at $12 per cubic yard? (Use the prismoidal formula.) Ans. $3,115.20.

(38) A cast-iron ball which will weigh 25 pounds is required. Figuring cast iron at .26 pound per cubic inch, what will be the diameter of the ball? Ans. 5.68 in., or $5\frac{11}{16}$ in., nearly.

(39) Estimating steel at .283 pound per cubic inch, what will be the weight per foot of the column shown in Fig. 16, in which (*a*) is an enlarged section of a "channel"? Neglect the curved corners and edges on the channels. Ans. 62.83 lb.

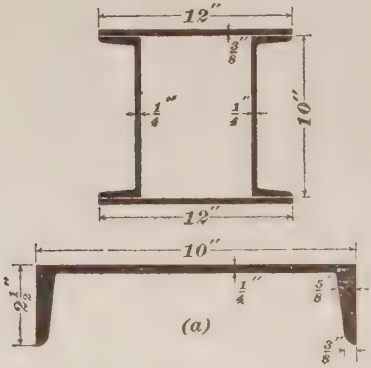


FIG. 16.

(40) The load on a column is 96,000 pounds. Allowing a safe pressure of 3 tons (of 2,000 lb.) per square foot on the soil, and 10 tons per square foot on the brick pier, what must be (*a*) the dimensions of the square footing area, and (*b*) the dimensions of the square column base?

Ans. { (*a*) 4 ft. square. (*b*) 2.19 ft. square.

(41) In a roof truss one of the tie-rods must sustain a load of 27,500 pounds. The safe stress being 10,000 pounds per square inch, what must be the diameter of the rod?

Ans. $1\frac{7}{8}$ in., nearly.

(42) A square pyramidal monument is 3 feet square at the base, 1 foot square at the top, and is 18 feet high. It is capped by a pyramid 1 foot square and 2 feet high. If the stone is granite, weighing 170 pounds per cubic foot, what is the total weight? Ans. 13,373⅓ lb.

ARCHITECTURAL ENGINEERING.

(ARTS. 1–134. SEC. 5.)

EXAMINATION QUESTIONS.

(1) At what point does the greatest bending moment occur in a cantilever beam?

(2) What live load would you use in designing the floor of a theater?

(3) In selecting **I** beams, what is considered good practice in regard to the depth, so as to avoid excessive deflection?

(4) By means of the method of the polygon of forces, determine the resultant of the several forces shown in Fig. 1.

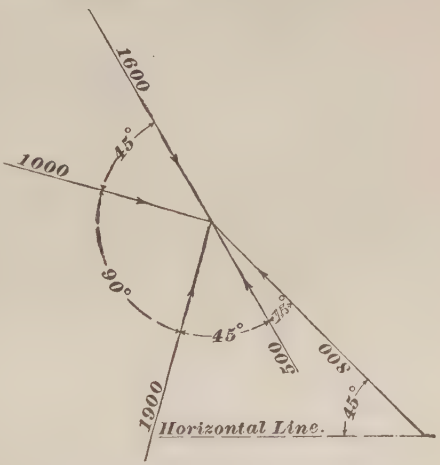


FIG. 1.

(5) Explain what is meant by the horizontal and vertical components of an oblique force.

(6) It is required to span an opening 25 feet wide in a solid brick wall 24 inches thick, using two **I** beams, side by side. The wall is laid in lime mortar, and the safe unit fiber stress of the material composing the **I** beams is 15,000 pounds; what should be the size of the beams?

Ans. { 12 in. 39.4 lb., or
{ 15 in. 41.2 lb.

For notice of copyright, see page immediately following the title page.

(7) Explain the use of separators placed between **I** beams.

(8) The span of a beam is 32 feet. For three-quarters of this distance from the left-hand support it is loaded with a uniformly distributed load of 6,000 pounds. At distances of 9 feet and 14 feet from the right-hand support are located concentrated loads of 4,500 pounds and 8,100 pounds, respectively. At what distance from the left-hand end does the shear change sign? Ans. 18 ft.

(9) In what way do the loads upon the foundations of an office building and a storage warehouse differ?

(10) A square yellow-pine column 18 feet long must support a load of 103,900 pounds; if a factor of safety of 5 is used, what will be the size of this column?

Ans. 12 in.×12 in.

(11) What is the shear between the points *c* and *b* on a beam loaded as shown in Fig. 2? Ans. 800 lb.

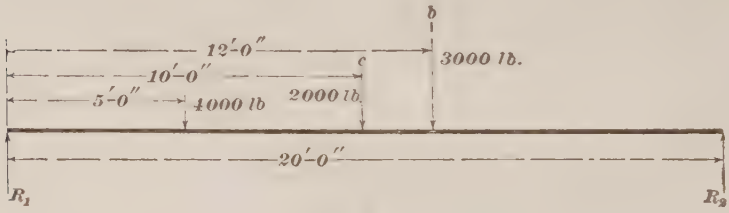


FIG. 2.

(12) The length of a beam is 50 feet, and it overhangs the right-hand support 10 feet; from the overhanging end there is suspended a weight of 10,000 pounds; 15 feet, 25 feet, and 28 feet from the left-hand support are loads of 9,000, 11,000, and 19,000 pounds, respectively. What is the right-hand reaction? Ans. 36,050 lb.

(13) Explain what is meant by a reaction.

(14) State the conditions demanded for good castings to be used in building operations.

(15) Explain wherein the design of the cast-iron column

shown in Fig. 3 is faulty. Redesign the cap and base to meet the requirements of good practice.

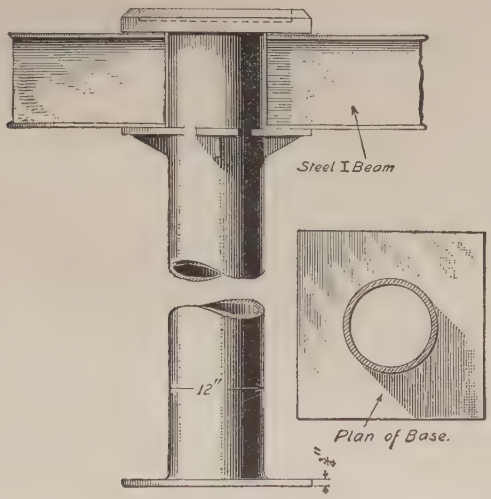


FIG. 3.

(16) What is the relation between the external forces acting on a beam, when the beam is in equilibrium ?

(17) The concentrated loads upon a beam are 8,000, 7,000, and 9,000 pounds. The reaction R_1 is 12,000 pounds. What is the magnitude of the reaction R_2 ? Ans. 12,000 lb.

(18) What safe uniformly distributed load will a granite lintel 20 inches deep and 25 inches wide sustain, the span being 5 feet ? Ans. 20 T.

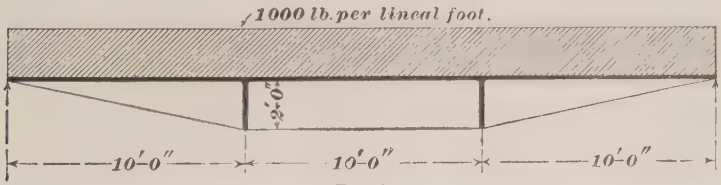


FIG. 4.

(19) In the trussed beam shown in Fig. 4, what is (*a*) the tension in the camber rod ? (*b*) the compression on the trussed beam ?

Ans. { (*a*) 56,089 lb.
(*b*) 55,000 lb.

(20) The area of a structural steel angle is 6 square inches. What will be the safe working tensile stress upon this angle, providing a factor of safety of 3 is adopted, and the ultimate tensile strength of the material is 60,000 pounds?

Ans. 120,000 lb.

(21) The compressive stress upon a short oak block 12 inches square is 135,360 pounds. What is the factor of safety? Ans. 3.83.

(22) What factor of safety would you use if you were designing a stone lintel to support a given load?

(23) When is any structure in equilibrium?

(24) A girder of 30-foot span is trussed at the center by camber rods and a strut. The depth of truss from the center of the girder to the center of the rods is 2 feet; if the beam is loaded with a uniformly distributed load of 2,500 pounds per lineal foot, (*a*) what is the stress on the rods? (*b*) What is the compressive stress on the beam? (*c*) What is the stress on the central strut?

Ans. { (*a*) 177,337 lb. (*b*) 175,781 lb. (*c*) 46,875 lb.

(25) A roof covering is composed of Spanish tile laid upon 3-inch spruce sheathing. Between the tile and sheathing there is placed 2 layers of roofing felt. What is the weight per square foot of this covering? Ans. 15 lb.

(26) Explain the difference between live and dead loads.

(27) Calculate the square of the radius of gyration for the section of a cylindrical column 10 inches outside diameter, metal $\frac{3}{4}$ inch thick. Ans. 10.77.

(28) What is the resisting moment of a $12'' \times 1''$ wrought-iron plate, using a unit fiber stress of 10,000 pounds?

Ans. 240,000 in.-lb.

(29) The span of the 15-inch steel **I** beams used in the floor of a fireproof building is 23 feet, and the beams are spaced 4 feet center to center. The live load is 200 pounds per square foot of floor surface. The floor to be supported

by a 4-inch brick segment arch, having a rise of 5 inches, laid in cement with the necessary concrete filling, over the top of which is laid a 1-inch yellow-pine floor. The flooring is nailed to $2'' \times 3''$ sleepers, embedded in the concrete. Using a unit fiber stress of 15,000 pounds, find the weight of beam to be used. In calculating the amount of the dead load, disregard the weight of the beams.

Ans. 15 in. = 56.9 lb.

(30) The length of a beam is 30 feet, and its only support is at the center; at the left-hand end is a load of 60 pounds, and 9 feet from the left-hand end is a load of 80 pounds. What will be the load required at the right-hand end to prevent the beam from rotating around its support? Ans. 92 lb.

(31) Explain what is meant by (*a*) neutral axis; (*b*) resisting moment.

(32) What would you consider a safe live load to be used in designing a country dwelling?

(33) Calculate the reactions R_1 and R_2 of a beam loaded as shown in Fig. 5.

Ans. $\begin{cases} R_1\ 26{,}857\frac{1}{7} \text{ lb.} \\ R_2\ 18{,}342\frac{6}{7} \text{ lb.} \end{cases}$

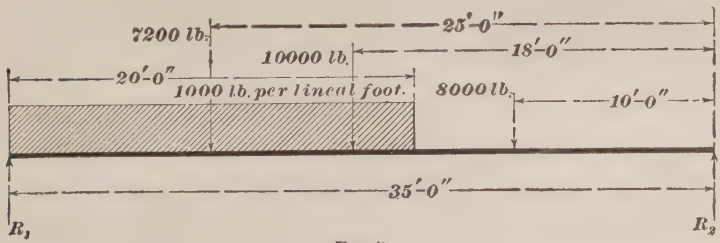


FIG. 5.

(34) When a beam is loaded with several concentrated loads, where does its greatest bending moment occur?

(35) State some of the advantages and disadvantages of cast-iron columns.

(36) A cantilever beam securely fastened into a wall extends 8 feet from the point of support; it is loaded with a

uniformly distributed load of 500 pounds per lineal foot. What is the maximum bending moment in foot-pounds?

Ans. 16,000 ft.-lb.

(37) The floor of a factory building is 60 ft. × 290 ft.; what will be the probable entire weight due to the live load upon this floor area? Ans. 2,610,000 lb.

(38) The uniformly distributed load upon a beam is 90,000 pounds; the beam is supported at both ends. What is the maximum shear upon the beam? Ans. 45,000 lb.

(39) What is meant by the shear on a beam?

(40) Explain why a factor of safety is used in designing any structure.

(41) A hollow cast-iron column is 6 inches in diameter outside, the metal is $\frac{3}{4}$ inch thick, and the length of the column 10 feet. Using a factor of safety of 6, what safe load will this column support? Ans. 78,000 lb.

(42) The stress upon a steel bar is 80,000 pounds, and the sectional area of the bar is 5 square inches. What is the unit stress? Ans. 16,000 lb.

(43) What safe load will a brick pier 3 ft. × 4 ft. × 10 ft. high support, providing the pier is laid in lime mortar?

Ans. 172,800 lb.

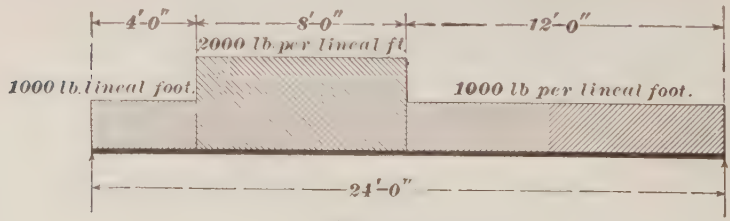


FIG. 6.

(44) A beam is loaded as shown in Fig. 6; (*a*) what in round numbers is the greatest bending moment, and (*b*) at what distance from the right-hand end does it occur?

Ans. { (*a*) 105,800 ft.-lb.
(*b*) 13 ft. 4 in.

(45) If a factor of safety of 5 is used, what must be the thickness of metal in a 16-inch column (outside diameter), 24 feet long, to carry a load of 421,000 pounds? Ans. 1 in.

(46) The footing under a brick pier rests upon a foundation soil of stiff clay; if the footing is 5 feet square, what load in pounds will it safely carry? Ans. 125,000 lb.

(47) In a beam uniformly loaded, and supported at the ends, where is the greatest bending moment?

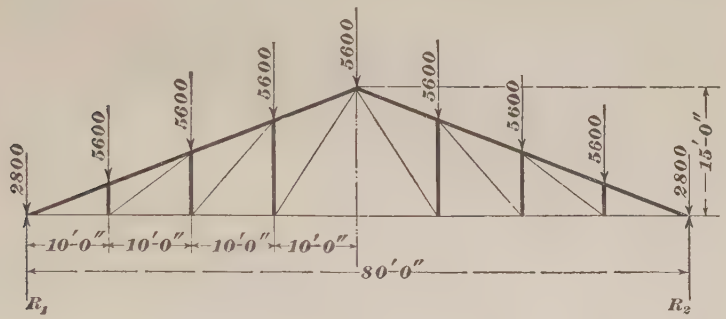


FIG. 7.

(48) Draw a dead-load stress diagram for the truss shown in Fig. 7.

(49) A beam has a span of 30 feet, and is loaded with a uniformly distributed load of 2,000 pounds per lineal foot; what is the greatest bending moment in inch-pounds upon the beam? Ans. 2,700,000 in.-lb.

(50) What safe uniformly distributed load will a $12'' \times 16''$ yellow-pine beam, 30 feet long, carry, using one-quarter of the value of the modulus of rupture for a working stress?

Ans. 20,800 lb.

(51) What will be the difference in weight between a $\frac{3}{16}$-inch thick slate roof, laid upon 1-inch hemlock sheathing, covered with two layers of roofing felt, and a 4-ply slag roof laid upon 2-inch spruce sheathing? Ans. $1\frac{3}{4}$ lb.

(52) A square granite capstone, on brickwork laid in Rosendale cement mortar, is required to support a load of 1,000,000 pounds. Compute the dimensions of the stone.

Ans. 6 ft. 10 in. square.

(53) Design the connection of an 8-inch I beam with a beam 15 inches in depth, making the top surface of the two beams flush, and using standard framing.

(54) A 12-inch I beam has an area of 14 square inches; what is its section modulus? Ans. 52.5.

(55) What will be the allowable load on a $4'' \times 12''$ spruce column 10 feet long, if a safety factor of 4 is used?

Ans. 25,200 lb.

(56) Calculate the full panel wind loads for a 100-foot span roof truss; the trusses are 20 feet apart from center to center, and have a rise of 6 inches per foot of run; the rafter members are supported at both ends, and have three intermediate supports, placed so as to divide them into four equal panels. Ans. 6,624 lb.

(57) Design a foundation pier to support a column which carries a load of 200,000 pounds; the soil is a stiff, dry clay, and the body of the foundation is a good rubble masonry laid in Portland cement mortar; the capstone under the column is granite.

(58) A tension member 15 feet long, under an excessive load is stretched $\frac{3}{16}$ of an inch; what is the unit strain on this rod? Ans. .00104 in.

(59) What size of steel I beam will be required to fulfil

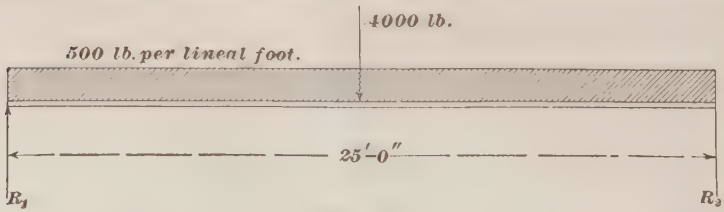


FIG. 8.

the conditions shown in Fig. 8, if a unit fiber stress of 15,000 pounds is used? Ans. 15 in. 41.2 lb.

(60) If a structural steel bar, which has an ultimate tensile strength of 60,000 pounds, is suspended from one end, how long must it be to break with its own weight?

Ans. 17,668 ft.

(61) Explain what is meant by (*a*) unit stress; (*b*) unit strain.

(62) What are Newton's three laws of motion?

(63) (*a*) What is the safe load on a cast-iron column 10 inches square, outside, and 20 feet long, if a factor of safety of 5 is used, the metal being 1 inch thick? (*b*) Design the base and also its foundation, using a limestone cap; the body of the foundation to be composed of brick masonry laid in Portland cement mortar; the footing of the foundation to be Portland cement concrete, resting on a good, dry clay, capable of supporting $2\frac{1}{2}$ tons per square foot.

Ans. (*a*) 268,700 lb.

(64) Draw the wind and dead load stress diagram for the truss shown in Fig. 9.

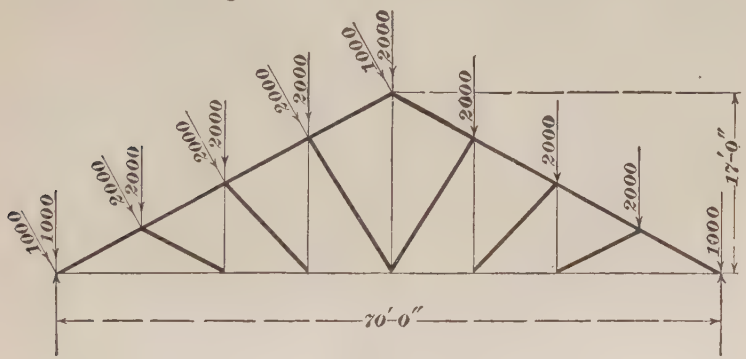


FIG. 9.

(65) Through what lever arm will 25 pounds be required to act, to produce a moment of 275 foot-pounds?

Ans. 11 ft.

(66) In a simple beam, what relation does the shear bear to the bending moment?

(67) What is considered as the maximum safe deflection for beams carrying plaster ceilings?

(68) What factor of safety would you deem advisable to use in structures composed of (*a*) structural steel? (*b*) wrought iron?

(69) Using an ultimate unit stress of 60,000 pounds, what will be the ultimate tensile strength of a $2'' \times \frac{1}{4}''$ structural-steel bar? Ans. 30,000 lb.

(70) Design a wooden roof truss, the frame diagram of which is shown in Fig. 10, disregarding the pressure due to

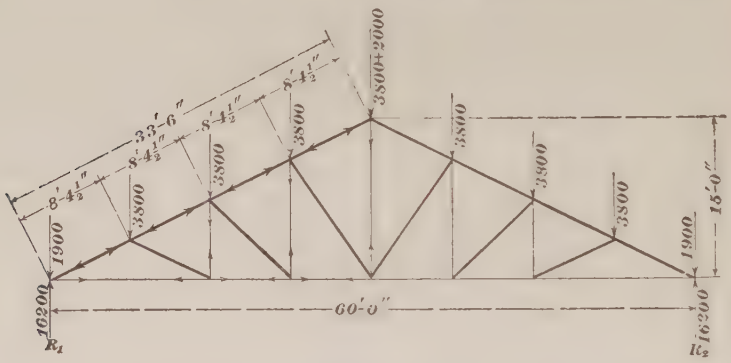


FIG. 10.

the wind. The truss is composed of yellow pine with wrought-iron tension rods, and a factor of safety of 5 is to be used.

(71) Explain the action of wind upon roofs.

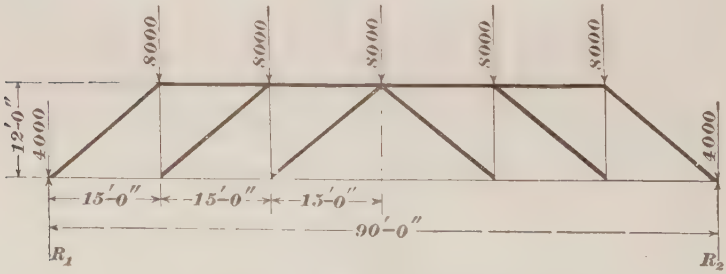


FIG. 11.

(72) Draw the vertical-load stress diagram for the truss shown in Fig. 11.

ARCHITECTURAL ENGINEERING.

(ARTS. 1–90. SEC. 6.)

EXAMINATION QUESTIONS.

(1) What is the moment of inertia of a circular area 4 inches in diameter (*a*) with respect to an axis through its center, and (*b*) with respect to a parallel axis 8 inches from its center?

Ans. { (*a*) 12.566. (*b*) 816.79.

(2) Calculate the moment of inertia of the column section shown in Fig. 1 (*a*) with respect to the axis *a b*, and (*b*) with respect to the axis *c d*. (*c*) What is the square of the least radius of gyration of the section?

Ans. { (*a*) 456.6912. (*b*) 123.3042. (*c*) 6.26.

FIG. 1.

(3) If made of structural steel with an ultimate compressive strength of 52,000 pounds per square inch, and a factor of safety of 5 is required, what safe load, in round numbers, will a column 18 feet long, with hinged ends and with the section shown in Fig. 1, carry?

Ans. 144,800 lb.

(4) A 15-inch 60-pound **I** beam 24 feet long carries a uniformly distributed load of 1,500 pounds per lineal foot What, in round numbers, is the greatest unit fiber stress?

Ans. 15,240 lb. per sq. in.

For notice of copyright, see page immediately following the title page.

(5) (*a*) What conditions should be considered in choosing a type of column for a given purpose? (*b*) Why is it sometimes better to use a column section in which the distribution of the material is not the most economical from a theoretical point of view?

(6) A plate girder supports a floor surface 24 feet by 18 feet, on which the total dead and live load is 400 pounds per square foot. If at each end, the girder is connected to a column with $\frac{3}{4}$-inch rivets through $\frac{5}{16}$-inch connection angles, and an allowable fiber stress in tension of 12,000 pounds per square inch is used, how many rivets are required to support each end of the girder?

Ans. 24 rivets.

(7) (*a*) What is the maximum allowable pitch of rivets in compression members? (*b*) What is the minimum distance that a $\frac{3}{4}$-inch rivet may be placed from the end of a $\frac{5}{16}$-inch plate? Ans. (*b*) $1\frac{3}{16}$ in.

(8) (*a*) What is understood by the term *camber* as applied to a roof truss? (*b*) What is the effect of camber on the strength of the members of a truss?

(9) A steel rod $7\frac{1}{2}$ inches long and $\frac{1}{2}$ inch in diameter is elongated .009 inch by a pull of 7,000 pounds; what, in round numbers, is the modulus of elasticity?

Ans. 29,709,000.

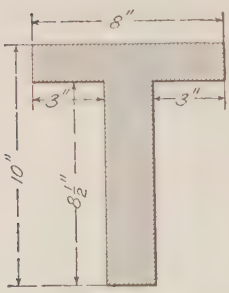


FIG. 2.

(10) (*a*) Calculate the moment of inertia of the section shown in Fig. 2 with respect to an axis parallel to the upper edge and passing through the center of gravity of the figure. (*b*) What is the least radius of gyration of the section?

Ans. { (*a*) 280.469. (*b*) 1.55.

(11) (*a*) What advantage does the box section plate girder have over the other forms? (*b*) What serious disadvantage does it have?

(12) (*a*) How is the distribution of the stresses in a plate girder assumed to differ from those in a beam composed of a single piece? (*b*) What part of the girder is assumed to resist the shearing stresses? (*c*) What part of the girder is assumed to resist the stresses due to the bending moment?

(13) Calculate the thickness of the web-plate for the plate girder in example 6, if the depth of the girder is 30 inches. Each pair of the end stiffeners is fastened to the web-plate by nine $\frac{3}{4}$-inch rivets, and the allowable resistance of the material to shearing is 12,000 pounds per square inch.

Ans. $\frac{3}{8}$ in.

(14) A uniformly loaded plate girder of 75-foot span has a flange with two $6'' \times 4'' \times \frac{1}{2}''$ angles and four $8'' \times \frac{3}{8}''$ cover-plates; there is a row of $\frac{3}{4}$-inch rivets joining these plates to each flange angle, and a single row of $\frac{3}{4}$-inch rivets joining the flange angles to the web-plate, the rivets being so arranged that the section to be deducted is that of two rivet holes for each of the flange plates and flange angles; beginning with the outside plate, what are the theoretical lengths of the plates?

Ans. { 1st plate, 27 ft. 9 in.
2d plate, 39 ft. 3 in.
3d plate, 48 ft. 1 in.
4th plate, 55 ft. 6 in.

(15) (*a*) What is the elastic limit? (*b*) When is a body said to have a permanent set? (*c*) Are the materials used in building construction thought to be perfectly elastic?

(16) A 10-inch 25-pound structural-steel **I** beam with a span of 14 feet supports a floor surface 24 inches wide, on which the total dead and live load is 150 pounds per square foot; what is the deflection of the beam? Ans. .073 in.

(17) The yellow-pine rafter member of a composite pin-connected roof truss is 52 feet long. It is divided into four equal panels, and the roof and wind exert a pressure equal to a uniformly distributed load of 600 pounds per lineal foot of the rafter. The stress diagram shows a maximum compressive stress on the rafter of 56,000 pounds; if the depth

of the rafter is 10 inches and a factor of safety of 4 is used, what must be the thickness? Ans. 10 in.

(18) A white-pine beam of rectangular cross-section carries a uniformly distributed load of 50 pounds per lineal foot. If the beam is 8 inches by 14 inches and 16 feet long, how much more will it deflect when the short side is vertical than when the long side is vertical? Ans. .0831 in.

(19) (*a*) What type of roof truss is most commonly used for buildings of moderate span? (*b*) What difficulty is encountered in constructing the stress diagram for this truss? (*c*) Explain the method by which the diagram is drawn so as to find the stresses at the joint where the difficulty occurs.

(20) Why are some of the members of a structural-steel roof truss made heavier than is demanded by the stresses they must withstand?

(21) (*a*) What is a flitch-plate girder? (*b*) What are some of its advantages in comparison with a simple steel beam? (*c*) In the design of a flitch-plate girder, what should be the relation between the different members?

(22) What is the reason for using flange plates of different lengths in the construction of a plate girder?

(23) Name the different methods that may be used in calculating the pitch of the rivets connecting the flange angles with the web-plate of a plate girder.

(24) By means of a diagram, determine the lengths of

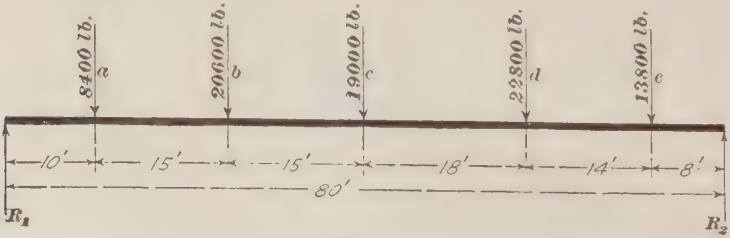


FIG. 3.

the flange plates for a plate girder loaded as shown in Fig. 3. The flange is made up of three $10'' \times \frac{1}{2}''$ plates and

two $4'' \times 4'' \times \frac{5}{8}''$ angles, connected as shown in Fig. 4.

Ans. { Outside plate, 36 ft. 6 in.
Middle plate, 49 ft. 9 in.
Inner plate, 63 ft. 0 in.

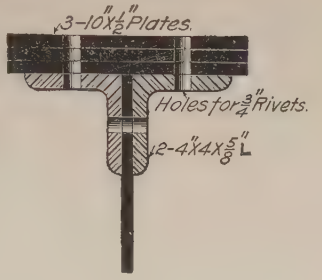


FIG. 4.

(25) The depth of the girder in the last example is 5 feet; (*a*) calculate the size of the angles required for the end stiffeners, if two angles are used and a compressive fiber stress of 12,000 pounds per square inch is allowed; (*b*) if the section cut out for fifteen holes for $\frac{3}{4}$-inch rivets is deducted from its total depth, calculate the thickness of the web-plate, allowing a shearing fiber stress of 12,000 pounds per square inch.

Ans. { (*a*) $2\frac{3}{4}'' \times 2\frac{3}{4}'' \times \frac{3}{8}''$ angles may be used.
(*b*) Calculated thickness, $\frac{3}{32}$ in.

(26) What is the net section of a $6'' \times 6'' \times \frac{7}{16}''$ angle from which is to be deducted the section cut out for two $\frac{7}{8}$-inch rivets? Ans. 4.185 sq. in.

(27) (*a*) What is considered the ratio of depth to span of a plate girder that should be allowed in the best practice? (*b*) If a smaller ratio is desired in order to meet conditions demanded by the construction of a building, what precaution is required in the design?

(28) (*a*) What is the usual practice in regard to rivet spacing at the joints and foot of a built-up column? (*b*) Give a practical rule for the diameter of rivets in built-up columns.

FIG. 5.

(29) What are some of the disadvantages of the Phœnix column section in building construction?

(30) With a factor of safety of 4, what, in round numbers, is the greatest allowable load on a structural-steel column 24 feet long with fixed ends and with the section shown in Fig. 5? Ans. 118,500 lb.

(31) (*a*) What rolled sections are most often used in the construction of structural-steel roof trusses of moderate span? (*b*) How are these sections connected?

(32) (*a*) What is gained by upsetting the ends of a tension member? (*b*) Would it be economical to upset the ends of very short tension members?

(33) What should be the relation between the size of the pins used in the eyes of tension members and the dimensions of the bars?

(34) Why is a lower factor of safety allowable in the design of a roof than in the design of a bridge?

(35) Find (*a*) the horizontal, (*b*) the vertical, and (*c*) the maximum bending moments on the pin shown in Fig. 6.

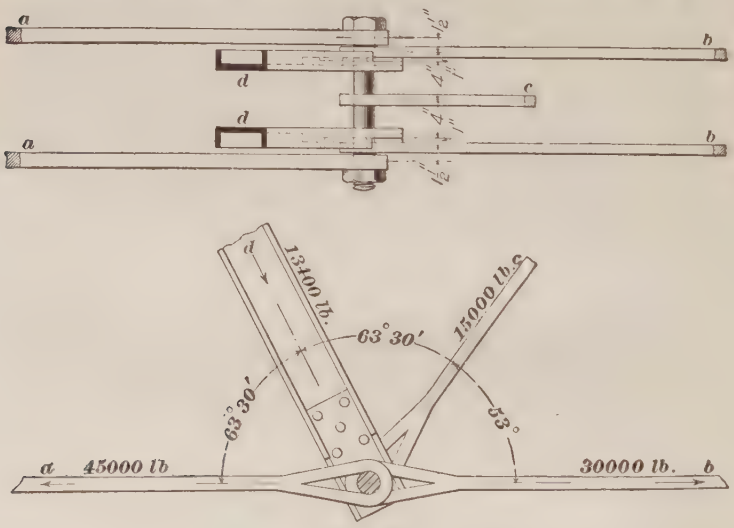


FIG. 6.

(*d*) If the allowable fiber stress is 20,000 pounds per square inch, what must be the diameter of the pin?

Ans. { (*a*) 59,250 in.-lb.
(*b*) 24,000 in.-lb.
(*c*) 63,926 in.-lb.
(*d*) $3\frac{1}{4}$ in.

(36) Fig. 7 shows the members meeting at a joint of the lower chord of a structural-steel roof truss, with the stress in each member. Assuming an allowable stress per square inch in tension of 12,000 pounds per square inch and a thick-

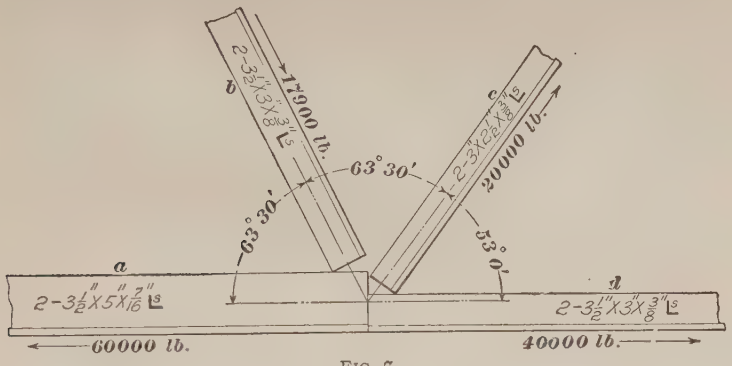


FIG. 7.

ness of gusset plate of $\frac{7}{16}$ inch, (*a*) calculate the number of $\frac{3}{4}$-inch rivets required in the end of each member; (*b*) make a sketch showing the gusset plate and the arrangement of the rivets. A splice plate may be used to connect the chord angles if required.

Ans. (*a*) { Member *a*, 9 rivets. Member *b*, 3 rivets. Member *c*, 3 rivets. Member *d*, 6 rivets.

(37) Calculate the moment of inertia, with respect to an axis 18 inches from its center, of the section of a hollow cylinder whose outside diameter is 12 inches and inside diameter 10 inches. Ans. 11,724.465.

(38) Two $5'' \times 3'' \times \frac{3}{4}''$ angles, placed back to back with the long legs parallel and $\frac{1}{2}$ inch apart, are to be used as a column with fixed ends; the length of the column is 22 feet. In round numbers, what load will the column carry with a factor of safety of 4? Ans. 72,000 lb.

(39) The reaction at the support of a plate girder is 156,000 pounds; the depth of the girder is 48 inches, and there are 14 holes for $\frac{3}{4}$-inch rivets to be deducted from the

width of the web-plate. If an allowable shearing stress of 13,000 pounds per square inch is used, what must be the thickness of the web? Ans. .336 in., say $\frac{3}{8}$ in.

(40) A plate girder, with a span of 84 feet, carries a uniformly distributed load of 3,000 pounds per lineal foot; the web-plate is $\frac{5}{16}$ inch thick, and it is divided into 14 equal panels; if $\frac{3}{4}$-inch rivets, spaced according to the direct vertical shear, are used, and the allowable stress in tension is 15,000 pounds per square inch, what should be the spacing, for the successive panels from the support to the center, of the rivets connecting the flange angles to the web?

Ans. { 1st panel, 3.43 in.
2d panel, 4 in.
3d panel, 4.8 in.
4th, 5th, 6th, and 7th panels, 6 in.

(41) A floor is to be supported by 12-inch 31½-pound I beams with a span of 20 feet spaced 24 inches between centers; the total dead and live load on the floor is to be 480 pounds per square foot. What will be the greatest deflection? Ans. .54 in.

(42) By means of the principle of moments, calculate the distance d of the neutral axis $a\,b$, of the section in Fig. 8, from the outer edge of the plate. Ans. 2.137 in.

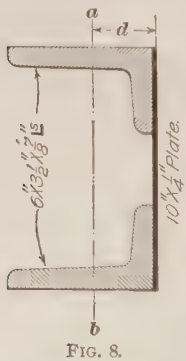


FIG. 8.

(43) What is the moment of inertia of the section shown in Fig. 8 with respect to the axis $a\,b$? Ans. 64.567.

(44) What is the section modulus with respect to an axis perpendicular to the web (*a*) of a 10-inch 33-pound I beam; (*b*) of a 12-inch 20-pound channel? Ans. { (*a*) 32.26. (*b*) 20.78.

(45) With a maximum fiber stress of 12,000 pounds per square inch, what is the resisting moment of each of the sections in the preceding example? Ans. { (*a*) 387,120 in.-lb. (*b*) 249,360 in.-lb.

(46) (*a*) Why should a greater factor of safety be used for long columns than for short ones? (*b*) Give a formula for the factor of safety to be used for any column with round or hinged ends.

(47) Describe Osborn's code of conventional signs for rivets.

(48) (*a*) In what ways may a riveted joint fail? (*b*) What relations are assumed between the tensile, compressive, and shearing strengths of the metal in computing the strength of rivets and riveted joints?

(49) What kinds of stresses are most likely to cause failure in pins?

(50) (*a*) What rule, in respect to the web-plate, is sometimes used in calculating the flange area of plate girders? (*b*) What precautions should be observed in splicing the web-plates of plate girders?

INDEX.

NOTE.—All items in this index refer first to the section (see the Preface) and then to the page of the section. Thus, "Angles 5 86" means that angles will be found on page 86 of section 5.

A.	Sec.	Page.
Abstract numbers	1	1
Accidental load	5	38
Acute angle	4	3
Addition	1	4
" of decimals	1	38
" of denominate numbers	2	15
" of fractions	1	27
" Rule for	1	8
" Sign of	1	4
" table	1	5
Adjacent angles	4	3
Aggregation, Symbols of	1	49
Altitude of parallelogram or trapezoid	4	29
" of prism or cylinder	4	45
" of pyramid or cone	4	47
" of triangle	4	8
Amount (Percentage)	2	2
Angle, Acute	4	3
" Definition of	4	2
" Inscribed	4	18
" Obtuse	4	3
" Vertex of	4	2
Angles	5	86
" Areas of: Table	6	137
" Connection	5	97
" Measurement of, by arcs	4	18
" Opposite	4	3
" or arcs, Measurement of	2	11
" Properties of: Table	6	138
" Properties of: Table	6	140
" Right	4	3
Antecedent of a ratio	2	55
Approximation, First	2	31
" Second	2	31
" Third	2	31
Arabic notation	1	2
Architectural engineering, Nature of	5	1
Arch, Span of	4	20
Arc of circle	4	16
" of circle, Length of	4	31
Area of any plane figure	4	39
" of circle	4	34
" of circular ring	4	35
" of ellipse	4	38
" of irregular figure	4	40
" of parallelogram	4	29
" of polygon	4	30
" of sector of circle	4	36
" of segment of circle	4	37
" of surface of frustum	4	49
" of surface of prism or cylinder	4	45
" of surface of pyramid or cone	4	47
" of surface of sphere	4	53
" of trapezoid	4	29
" of triangle	4	27
Arithmetic, Definition of	1	1
" Fundamental processes of	1	4
Avoirdupois weight	2	9
Axis, Neutral	6	1
" Neutral	5	76
" of symmetry	4	54

B.	Sec.	Page.
Base of triangle	4	8
" (Percentage)	2	2
Beam, Bending moments in	5	78
" Continuous	5	62
" Definition of	5	62
" Deflection of	6	102

	Sec.	Page.
Beam, Fixed	5	62
" girders	5	96
" Relation between depth of, and span	5	94
" Simple	5	62
" Span of	5	62
" Stone	5	99
" Strength of	5	84
Beams, Stresses in	5	69
" Trussed	5	101
" with one strut, Stresses in	5	102
" with two struts, Stresses in	5	103
Bearing value of rivets	6	45
" value of rivets	6	136
" value of rivets, Table of	6	49
" value of rivets, Table of	6	136
Bending moment	5	74
" moment and moment of resistance, Relation between	6	14
" moments, General formulas for	5	81
" moments in shear, Relation between	5	79
" moments in simple beams	5	78
" stress	5	40
" stresses	5	74
Box column	6	31
Brace	1	49
"	3	2
Bracing of roof trusses, Lateral	6	131
Brackets	1	49
"	3	2
Broken line	4	1
Buckling of web-plate	6	63
Building materials, Deterioration of	5	17
" materials, Elasticity of	6	102
" materials, Weight of	5	27

C.

	Sec.	Page.
Camber in roof truss	6	110
Cancelation	1	19
" Rule for	1	21
Cantilever	5	62
Capacity, Measures of	2	10
Cast-iron columns	5	56
" columns, Danger to, from fire	5	61
" columns, Design of	5	59
" columns, Formula for	5	57
" columns, Inspection of	5	60
Cause and effect, Principle of	2	66
Center of circle	4	16
" of gravity	5	26
Center of gravity of plane figures	5	26
" of moments	5	18
Chain, Engineers'	2	9
" Gunter's	2	8
Channel, Approximate section modulus of	5	87
" column	6	31
Channels	5	86
" Elements of: Table	5	90
" Properties of: Table	6	144
Chord of circle	4	16
Cipher	1	2
Circle, Arc of	4	16
" Area of	4	34
" Center of	4	16
" Chord of	4	16
" Circumference of	4	16
" Definition of	4	16
" Diameter of	4	16
" Radius of	4	16
" Relation between circumference and diameter of	4	31
" Sector of	4	17
" Segment of	4	16
Circles, Concentric	4	20
" Equality of	4	17
Circular measure, Explanation of	4	17
" ring, Area of	4	35
Circumference and diameter of circle, Relation between	4	31
" of circle	4	16
Circumscribed polygon	4	21
Column, Box	6	31
" Channel	6	31
" Grey	6	32
" Keystone octagonal	6	32
" Larimer	6	31
" Latticed angle	6	32
" sections used in building construction	6	30
" splices and connections	6	34
" used by Pa. R. R	6	32
" Z-bar	6	31
Columns	5	53
" and connections, Details of	6	33
" Cast-iron	5	56
" Conditions which affect the choice of	6	26
" Factors of safety for	6	25
" Formulas for	6	18
" Long	5	54
" Method of securing the ends of	5	55
" Rivet spacing in	6	33
" Round-ended	6	17

	Sec.	Page.
Columns, Short	5	53
" Types of	6	16
" with eccentric loads	6	22
" with fixed ends	6	17
" with fixed ends	6	21
" with flat ends	6	17
" with flat ends	6	20
" with hinged ends	6	17
" with hinged ends	6	18
" Wood	5	54
Common denominator	1	26
Components of a force	5	15
Composite pin-connected roof truss, Design of	6	118
Composition of forces	5	6
Compound denominate number	2	8
" proportion	2	67
" proportion, Rule for	2	68
Compression members of roof truss	6	132
Compressive stress	5	39
Concentric circles	4	20
Concrete numbers	1	1
Conditions of equilibrium	6	17
Cone	4	47
" Altitude of	4	47
" Frustum of	4	49
" Right	4	47
" Slant height of	4	47
" Surface of	4	47
" Volume of	4	48
Connection angles	5	97
Consequent of a ratio	2	55
Contact, Point of	4	19
Continuous beam	5	62
Conventional signs for rivets	6	41
" signs for rolled shapes	6	42
Conversion tables	4	25
Couplet of a proportion	2	59
" (Ratio)	2	55
Cross-section of a solid	4	51
Cube	4	44
" root	2	23
" root	2	26
" root	2	37
" root	2	37
" root, Illustrative example	2	37
" root, Illustrative example	2	38
" root, Illustrative example	2	39
" root, Method of extracting	2	36
" root, Table method of extracting	2	40
" root, When exact, can be found	2	39
Cubes, Table of	2	41
Cubical contents of solid	4	44
Cubic foot, Definition of	4	45
Cubic inch, Definition of	4	45
" measure	2	9
Curved line	4	1
Cylinder	4	45
" Area of surface of	4	45
" Circular	4	45
" Right	4	45
" Volume of	4	46

D.

	Sec.	Page.
Dates, To find interval of time between	2	18
Dead load	5	27
Decagon	4	6
Decimal point	1	36
" point, Location of	2	32
" To express approximately as a fraction having a given denominator	1	48
" To raise a whole number or, to any power	2	25
" To reduce a fraction to a	1	46
" To reduce, to a fraction	1	47
Decimals	1	35
" Addition of	1	38
" Division of	1	42
" how read	1	36
" Multiplication of	1	40
" Reduction of	1	46
" Subtraction of	1	39
Deflection of beams	6	102
" of beams	6	104
" of floorbeams	5	110
Denominate number, Compound	2	8
" numbers	2	7
" numbers, Addition of	2	15
" numbers, Division of	2	20
" numbers, Multiplication of	2	19
" numbers, Reduction of	2	12
" numbers, Simple	2	7
" numbers, Subtraction of	2	17
" numbers, To reduce, to higher denominations	2	13
" numbers, To reduce, to lower denominations	2	12
Denominator	1	22
Depth of beam and span, Relation between	5	94
Design for a large building	5	142
" of cast-iron columns	5	59

	Sec.	Page.
Design of composite pin-connected roof truss	6	118
" of foundations	5	50
" of roof truss, General notes regarding	6	131
" of structural-steel roof truss	6	124
Designing the members of a truss	5	139
Details of structural columns and connections	6	33
Deterioration of material	5	47
Diagonal of quadrilateral	4	29
Diagram for simple frames	5	116
" for small roof truss	5	121
" Frame	5	112
" how lettered	5	113
" Stress	5	112
Diameter and circumference of circle, Relation between	4	31
" of circle	4	16
" of circle determined from area	4	35
" of sphere determined from volume	4	54
Difference	1	9
" First	2	42
" (Percentage)	2	2
" Second	2	42
Digits	1	2
Direct proportion	2	59
" ratio	2	55
Dividend	1	16
Division	1	16
" of decimals	1	42
" of denominate numbers	2	20
" of fractions	1	32
" Rule for	1	18
" Sign of	1	16
Divisor	1	16
Dodecagon	4	6
Dollars and cents, how written decimally	1	49
" Sign for	1	48
Drawings, how dimensioned	4	24
Dry measure	2	10
Dynamics	5	3

E.	Sec.	Page.
Eccentric loads on columns	6	22
Elasticity, Definition of	6	102
" Modulus of	6	103
" of building materials	6	102
Elastic limit, Definition of	6	102
Elementary area, Definition of	6	8
Elements of usual sections: Table	6	135
Ellipse	4	37
" Area of	4	38
" Perimeter of	4	38
Elongation, Ultimate	5	42
Engineers' chain	2	9
Equality, Sign of	1	4
Equiangular polygon	4	7
Equilateral polygon	4	6
" triangle	4	8
Equilibrium	5	3
"	5	16
" of moments	5	19
" Stable	5	16
" Unstable	5	16
Evolution	2	25
Exponent	2	23
Extension, Measures of	2	8
Extremes (Proportion)	2	59

F.	Sec.	Page.
Factor of ignorance	5	46
" of safety	5	46
" Prime	1	20
Factors	1	19
" Irrational	2	28
" of safety for columns	6	25
" of safety for roof truss	6	132
" of safety, Table of	5	47
" Rational	2	28
Fathom	2	11
Fatigue of metals	5	47
Fifth, Higher roots than	2	52
" powers, Table of	2	41
" root	2	45
" root, Illustrative example	2	45
" root, Illustrative example	2	50
" root, Illustrative example	2	51
" root, Illustrative example	2	52
" root, Method of extracting	2	45
" root, Table method of extracting	2	52
Figure, Plane	4	6
Figures	1	2
" Local or relative values of	1	2
" Significant	2	26
" Similar	4	55
" Simple value of	1	2
" Symmetrical	4	54
" To find number of, in square root of a number	2	27
Filler for plate girder	6	57
Fink truss, Determination of stresses in	6	110
First approximation	2	31
" approximation, Factors to use in	2	33

	Sec.	Page.
First difference	2	42
Fixed beam	5	62
Flange angle	6	57
" plate	6	57
" plates for girder with concentrated loads in uniformly distributed load	6	85
" plates, Graphical method of determining length of	6	74
" plates, Length of	6	72
" plates, Rivet spacing in	6	96
" stresses	6	68
Flanges of girder, how proportioned	6	69
" of girder with concentrated loads, Graphical determination of length of	6	78
" of plate girder	6	67
Flitch-plate girder	6	106
Floorbeams, Deflection of	5	110
Foot-pound	5	18
Force, Components of	5	18
" Definition of	5	3
" Effect of	5	3
" Lever arm of	5	15
" Representation of	5	6
Forces, Composition of	5	6
" in equilibrium	5	16
" Parallelogram of	5	6
" Polygon of	5	11
" Resolution of	5	14
" Triangle of	5	9
Formula, Definition of	3	1
" Prismoidal	4	51
" Rankine-Gordon	3	10
Formulas for steel columns	6	18
Foundation materials, Strength of	5	48
Foundations	5	48
" Design of	5	50
Fourth power	2	23
" powers, Table of	2	41
" root	2	44
" root, Method of extracting	2	44
Fraction	1	22
" Improper	1	23
" Lowest terms of	1	25
" Proper	1	23
" Terms of	1	23
" To invert a	1	33
" To raise to any power	2	25
" To reduce a decimal to	1	47
" To reduce, to a decimal	1	46
Fraction, To reduce, to a higher term	1	23
" To reduce, to an equal fraction with a given denominator	1	24
" To reduce, to lower terms	1	24
" Value of	1	23
Fractions, Addition of	1	27
" Division of	1	32
" Multiplication of	1	31
" Reduction of	1	23
" Roots of	2	43
" Subtraction of	1	29
" To reduce, to common denominator	1	27
Frame and stress diagrams, how lettered	5	113
Frame diagram	5	112
Framing for I-beam connections	5	98
Frustum of pyramid or cone	4	49
" of pyramid or cone, Area of surface of	4	40
" of pyramid or cone, Volume of	4	50
Fulcrum	5	23

G.

	Sec.	Page.
Gain or loss per cent	2	6
Gallon, Cubic inches in	2	11
" Weight of	2	11
Gallons in cubic foot	2	11
Geometry, Definition of	4	1
Girder, Depth of	6	60
" design, Practical problems in	6	97
" Flanges of	6	67
" Flitch-plate	6	106
" Plate	6	57
" Sandwiched	6	106
Girders, Beam	5	96
" Rivet spacing in	6	90
" Usual sections of	6	58
" with concentrated loads, Application of graphical method to	6	78
Graphical determination of flange plates of girder with concentrated loads	6	78
" determination of length of flange plates	6	74
" diagram for jib crane	5	122
" diagram for simple frames	5	116

	Sec.	Page.
Graphical diagram for small roof truss	5	121
" diagram for wooden truss, 80-foot span	5	133
" diagram of roof truss, for church	5	126
" diagram of roof truss, 40-foot span	5	123
" method of computing stresses	5	105
" method of locating neutral axis	6	5
" methods of computing moments of inertia	5	11
" statics	5	111
Gravity, Center of	5	26
Grey column	6	32
Gunter's chain	2	8
Gusset plate	6	128
Gyration, Radius of	5	57
" Radius of	6	15

H.	Sec.	Page.
Heptagon	4	6
Hexagon	4	6
Horizontal line	4	2
Hypotenuse	4	8

I.	Sec.	Page.
I beam, Approximate section modulus of	5	87
" beam connections, Standard framing for	5	98
" beams	5	86
" beams, Elements of: Table	5	89
" beams, Properties of: Table	6	143
" beams, Separators for	5	94
Improper fractions, To reduce, to mixed numbers	1	25
Inches, To reduce, to decimal parts of a foot	1	47
Index of a root	2	26
Inertia, Moment of	6	8
Inscribed angle	4	18
" polygon	4	21
Inspection of cast-iron columns	5	60
Integer	1	2
Intensity of stress	5	41
Intersection, Point of	4	2
Inverse proportion	2	59
" proportion	2	62
" ratio	2	55
Involution	2	23
Irrational factors	2	28
Isosceles triangle	4	8

J.	Sec.	Page.
Jib crane, Graphical diagram for	5	122

K.	Sec.	Page.
Keystone octagonal column	6	32

L.	Sec.	Page.
Larimer column	6	31
Lateral bracing of roof truss	6	131
Latticed angle column	6	32
Laws of motion, Newton's	5	4
League	2	11
Least common denominator, To find	1	26
Lever	5	23
" arm of force	5	18
" Principle of	5	24
Like numbers	1	1
Limit, Elastic	6	102
Line	4	1
" Broken	4	1
" Curved	4	1
" Horizontal	4	2
" Right	4	1
" Straight	4	1
" Vertical	4	2
Linear measure	2	8
" measure, Surveyors'	2	8
Lines, Parallel	4	1
Liquid measures	2	10
Live load	5	31
Load, Accidental	5	38
" Dead	5	27
" Live	5	31
Loads carried by structures	5	27
" Moments due to	5	78
" Snow and wind	5	33
Local and relative values of figures	1	2
Location of decimal point	2	32
Long columns	5	54
Longitudinal section of solid	4	51
Long-ton table	2	10
Loss or gain per cent	2	6

M.	Sec.	Page.
Materials, Elasticity of	6	102
Maximum shear	5	72
Means (Proportion)	2	59
Measure, Definition of	2	8
" Dry	2	10
" Linear	2	8
" of angles or arcs	2	11
" of money	2	11
" of time	2	11
" Square	2	9
" Surveyors' linear	2	8
" Surveyors' square	2	9

	Sec.	Page.
Measures, Classification of	2	8
" Cubic	2	9
" Liquid	2	10
" Miscellaneous	2	11
" of capacity	2	10
" of extension	2	8
" of weight	2	9
" Standard units of	2	8
Mechanics, Definition of	5	3
" Elements of	5	3
Mensuration, Definition of	4	23
" of plane surfaces	4	26
" of solids	4	44
Meter	2	11
Method of extracting cube root	2	36
" of extracting fifth root	2	45
" of extracting fourth root	2	44
" of extracting square root	2	27
Minuend	1	9
Minus	1	9
Mixed number	1	23
" number, To reduce, to improper fraction	1	25
Moduli of elasticity: Table	6	148
Modulus of elasticity	6	103
" of rupture	5	42
" of section	5	84
Moment, Bending	5	74
" of a force	5	17
" of inertia	6	8
" of inertia determined by graphical methods	6	11
" of inertia, Rules and formulas for	6	10
" of resistance and bending moment, Relation between	6	14
" Resisting	5	75
" Resisting	5	84
" Resultant	5	20
Moments, Algebraic sum of	5	20
" Center or origin of	5	18
" due to loads, Effect of	5	78
" Equilibrium of	5	19
" how expressed	5	19
" Positive and negative	5	20
Money, Measure of	2	11
" U. S.	2	11
Motion	5	3
" Newton's laws of	5	4
Multiplicand	1	11
Multiplication	1	11
" of decimals	1	40
" of denominate numbers	2	19
" of fractions	1	31
Multiplication, Rule for	1	14
" Sign of	1	11
" table	1	12
Multiplier	1	11

N.

	Sec.	Page.
Naught	1	2
Negative moments	5	20
" shear	5	72
Neutral axis	5	76
" axis	6	1
" axis determined by a graphical method	6	5
" axis determined by a principle of moments	6	2
" axis, Location of	6	1
Newton's laws of motion	5	4
Notation	1	1
" Arabic	1	2
Number	1	1
" Abstract	1	1
" Concrete	1	1
" Denominate	2	7
" having same significant parts	2	26
" Mixed	1	23
" Prime	1	20
" Unit of	1	1
Numbers, Like	1	1
" Reading	1	3
" Unlike	1	1
Numeration	1	1
"	1	22

O.

	Sec.	Page.
Oblique triangle	4	8
Obtuse angle	4	3
Octagon	4	6
Origin of moments	5	18
Osborn's code for rivets	6	41

P.

	Sec.	Page.
Packing piece for plate girder	6	57
Panel points	5	113
Parallel lines	4	1
Parallelogram	4	28
" Area of	4	29
" of forces	5	6
Parallelopiped	4	44
Parenthesis	1	49
"	3	2
Parts, Significant	2	26
Pentagon	4	6
Percentage	2	1
"	2	2
" Meaning of	2	1
Per cent., Sign of	2	1
Perfect powers	2	28

	Sec.	Page.
Perimeter, Definition of	4	6
" of ellipse	4	38
Permanent set	6	102
Perpendicular	4	1
Phœnix column	6	32
Pin subjected to bending stresses	6	50
Pins and eyes of trusses	6	133
" Resultant moment of stresses on	6	52
" Strength of	6	43
" Table of resisting moments of	6	52
" Table of resisting moments of	6	149
Plane figure, Area of	4	39
" figures	4	6
" figures, Center of gravity of	5	26
" section of solid	4	51
" surface	4	6
" surfaces, Mensuration of	4	26
Plate girder	6	57
" girder, Depth of	6	60
" girder design, Practical problems in	6	97
" girder, Flanges of	6	67
" girder, General construction of	6	57
" girder, Stiffness of	6	57
" girders, Principles of design of	6	59
" girders, Usual sections of	6	58
Plus	1	4
Point	4	1
" of contact	4	19
" of intersection	4	2
" of tangency	4	19
Polonceau stress	6	110
Polygon	4	6
" Area of	4	30
" Equiangular	4	7
" Equilateral	4	6
" Inscribed and circumscribed	4	21
" of forces	5	11
" Regular	4	7
" Sum of angles of	4	7
Positive and negative moments	5	20
" and negative shear	5	72
Power of a number	2	22
Prime factor	1	20
" number	1	20
Primes and subscripts	3	5
Principles of successful design	5	2
Prism, Area of surface of	4	45
" Definition of	4	44
" Right	4	45
Prism, Volume of	4	46
Prismoidal formula	4	51
Prismoid, Definition of	4	51
" Volume of	4	51
Product	1	11
Proper fractions	1	23
Proportion	2	58
" Compound	2	67
" Direct	2	59
" how read	2	58
" how written	2	58
" Inverse	2	59
" Inverse	2	62
" Operations in	2	60
" Powers and roots in	2	64
" Rules for	2	59
" Simple	2	67
Pyramid	4	47
" Altitude of	4	47
" Frustum of	4	49
" Right	4	47
" Slant height of	4	47
" Surface of	4	47
" Volume of	4	48

Q.

	Sec.	Page.
Quadrant, Definition of	4	17
Quadrilateral	4	6
" Diagonal of	4	29
Quantity, Meaning of term	3	1
Quotient	1	16

R.

	Sec.	Page.
Radical sign	2	26
" sign	3	2
Radii of gyration for two angles	6	145
Radius of circle	4	16
" of gyration	5	57
" of gyration	6	14
Rankine-Gordon formula	3	10
Rate (Per cent.)	2	2
Ratio	2	54
" Direct	2	55
" Inverse	2	55
" Operations upon	2	57
" Reciprocal	2	55
" Terms of	2	55
" To invert	2	56
" Value of	2	55
Rational factors	2	28
Reactions	5	62
" Relation between	5	63
Reciprocal ratio	2	58
Rectangle	4	28
Reduction of decimals	1	46
" of denominate numbers	2	12
" of fractions	1	23

	Sec.	Page.
Regular polygon	4	7
Relative and local values of figures	1	2
Remainder	1	9
Resisting inches	5	84
" moment	5	75
" moment	5	84
" moment	6	14
" moments of pins, Table of	6	52
" moments of pins, Table of	6	149
Resultant moment of several stresses on pin	6	52
" moments	5	20
" of forces	5	7
" of forces	5	14
" of several forces	5	10
Rhomboid	4	28
Rhombus	4	28
Right angle	4	3
" angled triangle	4	8
" pyramid or cone	4	47
" section of a solid	4	51
" triangle, Application of, to roofs	4	12
Riveted joints, Failure of	6	44
Rivet signs	6	41
" spacing in girder, Effect of vertical stress upon	6	92
" spacing in girders	6	90
" spacing in the flange plates	6	96
Rivets and end angles or stiffeners over abutment	6	90
" and pins, Strength of	6	43
" and rivet spacing in columns	6	33
" Bearing value of	6	45
" connecting flange angles with web	6	90
" in double shear	6	45
" in ordinary bearing	6	45
" in single shear	6	45
" in stiffeners between abutments	6	90
" in web bearing	6	45
" spaced according to bending moment	6	93
" spaced according to direct vertical shear	6	96
Rolled shapes, Conventional signs for	6	42
Roof truss, Composite members of	6	132
" truss, Composite pin-connected, Design of	6	118
" truss, Diagram for	5	121
" truss, Factors of safety for	6	132
Roof truss, General notes regarding design of	6	131
" truss, Lateral bracing of	6	131
" truss, Structural-steel, Design of	6	124
" truss, Tension members of	6	132
" truss with 40-foot span	5	123
" trusses	6	110
" trusses, Details and design of	6	134
" trusses, Pins and eyes of	6	133
Root, Cube	2	23
" Cube	2	26
" Cube	2	36
" cube, Importance of	2	39
" cube, Method of extracting	2	36
" Fourth	2	23
" Fourth	2	44
" fourth, Method of extracting	2	44
" fifth, Method of extracting	2	44
" fifth, Table method of extracting	2	52
" higher than fifth	2	52
" Index of	2	26
" of fractions	2	43
" of fractions, Rule for	2	43
" Second	2	23
" Square	2	23
" square and cube, Table method of extracting	2	40
" square, Importance of	2	39
" square, Method of extracting	2	27
Rule for addition	1	8
" for cancelation	1	21
" for compound proportion	2	68
" for division	1	18
" for extracting roots of fractions	2	43
" for multiplication	1	14
" for subtraction	1	10
" of three	2	59
Rules for percentage	2	2
Rupture, Modulus of	5	42

S.

	Sec.	Page.
Safety, Factor of	5	46
Sandwiched girder	6	106
Scalene triangle	4	8
Scales, Use of	4	24
Score	2	11
Second approximation	2	31
" difference	2	42
" root	2	23
Section modulus	5	84

	Sec.	Page.
Section modulus of channel, Approximate rule for	5	87
" modulus of I beam, Approximate rule for	5	87
Sections of a solid	4	51
" of structural material, Elements of	6	135
" used in plate-girder construction	6	58
Sector of circle	4	17
" of circle, Area of	4	36
Segment of circle	4	16
" of circle, Area of	4	37
" of circle, Formulas for radius, chord, and height of	4	33
Semicircle	4	18
Semi-circumference	4	18
Separators for I beams	5	94
Shear	5	70
" and bending moment, Relation between	5	79
" in a simple beam	5	70
" Maximum	5	72
" Positive and negative	5	72
" Vertical	5	72
Shearing stress	5	39
" stresses in web-plate	6	60
Short columns	5	53
Sign for dollars	1	18
" of addition	1	4
" of division	1	16
" of equality	1	4
" of multiplication	1	11
" of per cent	2	1
" of subtraction	1	9
" Radical	2	26
" Radical	3	2
Significant figures	2	26
" figures in root	2	32
" figures, number to use	2	29
" numbers, Position of decimal in	2	27
Similar figures	4	55
" figures, Relations of areas and volumes of	4	55
" figures, Relations of areas and volumes of	4	56
" triangles	4	18
Simple beam	5	62
" denominate number	2	7
" proportion	2	67
Slant height of pyramid or cone	4	47
Snow and wind loads	5	33
Solid, Convex surface of	4	44
" Cubical contents of	4	44
" Definition of	4	44
Solid, Entire surface of	4	44
" Faces and edges of	4	44
" Sections of	4	51
" Volume of	4	44
Solids, Mensuration of	4	44
Spacing stiffeners, Practical rule for	6	65
Span and depth of beam, Relation between	5	94
" of arch	4	20
" of simple beam	5	62
Sphere	4	53
" Area of surface of	4	53
" Diameter of, determined from volume	4	54
" Volume of	4	53
Splices and connections for columns	6	34
Square	4	28
"	2	23
" foot, Definition of	4	26
" inch, Definition of	4	26
" measure	2	9
" measure, Surveyors'	2	9
" root	2	23
" root	2	27
" root, Examples in	2	35
" root, Illustrative example	2	31
" root, Illustrative example	2	32
" root, Illustrative example	2	34
" root, Illustrative example	2	35
" root, Importance of	2	39
" root, Table method of extracting	2	40
Squares, Table of	2	41
Stable equilibrium	5	16
Standard units of various measures	2	8
Statics	5	3
Steel beams	5	86
" beams, Strength of	5	87
" beams, Varieties of	5	86
" columns, Forms of	6	26
" columns, Formulas for	6	18
" columns, Types of	6	16
Stiffeners for plate girder	6	57
" of girder, Distribution of	6	63
" Practical rule for spacing	6	65
Stiffness, Definition of	6	104
" of plate girder	6	47
Stone beams, Strength of	5	99
Straight line	4	1
Strain, Definition of	5	39
" Definition of	5	41
" Unit	5	41
Strength of beams	5	84

	Sec.	Page.
Strength of building materials..	5	42
" of foundation materials	5	48
" of rivets and pins	6	43
" of steel beams	5	87
" Ultimate	5	42
Stress and frame diagrams, Order of letters in	5	115
" Compressive	5	39
" Definition of	5	39
" diagram	5	112
" Intensity of	5	41
" Shearing	5	39
" Tensile	5	39
" Transverse or bending	5	40
" Twisting	5	40
" Unit	5	41
Stresses and flange of girder	6	68
" Bending	5	74
" Graphical method of computing	5	105
" in beams	5	69
" in beams with one strut	5	102
" in beams with two struts	5	103
" in Fink truss, Determining	6	110
Subscripts and primes	3	5
Subtraction	1	9
" of decimals	1	39
" of denominate numbers	2	17
" of fractions	1	29
" Rule for	1	10
" Sign of	1	9
Subtrahend	1	9
Surds	2	28
Surface, Definition of	4	6
" of prism or cylinder	4	45
" of solid	4	44
" of sphere, Area of	4	53
" Plane	4	6
Surfaces, Mensuration of	4	26
Surveyors' linear measure	2	8
" square measure	2	9
Symbols of aggregation	1	49
" of aggregation, Order of precedence in	1	49
Symmetrical figures	4	54
Symmetry, Axis of	4	54

T.	Sec.	Page.
Table of squares, cubes, fourth and fifth powers	2	41
Tangency, Point of	4	19
Tangent to circle	4	19
Tees	5	86
Tensile stress	5	39
Tension, members of roof truss	6	132
Terms of a fraction	1	23
" of a ratio	2	55
Third approximation	2	31
Time, Measure of	2	11
Torsion	5	40
Transverse stress	5	40
Trapezium	4	28
Trapezoid	4	28
" Area of	4	29
Triangle	4	6
" Area of	1	27
" Equilateral	4	8
" General properties of	4	9
" Isosceles	4	8
" Oblique	4	8
" of forces	5	9
" Right-angled	4	8
" Scalene	4	8
Triangles, Equality of	4	13
" Similar	4	13
Troy weight	2	10
Truss, Designing the members of	5	139
" for church roof	5	126
Trussed beams	5	101
Trusses, Members in, subjected to transverse and direct stresses	6	133
" Members in, subjected to transverse stresses	6	133
" Pins and eyes of	6	133
" Roof	6	110
Twisting stress	5	40

U.	Sec.	Page.
Ultimate elongation	5	42
" strength	5	42
Unit	1	1
" of a number	1	1
" square	4	26
" strain	5	41
" stress	5	41
U. S. money	2	11
Unlike numbers	1	1
Unstable equilibrium	5	16

V.	Sec.	Page.
Value of a fraction	1	23
" of a ratio	2	55
" of rivets, Table of	6	19
" of rivets, Table of	6	136
Vary, Meaning of the term	2	55
Vertex of angle	4	2
Vertical line	4	2
" shear	5	72
" stress in flange, Effect of	6	92
Vinculum	1	19

	Sec.	Page.
Vinculum	3	2
Volume of frustum of pyramid or cone	4	50
" of prismoid	4	51
" of prism or cylinder	4	46
" of pyramid or cone	4	48
" of solid	4	44
" of sphere	4	53
" of wedge	4	50

W.

	Sec.	Page.
Web-plate, Buckling of	6	63
" of girder	6	57
" of girder, Shearing stresses in	6	60
" of girder, Thickness of	6	61
Wedge, Definition of	4	50
" Volume of	4	50
Weight, Avoirdupois	2	9
" Measures of	2	9
" of building materials	5	27
Wind and snow loads	5	33
" diagram	5	129
Wood columns, Formula for	5	54
Wooden truss, 80-foot span	5	123
Working drawings, how dimensioned	4	24
" drawings, how made	4	23

Z.

	Sec.	Page.
Z-bar column	6	31
Z bars, Properties of: Table	6	141
Zero	1	2

Y0-CIN-132